1. 对于一组正交归一完备的基{ $|\psi_n\rangle$ }， 可以定义力学量算符$\hat{A}$的迹(trace):

$$\mathrm{tr}\hat{A} = \sum_n \langle\psi_n|\hat{A}|\psi_n\rangle$$

a. 在这组基确定的矩阵表象下，简述算符迹的计算方法

b. 设体系的量子态由$|\Psi\rangle$表示，定义体系密度算符为

$$\hat{\rho} = |\Psi\rangle\langle\Psi|$$

证明该量子态下力学量$\hat{A}$的期望值满足如下关系

$$\langle\Psi|\hat{A}|\Psi\rangle = \mathrm{tr}\left(\hat{\rho}\hat{A}\right) = \mathrm{tr}\left(\hat{A}\hat{\rho}\right)$$

c. 证明任意算符的迹在不同的分离谱表象下不变，即

$$\mathrm{tr}\hat{A} = \sum_n \langle\psi_n|\hat{A}|\psi_n\rangle = \sum_\alpha \langle\phi_\alpha|\hat{A}|\phi_\alpha\rangle$$

解:

a. $\hat{A}$在$\{|\psi_n\rangle\}$表象下矩阵元为 $A_{mn} = \langle\psi_m|\hat{A}|\psi_n\rangle$，则 $\mathrm{tr}(\hat{A}) = \sum_n A_{nn}$

b ① 首先证明 $\mathrm{tr}(\hat{C}\hat{D}) = \mathrm{tr}(\hat{D}\hat{C})$

$$\mathrm{tr}(\hat{C}\hat{D}) = \sum_n \langle\psi_n|\hat{C}\hat{D}|\psi_n\rangle = \sum_{mn}\langle\psi_n|\hat{C}|\psi_m\rangle\langle\psi_m|\hat{D}|\psi_n\rangle$$

$$= \sum_{mn}\langle\psi_m|\hat{D}|\psi_n\rangle\langle\psi_n|\hat{C}|\psi_m\rangle = \sum_m\langle\psi_m|\hat{D}\hat{C}|\psi_m\rangle$$

$$= \mathrm{tr}(\hat{D}\hat{C})$$

② 设 $|\psi\rangle = \sum_n C_n|\psi_n\rangle$

$$\mathrm{tr}(\hat{\rho}\hat{A}) = \sum_n\langle\psi_n|\hat{\rho}\hat{A}|\psi_n\rangle$$

$$= \sum_n\langle\psi_n|\psi\rangle\langle\psi|\hat{A}|\psi_n\rangle$$

$$= \sum_n\langle\psi|\hat{A}|\psi_n\rangle\langle\psi_n|\psi\rangle$$

$$= \langle\psi|\hat{A}|\psi\rangle$$

而由① $\mathrm{tr}(\hat{\rho}\hat{A}) = \mathrm{tr}(\hat{A}\hat{\rho})$

即 $\langle\psi|\hat{A}|\psi\rangle = \mathrm{tr}(\hat{\rho}\hat{A}) = \mathrm{tr}(\hat{A}\hat{\rho})$

c. $\sum_\alpha\langle\varphi_\alpha|\hat{A}|\varphi_\alpha\rangle = \sum_\alpha\sum_{mn}\langle\varphi_\alpha|\psi_m\rangle\langle\psi_m|\hat{A}|\psi_n\rangle\langle\psi_n|\varphi_\alpha\rangle$

$= \sum_\alpha\sum_{mn}\langle\psi_n|\varphi_\alpha\rangle\langle\varphi_\alpha|\psi_m\rangle\langle\psi_m|\hat{A}|\psi_n\rangle = \sum_{mn}\langle\psi_n|\psi_m\rangle\langle\psi_m|\hat{A}|\psi_n\rangle$

$= \sum_{mn}\langle\psi_m|\hat{A}|\psi_n\rangle\delta_{mn} = \sum_n\langle\psi_n|\hat{A}|\psi_n\rangle$ 得证。

2. 力学量$\hat{A}$的正交完备基$\{|1\rangle,|2\rangle\}$定义为表象 I。在此表象下力学量$\hat{B}$的矩阵表示为$\begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$，力学量$\hat{A}$的矩阵表示为$\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$

a. 试求力学量$\hat{B}$的本征值, 并写出其本征态在表象 I 下的表示。
b. 验证力学量$\hat{B}$的本征态也构成一组正交完备基，由此可以定义表象 II。
c. 试求力学量$\hat{B}$的本征态在表象 II 下的表示。
d. 请写出由表象 I 到表象 II 的变换矩阵
e. 请写出力学量$\hat{A}$在表象 II 下的矩阵表示

解: a. 令 $\det\begin{pmatrix} \lambda & -\frac{1}{2} \\ -\frac{1}{2} & \lambda \end{pmatrix}=0 \Rightarrow \lambda_1=\frac{1}{2}$ 或 $\lambda_2=-\frac{1}{2}$

① $\lambda_1=\frac{1}{2}$ 时令 $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=\frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix}$ 得归一化本征矢 $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\triangleq|\alpha\rangle$

