期末考试对策

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前言：只是供大家复习时的参考提纲，不保熟。请大家回忆以下内容的推导并熟记。打*号内容与数理方程重复或并不一定要求推导。

——From 电动力学对策组

- 混合积：$(\vec{f}\times\vec{g})\cdot\vec{h}=\vec{f}\cdot(\vec{g}\times\vec{h});\vec{f}\times(\vec{g}\times\vec{h})=(\vec{f}\cdot\vec{h})\vec{g}-(\vec{f}\cdot\vec{g})\vec{h}=\vec{f}\cdot(\vec{h}\vec{g}-\vec{g}\vec{h})$　双点乘：$\overleftrightarrow{T}:\overleftrightarrow{S}=T_{ik}S_{ki}$
- 四维Minkowski张量结论(未采用复数而采用爱因斯坦求和标记)：
  - 反对称张量：$A^{\alpha\beta}=\begin{pmatrix}0 & p_1 & p_2 & p_3\\ -p_1 & 0 & a_3 & -a_2\\ -p_2 & -a_3 & 0 & a_1\\ -p_3 & a_2 & -a_1 & 0\end{pmatrix}=\{\vec{p},\vec{a}\},A_{\alpha\beta}=\{-\vec{p},\vec{a}\},A^{*\alpha\beta}=\frac{1}{2!}\varepsilon^{\alpha\beta\mu\nu}A_{\mu\nu}=\{\vec{a},-\vec{p}\}$
  - $T^{\alpha\beta}=\begin{pmatrix}\varphi & \vec{p}\\ \vec{q} & \overleftrightarrow{T}\end{pmatrix}\Rightarrow X^{\alpha}=\partial_{\beta}T^{\alpha\beta}\begin{cases}X^0=\partial_0\varphi+\nabla\cdot\vec{p}\\ X^i=\partial_0 q_i+\partial_j T^{ij}\end{cases}$
    - T对称：$X=\begin{pmatrix}\partial_0\varphi+\nabla\cdot\vec{p}\\ \partial_0\vec{p}+\nabla\cdot\overleftrightarrow{T}\end{pmatrix}$　T反对称：$X=\begin{pmatrix}\nabla\cdot\vec{p}\\ -\partial_0\vec{p}+\nabla\times\vec{a}\end{pmatrix}$
- 常用结论：$\det(\overleftrightarrow{I}+\vec{f}\vec{g})=1+\vec{f}\cdot\vec{g},\nabla\varphi=\nabla\cdot(\varphi\overleftrightarrow{I})$
  - $\nabla(\vec{f}\cdot\vec{g})=(\nabla\vec{f})\cdot\vec{g}+(\nabla\vec{g})\cdot\vec{f}=\vec{f}\cdot\nabla\vec{g}+\vec{g}\cdot\nabla\vec{f}+\vec{f}\times(\nabla\times\vec{g})+\vec{g}\times(\nabla\times\vec{f})$
  - $\nabla\times(\vec{f}\times\vec{g})=\nabla\cdot(\vec{g}\vec{f}-\vec{f}\vec{g})=(\nabla\cdot\vec{g}+\vec{g}\cdot\nabla)\vec{f}-(\nabla\cdot\vec{f}+\vec{f}\cdot\nabla)\vec{g}$
  - 若$\nabla\times\vec{E}=0,\overleftrightarrow{T}=\mathrm{Const},$则$\nabla(\overleftrightarrow{T}:\nabla\vec{E})=\overleftrightarrow{T}:\nabla\nabla\vec{E}$
- 链式法则：令$tr=tr(\vec{r}),\varphi=\varphi(tr),\vec{A}=\vec{A}(tr)$
  - $\nabla\varphi(tr)=\dot{\varphi}\nabla tr,\nabla\vec{A}(tr)=(\nabla tr)\dot{\vec{A}},\nabla\cdot\vec{A}(tr)=(\nabla tr)\cdot\dot{\vec{A}},\nabla\times\vec{A}(tr)=\nabla tr\times\dot{\vec{A}}$
- Taylor展开：$\varphi(\vec{r}+\vec{\varepsilon})=e^{\vec{\varepsilon}\cdot\nabla}\varphi(\vec{r})=\left[1+\vec{\varepsilon}\cdot\nabla+\frac{1}{2!}(\vec{\varepsilon}\cdot\nabla)^2+\cdots\right]\varphi(\vec{r}),\vec{f}(\vec{r}+\vec{\varepsilon})=e^{\vec{\varepsilon}\cdot\nabla}\vec{f}(\vec{r})$
- 散度积分：$\int_V dV\,\nabla\varphi=\oint_{\partial V}d\vec{\sigma}\,\varphi;\int_V dV(\nabla\times\vec{A})=\oint_{\partial V}d\vec{\sigma}\times\vec{A};\int_V dV\,\nabla\cdot\overleftrightarrow{T}=\oint_{\partial V}d\vec{\sigma}\cdot\overleftrightarrow{T}$
- Green公式：$\begin{cases}\int_V dV[\varphi\nabla^2\psi+\nabla\varphi\cdot\nabla\psi]=\oint_{\partial V}d\vec{\sigma}\cdot\varphi\nabla\psi=\oint_{\partial V}\varphi\frac{\partial\psi}{\partial n}d\sigma\\ \int_V dV[\varphi\nabla^2\psi-\psi\nabla^2\varphi]=\oint_{\partial V}d\vec{\sigma}\cdot(\varphi\nabla\psi-\psi\nabla\varphi)\\ \int_V dV[\varphi\nabla^2\varphi+(\nabla\varphi)^2]=\oint_{\partial V}d\vec{\sigma}\cdot\varphi\nabla\varphi\end{cases}$
- 旋度积分：$\int_\Sigma(d\vec{\sigma}\times\nabla)=\oint_{\partial\Sigma}d\vec{l}$,后面加什么都成立
- 曲线坐标*
  - 拉梅系数 $\frac{\partial\vec{x}}{\partial x_\alpha}=H_\alpha\hat{e}_\alpha(\text{no sum})$　while $H_\alpha=\left|\frac{\partial\vec{x}}{\partial x_\alpha}\right|=\sqrt{\sum_{i=1}^3\left(\frac{\partial X_i}{\partial x_\alpha}\right)^2}$
  - 等x_α面法向：$\nabla x_\alpha=\frac{\partial x_\alpha}{\partial\vec{x}}=h_\alpha\hat{e}_\alpha$　while $H_\alpha h_\alpha=1$
