2017-2018学年第二学期
信号与系统 期末

一.

1. $y(t) = -2e^{-2t}u(t) + \delta(t) + e\delta'(t)$

2. 参考教材 P181　$X(k) = X(e^{j\Omega})\big|_{\Omega = k\frac{2\pi}{N}}$

$X(k) = NF_k$

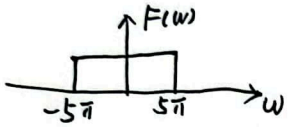
3. $x(t) = \sin(\pi t)u(t) - \sin(\pi t)u(t-1) = \sin(\pi t)u(t) * (\delta(t) + \delta(t-1))$

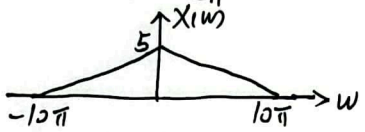
$\mathcal{L}[x(t)] = \frac{\pi}{s^2+\pi^2}(1+e^{-s})$　$\mathcal{F}[x(t)] = \frac{\pi}{\pi^2-w^2}(1+e^{-jw})$

4. 法① $x(t) = \frac{1-\cos(10\pi t)}{2\pi^2 t^2} \Rightarrow W_M = 10\pi \Rightarrow W_S = 2W_M = 20\pi$, $T_S = \frac{2\pi}{20\pi} = \frac{1}{10}$

法② $x(t) = \left[\frac{5\pi}{\pi}Sa(5\pi t)\right]^2$　记 $F(w) = \mathcal{F}\{\frac{5\pi}{\pi}Sa(5\pi t)\} = u(w+5\pi) - u(w-5\pi)$

则 $X(w) = \frac{1}{2\pi}F(w) * F(w) = \frac{1}{2\pi}\{(w+10\pi)[u(w+10\pi)-u(w)]$
$+(-w+10\pi)[u(w)-u(w-10\pi)]\}$





$W_M = 10\pi \Rightarrow W_S, T_S$ 可求

5. 记 $R_{2\tau}(t) = \begin{cases} 1, & |t|<\tau \\ 0, & |t|>\tau \end{cases}$　则 $x(t) = \frac{1}{2\tau}[1+\cos(\frac{\pi}{\tau}t)] \cdot R_{2\tau}(t)$

$\mathcal{F}\{\frac{1}{2\tau}[1+\cos(\frac{\pi}{\tau}t)]\} = \frac{1}{2\tau}[2\pi\delta(w) + \pi\delta(w-\frac{\pi}{\tau}) + \pi\delta(w+\frac{\pi}{\tau})]$，$\mathcal{F}\{R_{2\tau}(t)\} = 2\tau Sa(w\tau)$

$X(w) = \frac{1}{2\pi}2\tau Sa(w\tau) * \frac{1}{2\tau}[2\pi\delta(w) + \pi\delta(w-\frac{\pi}{2\tau}) + \pi\delta(w+\frac{\pi}{\tau})]$

$= Sa(w\tau) + \frac{1}{2}Sa((w-\frac{\pi}{\tau})\tau) + \frac{1}{2}Sa((w+\frac{\pi}{\tau})\tau) = \frac{Sa(w\tau)}{1-(w\tau/\pi)^2}$　教材 P231

6. $H(s) = \frac{2}{s+4} - \frac{1}{s+3}$，$h(t) = 2e^{-4t}u(t) + e^{-3t}u(-t)$

7. $F'(s) = \frac{1}{1+as+\cdots} \cdot \frac{-a}{s^2} = -\frac{1}{s} + \frac{1}{s+a} \Rightarrow -tf(t) = -u(t) + e^{-at}u(t) \Rightarrow f(t) = \frac{(1-e^{-at})u(t)}{t}$

$F(z) = \sum_{n=1}^{+\infty}\frac{(-1)^{n+1}a^n}{n}z^{-n} \Rightarrow f[n] = \frac{(-1)^{n+1}}{n}a^n u[n-1]$　教材 P207

8. 略，参考教材 P292

9. $H(s) = \frac{2}{s+3}$，$H(2) = \frac{2}{5} \Rightarrow y(t) = \frac{2}{5}e^{2t}$

10. 时移、频移性质的综合运用

$H(s) = \frac{-se^{-s}}{s^2+6s+10} = \frac{-(s+3)+3}{(s+3)^2+1} \cdot e^{-s}$

$\mathcal{L}\{\cos(t)u(t)\} = \frac{s}{s^2+1} \Rightarrow \mathcal{L}\{e^{-3t}\cos(t)u(t)\} = \frac{s+3}{(s+3)^2+1}$

同理，$\mathcal{L}\{e^{-3t}\sin(t)u(t)\} = \frac{1}{(s+3)^2+1}$

$h(t) = e^{-3(t-1)}[-\cos(t-1)+3\sin(t-1)]u(t-1)$

二.