  - 场：$\begin{cases}\nabla\varphi=\sum_{\alpha=1}^3\frac{1}{H_\alpha}\left(\frac{\partial\varphi}{\partial x_\alpha}\right)\hat{e}_\alpha\\ \nabla\cdot\vec{a}=\frac{1}{H_1H_2H_3}\left[\frac{\partial(a_1H_2H_3)}{\partial x_1}+\frac{\partial(a_2H_3H_1)}{\partial x_2}+\frac{\partial(a_3H_1H_2)}{\partial x_3}\right]\\ \nabla\times\vec{a}=\sum_{\alpha\beta\gamma=1}^3\varepsilon_{\alpha\beta\gamma}\left[\frac{\hat{e}_\alpha}{H_\beta H_\gamma}\left(\frac{\partial(a_\gamma H_\gamma)}{\partial x_\beta}\right)\right]\end{cases}\Leftarrow$ 三剑客 $\begin{cases}\nabla x_\alpha=\frac{\hat{e}_\alpha}{H_\alpha}\\ \nabla\times\frac{\hat{e}_\alpha}{H_\alpha}=0\\ \nabla\cdot\frac{H_\alpha\hat{e}_\alpha}{H}=0\end{cases}$
  - $\Rightarrow\Delta=\frac{1}{H_1H_2H_3}\left[\frac{\partial}{\partial x_1}\left(\frac{H_2H_3}{H_1}\left(\frac{\partial}{\partial x_1}\right)\right)+\frac{\partial}{\partial x_2}\left(\frac{H_3H_1}{H_2}\left(\frac{\partial}{\partial x_2}\right)\right)+\frac{\partial}{\partial x_3}\left(\frac{H_1H_2}{H_3}\left(\frac{\partial}{\partial x_3}\right)\right)\right]$
  - 球：$H_r=1,H_\theta=r,H_\varphi=r\sin\theta,H=r^2\sin\theta$ 柱：$H_s=H_z=1,H_\phi=s,H=s,$其实$H=J$(正交坐标下)
- δ函数*
  - $F[\delta(x)]=1,\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{i\lambda x}d\lambda=\delta(x),\int_{-\infty}^{+\infty}\cos\lambda x\,d\lambda=2\pi\delta(x),\delta'(-x)=-\delta(x),x\delta'(x)=-\delta(x),\delta(\alpha x)=\frac{1}{\alpha}\delta(x)$
  - $F[x^k]=i^k2\pi\delta^k(\lambda),F[\mathrm{sgn}x]=\frac{2}{i\lambda},F\left[\frac{1}{x^2}\right]=-\pi|\lambda|,F\left[\frac{1}{x^3}\right]=\frac{\pi}{2}\lambda|\lambda|i,F[H(x)]=\pi\delta(\lambda)-i\frac{1}{\lambda}$
  - $<f^{(n)}(x),\varphi(x)>=(-1)^n<f(x),\varphi^{(n)}(x)>$，$\delta^{(n)}(x)*f(x)=\delta(x)*f^{(n)}(x),L(f(x)*g(x))=(Lf)*g=g*Lg$
  - $\delta[u(x)]=\Sigma\frac{\delta(x-x_k)}{|u'(x_k)|}$,$x_k$为实轴上单零点
  - 高维δ函数
    - $\delta(\vec{x})=\frac{1}{|J|}\delta(\vec{\xi}),\Delta\frac{1}{r}=-4\pi\delta(\vec{r}),\nabla\frac{1}{R}=-4\pi\delta(\vec{R})$
    - $F[1]=(2\pi)^n\delta(\lambda),F[x_1^{\alpha_1}x_2^{\alpha_2}\ldots x_n^{\alpha_n}]=(2\pi)^n i^{\alpha_1+\cdots+\alpha_n}\frac{\partial^{\alpha_1+\cdots+\alpha_n}\delta(\lambda)}{\partial\lambda_1^{\alpha_1}\ldots\partial\lambda_n^{\alpha_n}}$
    - $F[\sin a\cdot x]=i2^{n-1}\pi^n(\delta(\lambda+a)-\delta(\lambda-a));F[\cos a\cdot x]=2^{n-1}\pi^n(\delta(\lambda+a)+\delta(\lambda-a))$
- Helmholtz定理
  - $\nabla\cdot\vec{F}(\vec{r})=D(\vec{r}),\nabla\times\vec{F}(\vec{r})=\vec{C}(\vec{r})$
  - 则在远场条件下：$\vec{F}=-\nabla\varphi+\nabla\times\vec{A}=-\nabla\frac{1}{4\pi}\int dV'\frac{D(\vec{r}')}{R}+\nabla\times\frac{1}{4\pi}\int dV'\frac{\vec{C}(\vec{r}')}{R}$
- Maxwell $\begin{cases}\boldsymbol{D}=\varepsilon_r\varepsilon_0\boldsymbol{E}\\ \boldsymbol{B}=\mu_r\mu_0\boldsymbol{H}\\ \boldsymbol{j_0}=\sigma(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}+\boldsymbol{K})\\ \nabla\cdot\vec{j}(\vec{r},t)=-\partial_t\rho(\vec{r},t)\\ \text{回忆一下辅助矢量和完备性}\end{cases}\rightarrow\begin{cases}\oiint\boldsymbol{D}\cdot d\boldsymbol{S}=q_0\\ \oint\boldsymbol{E}\cdot d\boldsymbol{l}=(-I_m)-\iint\frac{\partial\boldsymbol{B}}{\partial t}\cdot d\boldsymbol{S}\\ \oiint\boldsymbol{B}\cdot d\boldsymbol{S}=0(q_m)\\ \oint\boldsymbol{H}\cdot d\boldsymbol{l}=I_0+\iint\frac{\partial\boldsymbol{D}}{\partial t}\cdot d\boldsymbol{S}\end{cases}\Rightarrow\begin{cases}\nabla\cdot\boldsymbol{D}=\rho_{e0}\\ \nabla\times\boldsymbol{E}=(-\boldsymbol{j_m})-\frac{\partial\boldsymbol{B}}{\partial t}\\ \nabla\cdot\boldsymbol{B}=0(\rho_m)\\ \nabla\times\boldsymbol{H}=\boldsymbol{j_0}+\frac{\partial\boldsymbol{D}}{\partial t}\end{cases}$
- 边值关系法则：∇作用于场或者源：$\nabla\rightarrow\hat{n}_{1\rightarrow2},\vec{F}\rightarrow\vec{F}_2-\vec{F}_1,$其余场$\rightarrow0$，其余源：体密度$\rightarrow$面密度
- 电磁势
  - 广义势能：$U=e(\varphi-\vec{v}\cdot\vec{A})$　广义动量：$\vec{p}=\frac{\partial L}{\partial\vec{v}}=m\vec{v}+e\vec{A}$
  - 场 $\vec{E}=-\nabla\varphi-\partial_t\vec{A}$ and $\nabla\times\vec{A}=\vec{B}\Rightarrow$ 规范变换 $\varphi'=\varphi-\partial_t\Psi$、$\vec{A}'=\vec{A}+\nabla\Psi$
  - Coulomb规范：$\nabla\cdot\vec{A}=0$，Lorentz规范：$\nabla\cdot\vec{A}+\frac{1}{c^2}\partial_t\varphi=L=0$, d'Alembert算子：$\square\triangleq\nabla^2-\frac{1}{c^2}\partial_t^2$
  - 则真空场方程：$\begin{cases}\square\varphi+\partial_t L=-\frac{\rho}{\varepsilon_0}\\ \square\vec{A}-\nabla L=-\mu_0\vec{j}\end{cases}$
  - Colomb规范场方程：$\begin{cases}\Delta\varphi=-\frac{\rho}{\varepsilon_0}\\ \square\vec{A}=-\mu_0\vec{j}+\frac{1}{c^2}\nabla\partial_t\varphi\end{cases}\Rightarrow$(静态解)$\begin{cases}\varphi=\frac{1}{4\pi\varepsilon_0}\int dV'\frac{\rho(\vec{r}')}{R}\\ \vec{A}(\vec{r})=\frac{\mu_0}{4\pi}\int dV'\frac{\vec{j}(\vec{r}')}{R}\end{cases}$
  - Lorentz规范场方程：$\begin{cases}\square\varphi=-\frac{\rho}{\varepsilon_0}\\ \square\vec{A}=-\mu_0\vec{j}\end{cases}$
- 能量
  - Poynting定理：$\vec{E}\cdot\vec{j}=-\partial_t\omega-\nabla\cdot\vec{S}$,其具有解的不确定性
  - 线性无色散物质：$\vec{S}\triangleq\vec{E}\times\vec{H},\omega\triangleq\frac{1}{2}\vec{D}\cdot\vec{E}+\frac{1}{2}\vec{B}\cdot\vec{H}$
  - 能量中心：$\vec{R}_\varepsilon\triangleq\frac{\int dV\omega\vec{r}+\Sigma_k\varepsilon_k\vec{r}_k}{\int dV\omega+\Sigma_k\varepsilon_k}$,相对论下$\vec{v}=\frac{c^2\vec{p}}{\varepsilon}$,有$\vec{v}_\varepsilon=\frac{c^2\vec{P}_{tot}}{U_{tot}}$为能量中心速度

- 动量角动量
  - 动量密度：$\vec{g} \triangleq \varepsilon_0\vec{E}\times\vec{B} = \frac{\vec{S}}{c^2}$，动量流密度：$\overleftrightarrow{T} = \omega\overleftrightarrow{I} - \varepsilon_0(\vec{E}\vec{E} + c^2\vec{B}\vec{B}), \vec{F} = -\oint_{\partial V} d\vec{\sigma}\cdot\overleftrightarrow{T}$
    - 线性均匀物质：$\vec{g} = \vec{D}\times\vec{B}, \overleftrightarrow{T} = \omega\overleftrightarrow{I} - (\vec{D}\vec{E} + \vec{B}\vec{H})$，$\overleftrightarrow{T}$对称
    - 线性各向同性：$\vec{f_0} - \frac{1}{2}E^2\nabla\varepsilon - \frac{1}{2}H^2\nabla\mu$(对介质的力) $= -\partial_t\vec{g} - \nabla\cdot\overleftrightarrow{T}$
  - 角动量密度：$\vec{l}_{em} \triangleq \vec{r}\times\vec{g}$，角动量流密度：$\overleftrightarrow{R} = -\overleftrightarrow{T}\times\vec{r}$(线性，均匀各向同性)
- 静电场*
  - 格林倒易定律：$\int \varphi' dq = \int \varphi dq'$
  - 介质：$\vec{D} = \varepsilon_0\vec{E} + \vec{P} \Rightarrow \nabla\cdot(\varepsilon\nabla\varphi) = -\rho_0, \omega = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\varepsilon_0E^2 + \frac{1}{2}\vec{P}\cdot\vec{E}, W = \frac{1}{2}\int dV\,\rho_0\varphi$
  - 唯一性定理：$\int_V dV\,\nabla\cdot(\phi\varepsilon\nabla\phi) = \int_V dV\,\varepsilon(\nabla\phi)^2 = \oint_{\partial V} d\sigma\,\phi\varepsilon\frac{\partial\phi}{\partial n}$
    - Dirichlet边条：$\varphi(\vec{r}_s) = f(\vec{r}_s), (\vec{r}_s \in \partial V)$
    - Newman边条：$\frac{\partial\varphi}{\partial n}|_{\partial V} = g(\vec{r}_s)$
    - 导体边界：只需要知道每个导体边界的总电量即可
- 柱拉普拉斯方程的普遍解*：$u(r,\theta) = (A_0 + B_0\ln r)(C_0 + D_0\phi) + \Sigma_{n=1}^{+\infty}(C_nr^n + D_nr^{-n})(A_n\cos n\theta + B_n\sin n\theta)$
- 轴对称球Laplace方程的解*: $u(r,\theta) = \Sigma_{n=0}^{\infty}\left(C_nr^n + D_nr^{-(n+1)}\right)P_n(\cos\theta)$
- 伴随Legendre方程*：$[(1-x^2)y']' + \left(\lambda - \frac{m^2}{1-x^2}\right)y = 0$
  - 第一类伴随勒让德函数:$P_n^m(x) \triangleq (1-x^2)^{\frac{m}{2}}\frac{d^m}{dx^m}P_n(x)$,第二类 $Q_n^m(x) \triangleq (1-x^2)^{\frac{m}{2}}\frac{d^m}{dx^m}Q_n(x)$