$j\cdot Im[X(e^{j\Omega})]=\frac{1}{2}[e^{j\Omega}-e^{-j\Omega}-e^{2j\Omega}+e^{-2j\Omega}]\Rightarrow X_{odd}[n]=\frac{1}{2}(\delta[n+1]-\delta[n-1]-\delta[n+2]+\delta[n-2])$

$X[n]$在$n>0$时等于0 $\Rightarrow X_{even}[1]=\frac{1}{2}$, $X_{even}[2]=-\frac{1}{2}\Rightarrow X_{even}[-1]=\frac{1}{2}$, $X_{even}[-2]=-\frac{1}{2}$

$\Rightarrow X[-1]=\frac{1}{2}+\frac{1}{2}=1$, $X[-2]=-\frac{1}{2}-\frac{1}{2}=-1$

$\sum_{n=-\infty}^{+\infty}|X[n]|^2=X^2[0]+X^2[-1]+X^2[-2]=X^2[0]+1^2+(-1)^2=\frac{1}{2\pi}\int_0^{2\pi}|X(e^{j\Omega})|^2d\Omega=\frac{12\pi}{2\pi}=6$

$\Rightarrow X^2[0]=4$ 又因为$X[0]>0$, 故 $X[0]=2$

$\therefore X[n]=2\delta[n]+\delta[n+1]-\delta[n+2]$

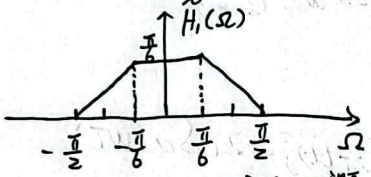
三. 参考2021-2022年的期末

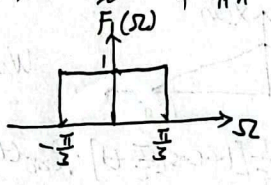
四.

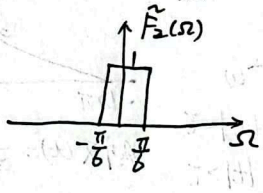
1. $x[n]=\delta[n]$时, $(-1)^n x[n]=\delta[n]$, $\delta[n]*h_1[n]=h_1[n]$, $h[n]=(-1)^n h_1[n]=(-1)^n\frac{\sin\frac{n\pi}{3}\sin\frac{n\pi}{6}}{\pi n^2}$

2. $h_1[n]=\pi\frac{\sin(n\pi/3)}{\pi n}\cdot\frac{\sin(n\pi/6)}{\pi n}$, 记$\tilde{F}_1(\Omega)=\mathcal{F}\{\frac{\sin(n\pi/3)}{\pi n}\}$, $\tilde{F}_2(\Omega)=\mathcal{F}\{\frac{\sin(n\pi/6)}{\pi n}\}$

则$\tilde{H}_1(\Omega)=\frac{\pi}{2\pi}\tilde{F}_1(\Omega)\circledast\tilde{F}_2(\Omega)$





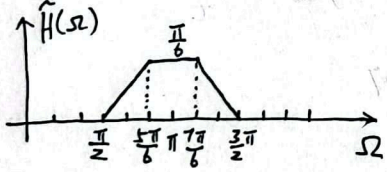


$(-1)^n=\cos(n\pi)=\frac{e^{jn\pi}+e^{-jn\pi}}{2}$,

$\mathcal{F}\{\cos(n\pi)\}=2\pi\sum_{m=-\infty}^{+\infty}\frac{1}{2}[\delta(\Omega-\pi-2m\pi)+\delta(\Omega+\pi-2m\pi)]$

$=2\pi\sum_{m=-\infty}^{+\infty}\delta(\Omega+\pi-2m\pi)$

$\tilde{H}(\Omega)=\frac{1}{2\pi}\tilde{H}_1(\Omega)\circledast 2\pi\sum_{m=-\infty}^{+\infty}\delta(\Omega+\pi-2m\pi)=\tilde{H}_1(\Omega)\circledast\sum_{m=-\infty}^{+\infty}\delta(\Omega+\pi-2m\pi)$



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3.

$x[n]=\cos(\frac{\pi n}{2})+1+\frac{1}{2}\cos(\frac{\pi n}{3})+\frac{1}{4}\cos(\frac{2\pi n}{3})+\sin(\frac{7\pi n}{12}-\frac{1}{12}\pi)$

$y[n]=\frac{1}{4}\cos(\frac{2\pi n}{3})\times\frac{1}{2}\times\frac{\pi}{6}+\sin(\frac{7\pi n}{12}-\frac{1}{12}\pi)\times\frac{1}{4}\times\frac{\pi}{6}$

$=\frac{\pi}{48}\cos(\frac{2\pi n}{3})+\frac{\pi}{24}\sin(\frac{7\pi n}{12}-\frac{1}{12}\pi)$