  - $\left||P_n^m(x)|\right|^2 = \frac{2}{2n+1}\frac{(n+m)!}{(n-m)!}$
  - 常见 $P_0^0(\cos\theta) = 1, P_1^0(\cos\theta) = \cos\theta, P_1^1(\cos\theta) = \sin\theta, P_2^0 = \frac{1}{4}(1+3\cos 2\theta), P_2^1(\cos\theta) = \frac{3}{2}\sin 2\theta, P_2^2(\cos\theta) = \frac{3}{2}(1-\cos 2\theta)$
  - 球函数 $Y_{nm}(\theta,\varphi) \triangleq \begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}P_n^m(\cos\theta), N_{n0}^2 = \frac{4\pi}{2n+1}, N_{nm}^2 = \frac{2\pi}{2n+1}\cdot\frac{(n+m)!}{(n-m)!}, m \geq 1$ 球谐函数：$Y_{lm} = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}\,P_l^m(\cos\theta)e^{im\phi}$
  - 球laplace一般解：$u(r,\theta,\varphi) = \Sigma_{n=0}^{\infty}\Sigma_{m=0}^{n}\begin{pmatrix}r^n\\ r^{-(n+1)}\end{pmatrix}P_n^m(\cos\theta)\begin{pmatrix}\cos m\varphi\\ \sin m\varphi\end{pmatrix}$
- Green基本解*:
  - 格林公式 三维$\begin{cases}\oiint_S u\frac{\partial v}{\partial n}dS = \iiint_V u\Delta v dV + \iiint_V \nabla u\cdot\nabla v\,dV\\ \oiint_S\left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right)dS = \iiint_V(u\Delta v - v\Delta u)dV\end{cases}$,二维$\begin{cases}\oint_l u\frac{\partial v}{\partial n}dl = \iint_D u\Delta v dA + \iint_D \nabla u\cdot\nabla v dA\\ \oint_l\left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right)dl = \iint_D(u\Delta v - v\Delta u)dA\end{cases}$
  - $\Delta_3U = \delta(x,y,z), U = -\frac{1}{4\pi r}, \Delta_2U = \delta(x,y), U = -\frac{1}{2\pi}\ln\frac{1}{r}$
  - $\Delta_2u + k^2u = 0, U = \frac{1}{4}N_0(kr), \Delta_2u - k^2 = 0, U = -\frac{1}{2\pi}K_0(kr)$
  - $\Delta_3u + k^2u = 0, U = -\frac{e^{-ikr}}{4\pi r}$ or $-\frac{\cos kr}{4\pi r}, \Delta_3u - k^2u = 0, U = -\frac{e^{-kr}}{4\pi r}$
  - 基本解：$u(M) = \int_V f(M_0)GdM_0 + \frac{1}{\beta}\oint_{\partial V}\varphi(M_0)GdS_0 = \int_V f(M_0)GdM_0 - \frac{1}{\alpha}\oint_{\partial V}\frac{\varphi(M_0)\,\partial G}{\partial n_0}dS_0$,(二维相同形式)
    - 二类边值推广格林函数，$u(M) = \int_V f(M_0)GdM_0 + \oint_{\partial V}\varphi(M_0)GdS_0 + C$ (补加项$\frac{1}{V}$)
    - (补加边界$-\frac{1}{S}$),$u(M) = \int_V f(M_0)G(M_0;M)dM_0 + \oint_{\partial V}\frac{\partial\varphi(M_0)}{\partial n_0}G(M_0;M)dS_0 + C$
  - 夹角公式：$\cos\psi = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\varphi - \varphi_0)$
  - 圆内格林函数：$G(M;M_0) = \frac{1}{2\pi}[\ln\frac{1}{r(M,M_0)} - \ln\frac{R}{\rho r(M,M_1)}]$
  - 本征函数展开：$\nabla^2\psi_n(M) + \lambda\psi(M) = 0; \psi(M_s)|_{\partial V} = 0$,则$G = \Sigma_n\frac{\psi_n^*(M')\psi_n(M)}{\lambda_n}$
- 电多极子
  - 电偶极子：$\vec{p} \triangleq \int\vec{r}'dq, \varphi(\vec{r}) = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0r^3}, \vec{E}(\vec{r}) = \frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{4\pi\varepsilon_0r^3}, U = -\vec{p}\cdot\vec{E}_e, \vec{F} = \vec{p}\cdot\nabla\vec{E}_e, \tau = \vec{p}\times\vec{E}_e$
  - 电四极子：$\overleftrightarrow{D} = 3\overleftrightarrow{D}' - Tr(\overleftrightarrow{D}')\overleftrightarrow{I} \triangleq \int(3\vec{r}'\vec{r}' - r'^2\overleftrightarrow{I})dq, \varphi(\vec{r}) = \frac{1}{8\pi\varepsilon_0r^3}\overleftrightarrow{D}:\hat{r}\hat{r}, \vec{E}(\vec{r}) = \frac{5(\overleftrightarrow{D}:\vec{r}\vec{r})\vec{r}}{8\pi\varepsilon_0r^7} - \frac{\vec{r}\cdot\overleftrightarrow{D}}{4\pi\varepsilon_0r^5}$(留意的对称性),$U = -\frac{1}{6}\overleftrightarrow{D}:\nabla\vec{E}_e, F = \frac{1}{6}\overleftrightarrow{D}:\nabla\nabla\vec{E}_e, \vec{\tau} = \frac{1}{3}(\overleftrightarrow{D}\cdot\nabla)\times\vec{E}_e$
  - 球展开：$\frac{1}{R} = \frac{1}{r_>}\Sigma_{l=0}^{\infty}\left(\frac{r_<}{r_>}\right)^l P_l(\hat{r}\cdot\hat{r}') = \frac{1}{r_>}\Sigma_{l=0}^{\infty}\left(\frac{r_<}{r_>}\right)^l\frac{4\pi}{2l+1}\Sigma_{m=-l}^{l}Y_{lm}^*(\Omega')Y_{lm}(\Omega)$
    - 球内展开：$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\Sigma_{l=0}^{\infty}\Sigma_{m=-l}^{l}A_{lm}r^lY_{lm}(\Omega)$ where $A_{lm} = \frac{4\pi}{2l+1}\int\frac{Y_{lm}^*(\Omega')}{r^{l+1\prime}}dq$
    - 球外展开：$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\Sigma_{l=0}^{\infty}\Sigma_{m=-l}^{l}\frac{B_{lm}}{r^{l+1}}Y_{lm}(\Omega)$ where $B_{lm} = \frac{4\pi}{2l+1}\int r'^lY_{lm}^*(\Omega')dq$
- 静磁场*
  - 能量：$\omega = \frac{1}{2}\vec{B}\cdot\vec{H} = \frac{B^2}{2\mu_0} - \frac{1}{2}\vec{M}\cdot\vec{B}, W = \frac{1}{2}\int dV\,(\vec{j_0}\cdot\vec{A})$,对固有磁矩的物体，力学势能：$U = -W$ (因为电池做功 = 磁能增加和安培力功的和)
  - 自感/互感系数：$W = \frac{1}{2}\Sigma_{ik}L_{ik}I_iI_k$, where $L_{ik} = \frac{\mu_0}{4\pi}\oint_{Ci}\oint_{Ck}\frac{\vec{dl_i}\cdot\vec{dl_k}}{R_{ik}}$
  - 二维二分量：$\nabla^2A = -\mu j_0$,边值关系：$A_1 = A_2, \frac{1}{\mu_2}\left(\frac{\partial A}{\partial n}\right)_2 - \frac{1}{\mu_1}\left(\frac{\partial A}{\partial n}\right)_1 = -i_0$
- 对于稳恒电流重要的积分*：$\int dV\,\vec{j} = \int dV\,\nabla\cdot(\vec{j}\vec{r}) + \frac{\partial\rho}{\partial t}\vec{r} = \dot{\vec{p}}, \int dV\,\vec{j}\vec{r} = \vec{m}\times\overleftrightarrow{I} + \frac{1}{6}\dot{\overleftrightarrow{D}} + \frac{1}{6}\dot{g}\overleftrightarrow{I}$(考虑$\nabla\cdot(\vec{j}\vec{r}\vec{r})$)
  - Where $\vec{m} = \frac{1}{2}\int dV\,(\vec{r}\times\vec{j})$(与原点无关), $g = \int dV\,\rho r^2$
  - $\vec{A}(\vec{r}) = \frac{\mu_0\vec{m}\times\vec{r}}{4\pi r^3}$ and $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$ or $\vec{B}(\vec{r}) = -\nabla\psi(\vec{r}), \psi(\vec{r}) = \frac{\mu_0\vec{m}\cdot\vec{r}}{4\pi r^3}, W = \vec{m}\cdot\vec{B_e},, U = -W_{外}$
  - $\vec{F} = \vec{m}\cdot(\nabla\vec{B}_e), \vec{\tau} = \vec{m}\times\vec{B}_e(\vec{r})$
- 磁标势*
  - 在不包含电流的单连通区域有：$\psi(\vec{r}) = -\frac{I}{4\pi}\int_\Sigma -\hat{R}\cdot\frac{d\vec{\sigma}'}{R^2}, \vec{H} = -\nabla\psi$
  - 对存在介质处：$\nabla\cdot\vec{H} = -\nabla\cdot\vec{M} = \rho^*, \nabla^2\psi = \nabla\cdot\vec{M} = -\rho^*$ 则边界条件$\begin{cases}\psi_1 = \psi_2\\ \frac{\partial\psi_1}{\partial n} - \frac{\partial\psi_2}{\partial n} = \hat{n}\cdot(\vec{M_1} - \vec{M_2})\end{cases}$, $\vec{B} = \mu_0(\vec{H} + \vec{M})$
  - 对简单介质(非永磁体)：则边界条件$\begin{cases}\psi_1 = \psi_2\\ \mu_1\frac{\partial\psi_1}{\partial n} - \mu_2\frac{\partial\psi_2}{\partial n} = 0\end{cases}$, $\vec{B} = \mu\vec{H}$
- 时谐电磁波
  - 真空波动方程：$\Box\vec{E} = 0 = \Box\vec{B}$, 偏振度$\tilde{R} \triangleq \frac{\widetilde{E_{02}}}{\widetilde{E_{01}}} = \frac{A_2}{A_1}e^{i\delta}$ 圆偏振基（沿着z正传播）$\begin{cases}RCP: \hat{e}_+ = \frac{\hat{e_1} + i\hat{e_2}}{\sqrt{2}}\\ LCP: \hat{e}_- = \frac{\hat{e_1} - i\hat{e_2}}{\sqrt{2}}\end{cases}$
  - 时谐场均值定理：$< Re\vec{f}\cdot Re\vec{g} > = \frac{1}{2}Re\left(\vec{f_0^*}\cdot\vec{g_0}\right), < Re\vec{f}\times Re\vec{g} > = \frac{1}{2}Re\left(\vec{f_0^*}\times\vec{g_0}\right)$
  - 绝缘均匀介质
    - 折射率：$n = c\sqrt{\mu\varepsilon} \approx \sqrt{\varepsilon_r}$，固有阻抗:$Z \triangleq \sqrt{\frac{\mu}{\varepsilon}}, k = \frac{\omega}{c}n, \Rightarrow \vec{H} = \frac{\vec{k}\times\vec{E}}{\omega\mu} = \frac{\hat{k}\times\vec{E}}{Z}, \vec{E} = \frac{\vec{H}\times\vec{k}}{\omega\varepsilon}$
    - 则$< \omega > = \frac{1}{2}\varepsilon\left|\vec{E_0}\right|^2, < \vec{S} > = < \omega > \vec{v_p}$，$\overleftrightarrow{T} = \omega\,\hat{k}\hat{k}$
    - 菲涅尔公式:
      - s分量：$\tilde{r}_s = \frac{n_1\cos i_1 - n_2\cos i_2}{n_1\cos i_1 + n_2\cos i_2}$; $\tilde{t}_s = \frac{2n_1\cos i_1}{n_1\cos i_1 + n_2\cos i_2}$; E垂直于入射面.
      - P分量：$\tilde{r}_p = \frac{n_2\cos i_1 - n_1\cos i_2}{n_2\cos i_1 + n_1\cos i_2}; \tilde{t}_p = \frac{2n_1\cos i_1}{n_2\cos i_1 + n_1\cos i_2}$;
      - 强度透射率: $T = \frac{n_2}{n_1}|\tilde{t}|^2$ 光束截面比: $\frac{S_2}{S_1} = \frac{\cos i_2}{\cos i_1}$； $I = \frac{1}{2}cn\varepsilon_0E_0^2$

        - 斯托克斯倒逆关系：$\tilde{r}^2+\tilde{t}\tilde{t}'=1$ and $\tilde{r}'=-\tilde{r}$
        - 光疏入射至光密时，反射波电场垂直入射面分量存在半波损失
        - 全反射相位差：$\delta_p=2\arctan\left(\frac{n_1}{n_2}\left(\frac{\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2 i_1-1}}{\cos i_1}\right)\right)$； $\delta_s=2\arctan\left(\frac{n_2}{n_1}\left(\frac{\sqrt{\left(\frac{n_1}{n_2}\right)^2\sin^2 i_1-1}}{\cos i_1}\right)\right)$；
        - 隐失波: 衰减度$\kappa=\frac{2\pi}{\lambda_1}\sqrt{\sin^2 i_1-\sin^2 i_c}$ and $d_z=\frac{1}{\kappa}$(有效穿透深度)
    - 导体
        - 复介电常数：$\tilde{\varepsilon}\triangleq\varepsilon+\frac{i\sigma}{\omega}$,则替换后方程形式一致，即$\vec{k}=\vec{\beta}+i\vec{\alpha}$且$\beta^2-\alpha^2=\omega^2\mu\varepsilon,\vec{\alpha}\cdot\vec{\beta}=\frac{1}{2}\omega\mu\sigma$
        - 良导体近似下：$\beta_z\approx\alpha\approx\sqrt{\frac{\omega\mu\sigma}{2}}$,正入射下$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)^{\frac{1}{2}},\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)^{\frac{1}{2}}$
        - 良导体正入射：$H''_0=\sqrt{\frac{\sigma}{\omega\mu}}E''_0e^{\frac{i\pi}{4}},T\approx\sqrt{\frac{8\omega\varepsilon_1}{\sigma}}$则导体中等效面电流$i=H_0''e^{-i\omega t}$,平均焦耳功率$\overline{P}=\frac{\alpha}{2\sigma}i_0^2A$
    - 谐振腔与波导管
        - 一般理想导体作为边界的边界条件：$\frac{1}{E}\frac{\partial E}{\partial n}=-\frac{2}{\rho\text{（边界曲率）}},E_\tau=0$,满足波动方程$\nabla^2\vec{E}+\omega^2\varepsilon\mu\vec{E}=0$
        - 谐振腔下截至频率：$f_{min}=\frac{v}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$,波导管：$\omega_{min}=\frac{\pi}{\sqrt{\varepsilon\mu}\max(a,b)}$，

          矩形下：$k_3=\sqrt{\omega^2\varepsilon\mu-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2}$
        - 一般波导管*
            - $\nabla=\hat{x}\partial_x+\hat{y}\partial_y+\hat{z}\partial_z=\nabla_2+ik_3\hat{z},\phi=k_3z-\omega t$
            - TM:$(\Delta_2+\gamma^2)E_z=0,E_z|_s=0(\gamma^2\triangleq\omega^2\mu\varepsilon-k_3^2>0),\vec{E}=\left(\frac{ik_3}{\gamma^2}\nabla_2E_z+E_z\hat{z}\right)e^{i\phi},\vec{H}=\frac{\omega\varepsilon}{k_3}\hat{z}\times\vec{E}$
            - TE:$(\Delta_2+\gamma^2)H_z=0,\frac{\partial H_z}{\partial n}|_s=0,\vec{H}=\left(\frac{ik_3}{\gamma^2}\nabla_2H_z+H_z\hat{z}\right)e^{i\phi},\vec{E}=-\frac{\omega\mu}{k_3}\hat{z}\times\vec{H}$
- 电磁波的传播
    - 推迟势：$\varphi(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\int dV'\frac{\rho(\vec{r}',t_r)}{R},\vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int dV'\frac{\vec{j}(\vec{r}',t_r)}{R}$, where $t_r\triangleq t-\frac{R}{c}$
    - $f(\vec{r}',t_r)\Rightarrow\frac{\partial f}{\partial t}=\frac{\partial f}{\partial t_r},\nabla f=-\frac{\hat{R}}{c}\frac{\partial f}{\partial t_r}=-\frac{\hat{R}}{c}\dot{f},\rho(\vec{r})=-\vec{p}\cdot\nabla\delta(\vec{r}-\vec{r}_0)$ $(\vec{j}(\vec{r}',t)=\dot{\vec{p}}(t)\delta(\vec{r}-\vec{r}'))$
    - 则$\vec{E}=\frac{1}{4\pi\varepsilon_0}\int dV'\left[\frac{\rho\hat{R}}{R^2}+\frac{\dot{\rho}\hat{R}}{cR}-\frac{\dot{\vec{j}}}{c^2R}\right],\vec{B}=\frac{\mu_0}{4\pi}\int dV'\left[\frac{\vec{j}}{R^2}+\frac{\dot{\vec{j}}}{cR}\right]\times\hat{R}$(都在推迟时刻！)
- 时谐振电流的场：
    - $i\omega\rho=\nabla\cdot\vec{j},\vec{A}(\vec{r},t)=\frac{\mu_0}{4\pi}\int dV'\frac{j(\vec{r}',t)}{R}e^{ikR},\varphi(\vec{r},t)=-\frac{ic^2}{\omega}\nabla\cdot\vec{A}(\vec{r},t)$
    - 辐射场近似：远区：$r'\ll r,kr'\le 1\Rightarrow\frac{1}{R}\approx\frac{1}{r},kR\approx\vec{k}\cdot\vec{R}$
    - 电偶极辐射
        - $\vec{A}=\frac{\mu_0e^{ikr}}{4\pi r}\dot{\vec{p}}(t),\vec{B}=\frac{\mu_0}{4\pi cr}e^{ikr}\ddot{\vec{p}}\times\hat{r},\vec{E}=\frac{\mu_0}{4\pi r}e^{ikr}(\ddot{\vec{p}}\times\hat{r})\times\hat{r},\frac{d\overline{P}}{d\Omega}=\frac{\mu_0}{32\pi^2c}\left|\ddot{\vec{p}}\times\hat{r}\right|^2,\overline{P}=\frac{\mu_0\left|\ddot{\vec{p}}\right|^2}{12\pi c}$
    - 磁偶极辐射*
        - $\vec{A}=\frac{\mu_0e^{ikr}}{4\pi cr}\dot{\vec{m}}(t)\times\hat{r},\vec{B}=\frac{\mu_0e^{ikr}}{4\pi c^2r}(\ddot{\vec{m}}\times\hat{r})\times\hat{r},\vec{E}=-\frac{\mu_0e^{ikr}}{4\pi cr}\ddot{\vec{m}}\times\hat{r},\frac{d\overline{P}}{d\Omega}=\frac{\mu_0\left|\ddot{\vec{m}}\times\hat{r}\right|^2}{32\pi^2c^3},\overline{P}=\frac{\mu_0\left|\ddot{\vec{m}}\right|^2}{12\pi c^3}$
    - 电四极辐射*
        - $\vec{A}=\frac{\mu_0e^{ikr}}{24\pi cr}\hat{r}\cdot\ddot{\overleftrightarrow{D}},\vec{B}=\frac{\mu_0e^{ikr}}{24\pi c^2r}\left(\hat{r}\cdot\dddot{\overleftrightarrow{D}}\right)\times\hat{r},\vec{E}=\frac{\mu_0e^{ikr}}{24\pi cr}\left[\left(\hat{r}\cdot\dddot{\overleftrightarrow{D}}\right)\times\hat{r}\right]\times\hat{r},\frac{d\overline{P}}{d\Omega}=\frac{\mu_0}{1152\pi^2c^3}\left|\hat{r}\cdot\dddot{\overleftrightarrow{D}}\times\hat{r}\right|^2$

          $\overline{P}=\frac{\mu_0}{1440\pi c^3}\dddot{\overleftrightarrow{D}}^*:\dddot{\overleftrightarrow{D}}$
    - 功率为以上之和，交叉项为零，对时间任意变化的，只需要去掉相因子，并把时间变为推迟时间即可
    - 天线辐射*：$\vec{A}(\vec{r})=\frac{\mu_0e^{ikr}}{4\pi r}\int I(z)e^{-ik\cdot r'}\hat{z}dz$,典型分布：$I_0(z)=I_a\sin\left[k\left(\frac{l}{2}-|z|\right)\right]=I_a\sin(m\pi-k|z|)$
- Lienard − wiechert 势 *
    - $\vec{A}(\vec{r},t)=\frac{\vec{\beta}^*}{c}\varphi(\vec{r},t)=\frac{e\vec{\beta}^*}{4\pi c\varepsilon_0}\frac{1}{\vec{R}^*\cdot\vec{n}^*}$, where $t^*=t-\frac{\left|\vec{r}-\vec{r}^*(t^*)\right|}{c}$,定义$\vec{n}^*\triangleq\hat{R}^*-\vec{\beta}^*,R^*\vec{n}^*=c\Delta t\hat{R}^*-\vec{v}^*\Delta t$
    - *量的导数：$\nabla t^*=-\frac{\vec{R}^*}{c\vec{R}^*\cdot\vec{n}^*},\partial_t t^*=\frac{R^*}{\vec{R}^*\cdot\vec{n}^*}$
    - 电磁场：$\vec{E}(\vec{r},t)=\frac{e}{4\pi\varepsilon_0}\frac{R^*}{\left(\vec{R}^*\cdot\vec{n}^*\right)^3}\left[\left(1-\beta^{*2}\right)\vec{n}^*+\frac{\vec{R}^*\times(\vec{n}^*\times\vec{a}^*)}{c^2}\right],\overrightarrow{cB}=\hat{R}^*\times\vec{E}(\vec{r},t)$
    - 切连科夫辐射：传播夹角(半锥角)：$\sin\theta=\cos\theta_c=\frac{c}{nv}$
    - 发散功率：$\frac{dP}{d\Omega}=\frac{e^2}{16\pi^2\varepsilon_0c^3}\frac{\left|\hat{R}^*\times(\vec{n}^*\times\vec{a}^*)\right|^2}{\left(\hat{R}^*\cdot\vec{n}^*\right)^5}=\frac{dP}{d\Omega}|_{观测者}$ $(\hat{R}^*\cdot\vec{n}^*)$
    - 低速情形：$P=\frac{e^2a^{*2}}{6\pi\varepsilon_0c^3},\vec{E}=\frac{e}{4\pi\varepsilon_0c^2R^*}\left(\vec{a}^*\times\hat{R}^*\right)\times\hat{R}^*,\vec{B}=\frac{1}{c}\hat{R}^*\times\vec{E}$
    - 高速情形：以β^*方向为极轴,$\vec{a}_\perp$为x轴（以下已省略*标记）
        - $\vec{v}$与$\vec{a}$平行：$\frac{dP}{d\Omega}=\frac{e^2a^2\sin^2\theta}{16\pi^2\varepsilon_0c^3(1-\beta\cos\theta)^5},P=\frac{\gamma^6e^2a^2}{6\pi\varepsilon_0c^3},\theta_{max}=\arccos\left[\frac{\left(\sqrt{1+15\beta^2}-1\right)}{3\beta}\right]\approx\frac{1}{2\gamma}$
        - $\vec{v}$与$\vec{a}$垂直：$\frac{dP}{d\Omega}=\frac{e^2a^2}{16\pi^2\varepsilon_0c^3}\frac{\left[(1-\beta\cos\theta)^2-(1-\beta^2)\sin^2\theta\cos^2\phi\right]}{(1-\beta\cos\theta)^5},P=\frac{\gamma^4e^2a^2}{6\pi\varepsilon_0c^3},\theta_{max}=0$
        - $\frac{dP}{d\Omega}|_{交叉}=\frac{e^2a_\perp a_\parallel}{8\pi^2\varepsilon_0c^3}\frac{\sin\theta\cos\phi(\beta-\cos\theta)}{(1-\beta\cos\theta)^5},P=P_\perp+P_\parallel$
- 相对论动力学方程:$\vec{F}=\gamma^3mc\left[\left(\vec{\beta}\cdot\dot{\vec{\beta}}\right)\vec{\beta}+\frac{\dot{\vec{\beta}}}{\gamma^2}\right]$
- 爱因斯坦求和约定与度规矩阵
    - $g^{\alpha\gamma}g_{\gamma\beta}\triangleq\delta^\alpha_\beta,g_{\alpha\beta}x^\beta=x_\alpha=(-ct,\vec{r})$
    - 等式中同一自由指标上升或下降，又或是单项式中一对哑指标一个上升一个下降，结果不变
    - 变换矩阵：$x'^\alpha=\Lambda^\alpha{}_\beta(\vec{v})x^\beta$,且有$\Lambda^Tg\Lambda=g$,若$\det\Lambda=+1$,则$\in SO_{1,3}$,且$\Lambda=e^\Omega$, where $\Lambda=\delta+\Omega(|\Omega|\ll 1)$
    - 几何关系：$x'_E=\frac{OE\cos\theta}{\gamma},ct'_F=\frac{OF\cos\theta}{\gamma}$, where $\beta=\tan\theta$
- 洛伦兹变换
    - 对逆变四-矢量：$\mathbb{X}'^\alpha=\Lambda^\alpha{}_\beta\mathbb{X}^\beta$, where $\Lambda^\alpha{}_\beta=\begin{pmatrix}\gamma&-\beta\gamma&0&0\\-\beta\gamma&\gamma&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$
    - 对称变换：$\begin{pmatrix}ct\\x\end{pmatrix}\mapsto\begin{pmatrix}ct'\\x'\end{pmatrix}=\begin{pmatrix}\cosh\xi&-\sinh\xi\\-\sinh\xi&\cosh\xi\end{pmatrix}\begin{pmatrix}ct\\x\end{pmatrix}$，where $\xi=\frac{1}{2}\ln\frac{1+\beta}{1-\beta}$
    - 速度变换：$s\to s'\begin{cases}v'_x=\frac{v_x-u}{1-\frac{u}{c^2}v_x}\\v'_y=\frac{v_y}{1-u\frac{v_x}{c^2}}\sqrt{1-u^2/c^2}\\v'_z=\frac{v_z}{1-u\frac{v_x}{c^2}}\sqrt{1-u^2/c^2}\end{cases}\Rightarrow$加速度变换$\begin{cases}a'_x=\frac{a_x}{\left[\gamma\left(1-\frac{vu_x}{c^2}\right)\right]^3}\\a'_y=\frac{1}{\gamma^2\left(1-\frac{vu_x}{c^2}\right)^3}\left[\left(1-\frac{vu_x}{c^2}\right)a_y+\frac{vu_ya_x}{c^2}\right]\\a'_z=\frac{1}{\gamma^2\left(1-\frac{vu_x}{c^2}\right)^3}\left[\left(1-\frac{vu_x}{c^2}\right)a_z+\frac{vu_za_x}{c^2}\right]\end{cases}$
    - 角度变换：$\begin{cases}\tan\theta'=\frac{\sqrt{1-\beta^2}\sin\theta}{\cos\theta-\beta}\\\cos\theta'=\frac{\cos\theta-\beta}{1-\beta\cos\theta}\end{cases}$

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    - 光的多普勒效应：$\begin{cases} \nu' = \frac{\sqrt{1-\beta^2}}{1+\beta\cos\theta'}\nu \\ \nu' = \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}\nu \end{cases}$(传播角是以观察者为准的)(从4 − 波矢得到)
- 电磁场变换
    - 4-电流密度：$j^\alpha = \rho_0 U^\alpha = (\rho c, \vec{j}) = \rho_0\gamma(c, \vec{u}) \Rightarrow \partial_\alpha j^\alpha = 0$
    - 4-波矢：$k^\alpha \triangleq \left(\frac{\omega}{c}, \vec{k}\right)$ $k^\alpha k_\alpha = 0$
    - 4-速度：$U^\alpha = \gamma(c, \vec{u})$, 4-动量：$p^\alpha = \left(\frac{\varepsilon}{c}, \vec{p}\right)$, 4 − 力：$K^\alpha = \gamma(\frac{1}{c}\vec{F}\cdot\vec{u}, \vec{F})$
    - 电磁场张量：$F^{\alpha\beta} = \left\{\frac{\vec{E}}{c}, \vec{B}\right\}$, $G^{\alpha\beta} = F^{*\alpha\beta}$
    - 场方程：$\begin{cases} \partial_\beta F^{\alpha\beta} = -\partial_\beta F^{\beta\alpha} = \mu_0 j^\alpha \\ \partial_\beta G^{\alpha\beta} = 0 \Rightarrow \partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \end{cases}$ ⇒ 两个不变量$\begin{cases} \mathcal{L}_{em} = -\frac{1}{4\mu_0}F_{\alpha\beta}F^{\alpha\beta} = \frac{1}{2}\varepsilon_0(E^2 - c^2B^2) \\ -\frac{c}{4}F_{\alpha\beta}G^{\alpha\beta} = \vec{E}\cdot\vec{B} \end{cases}$
    - 电磁场变换：$\begin{cases} \vec{E}'_\parallel = \vec{E}_\parallel, \vec{E}'_\perp = \gamma_0(\vec{E}_\perp + \overrightarrow{\beta_0} \times c\vec{B}) \\ \vec{B}'_\parallel = \vec{B}_\parallel, \vec{B}'_\perp = \gamma_0(\vec{B}_\perp - \overrightarrow{\beta_0} \times \frac{\vec{E}}{c}) \end{cases}$ ⇒ 低速$\begin{cases} \vec{E}' = \vec{E} + \overrightarrow{v_0} \times \vec{B} \\ \vec{B}' = \vec{B} - \overrightarrow{v_0} \times \frac{\vec{E}}{c^2} \end{cases}$
    - 4-力密度：$f^\mu = F^{\mu\alpha}j_\alpha = \left(\vec{E}\cdot\frac{\vec{j}}{c}, \rho\vec{E} + \vec{j}\times\vec{B}\right)$则 $f^\mu = -\partial_\nu T^{\mu\nu}$
    - 能量-动量密度张量：$T^{\mu\nu} = g^{\mu\nu}\mathcal{L}_{em} - \frac{1}{\mu_0}F^{\mu\alpha}F_\alpha{}^\nu = \begin{pmatrix} \omega & \frac{\vec{S}}{c} \\ \frac{\vec{S}}{c} & \overleftrightarrow{T} \end{pmatrix}$
    - 规范势：$A^\alpha = \left(\frac{\varphi}{c}, \vec{A}\right)$ 规范变换：$\tilde{A}^\alpha = A^\alpha + \partial^\alpha\psi$
    - 势方程：$\Box A^\alpha - \partial^\alpha(\partial_\beta A^\beta) = -\mu_0 j^\alpha$
    - 含介质情形*：$H^{\alpha\beta} = \{c\vec{D}, \vec{H}\}, M^{\alpha\beta} = \{-c\vec{P}, \vec{M}\} = \frac{1}{\mu_0}F^{\alpha\beta} - H^{\alpha\beta}$
- Euler-Lagrange 场方程
    - Euler-Lagrange 方程：$\frac{d}{d\tau}\frac{\partial L_\tau}{\partial u^\alpha} - \frac{\partial L_\tau}{\partial x^\alpha} = 0, L = \frac{d\tau}{dt}L_\tau, \frac{d}{dt}\frac{\partial L}{\partial \vec{v}} = \frac{\partial L}{\partial \vec{x}}$
        - 正则4-动量：$\pi_\alpha \triangleq \frac{\partial L_\tau}{\partial x^\alpha}$
        - 自由粒子，可取$L_\tau = \frac{1}{2}mu^\mu u_\mu$
        - 电磁场中带电粒子：$L_\tau = -mc\sqrt{-u^\mu u_\mu} + eu^\alpha A_\alpha \Rightarrow L = -\frac{mc^2}{\gamma} + e(\vec{v}\cdot\vec{A} - \phi)$
    - $Lagrange$ 密度：$L = \int \mathcal{L}d^3x\,; S[\phi] = \frac{1}{c}\int_{t_1}^{t_2}\mathcal{L}d^4x$
        - 方程：$\frac{\partial\mathcal{L}}{\partial\phi_\alpha} - \partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a)} = 0 \Rightarrow \Pi^{\mu\alpha} \triangleq \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_\alpha)}$
        - 电磁场：$\mathcal{L} = \mathcal{L}_{em} + \mathcal{L}_{emp}, \mathcal{L}_{emp} = A_\alpha j^\alpha$
        - 带电粒子作用量：$S = \frac{1}{c}\int \mathcal{L}_{emp}d^4x - \sum_n m_n c\int\sqrt{-u_n^\alpha u_{n\alpha}}\,d\tau_n$
            - Lorentz 方程：$\frac{dp_n^\alpha}{d\tau_n} = e_n F_n^{\alpha\beta}u_{n\beta}$