在表象I中 $|\alpha\rangle=\frac{1}{\sqrt{2}}|1\rangle+\frac{1}{\sqrt{2}}|2\rangle$

② $\lambda_2=-\frac{1}{2}$ 时, 令 $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}=-\frac{1}{2}\begin{pmatrix} a \\ b \end{pmatrix}$ 得归一化本征矢 $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\triangleq|\beta\rangle$

在表象I中, $|\beta\rangle=\frac{1}{\sqrt{2}}|1\rangle-\frac{1}{\sqrt{2}}|2\rangle$

b. 正交性: $\langle\alpha|\beta\rangle=\frac{1}{2}\left(\langle1|+\langle2|\right)\left(|1\rangle-|2\rangle\right)=0$

而$(\alpha,\beta)=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}(|1\rangle,|2\rangle)$

$\det\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)\neq0$, 保证了完备性, 综上, $|\alpha\rangle,|\beta\rangle$正交完备.

表象II: $\{|\alpha\rangle,|\beta\rangle\}$

c. $\hat{B}$的本征态 $|\alpha\rangle$表示为 $(1,0)^T$, $|\beta\rangle$表示为$(0,1)^T$

d. 变换矩阵为 $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}=\begin{pmatrix} |\alpha\rangle \\ |\beta\rangle \end{pmatrix}=S$

（上述变换矩阵对应的 $\hat{B}^{(\text{II})}$ 为 $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$

若对应 $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$，则相应的变换矩阵为 $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ ）

e. II表象下 $\hat{A}$ 对应矩阵

$$A^{(\text{II})} = SA^{(\text{I})}S^{+}$$
$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

3

a. 已知算符 $\hat{A}$ 在两个表象 F,G 之间的变换关系

$$\hat{A}^{(F)} = S\hat{A}^{(G)}S^{\dagger}$$

请证明

$$\left(e^{\hat{A}}\right)^{(F)} = S\left(e^{\hat{A}}\right)^{(G)}S^{\dagger}$$

b. 如算符 $\hat{A}$ 在某表象里的矩阵表示为 $\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$， 求 $\exp(\hat{A})$ 在该表象里的矩阵表示。

c. 如算符 $\hat{A}$ 在某表象里的矩阵表示为 $\begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$，求 $\exp(\hat{A})$ 在该表象里的矩阵表示。

解：

a. $$S\left(e^{\hat{A}}\right)^{(G)}S^{+}$$
$$= S\sum_{n}\frac{1}{n!}\left(A^{(G)}\right)^{n}S^{+}$$
$$= \sum_{n}\frac{1}{n!}\underbrace{(SA^{(G)}S^{+})(SA^{(G)}S^{+})\cdots(SA^{(G)}S^{+})}_{n\uparrow}$$
$$= \sum_{n}\frac{1}{n!}\left(A^{(F)}\right)^{n} = \left(e^{\hat{A}}\right)^{(F)}$$

b. $e^{\hat{A}} = \sum_n \frac{1}{n!} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}^n$

$= \sum_n \frac{1}{n!} \begin{pmatrix} \frac{1}{2^n} & 0 \\ 0 & \frac{1}{2^n} \end{pmatrix}$

$= \begin{pmatrix} \sum_n \frac{1}{n!} (\frac{1}{2})^n & 0 \\ 0 & \sum_n \frac{1}{n!} (\frac{1}{2})^n \end{pmatrix} = \begin{pmatrix} e^{\frac{1}{2}} & 0 \\ 0 & e^{\frac{1}{2}} \end{pmatrix}$

c. 首先对角化 $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

利用第2题的结论.

$\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

定义 $S = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

则 $e^{(A)} = \sum_n \frac{1}{n!} \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}^n = \sum_n \frac{1}{n!} \left(S \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} S^+ \right)^n$

$= S \sum_n \frac{1}{n!} \begin{pmatrix} \frac{1}{2^n} & 0 \\ 0 & \frac{1}{(-2)^n} \end{pmatrix} S^+ = S \begin{pmatrix} e^{\frac{1}{2}} & 0 \\ 0 & e^{-\frac{1}{2}} \end{pmatrix} S^+$

$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} e^{\frac{1}{2}} & 0 \\ 0 & e^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

$= \frac{1}{2} \begin{pmatrix} e^{\frac{1}{2}} + e^{-\frac{1}{2}} & e^{\frac{1}{2}} - e^{-\frac{1}{2}} \\ e^{\frac{1}{2}} - e^{-\frac{1}{2}} & e^{\frac{1}{2}} + e^{-\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \cosh\frac{1}{2} & \sinh\frac{1}{2} \\ \sinh\frac{1}{2} & \cosh\frac{1}{2} \end{pmatrix}$

4. 体系哈密顿量$\hat{H}$在表象 I 中矩阵为$\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}$，力学量$\hat{A}$的矩阵表示为$\begin{pmatrix} -1 & 0 & -3i \\ 0 & 2 & 0 \\ 3i & 0 & -1 \end{pmatrix}$。

a. 证明$\hat{H}$和$\hat{A}$对易。

b. 求$\hat{H}$和$\hat{A}$的共同本征态。如利用共同本征态构建新的表象 II，说明如何用本征值标定表象 II 的基。

c. 如力学量$\hat{B}$在表象 I 中的矩阵表示为$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$，写出其在表象 II 中的矩阵。

解：设 $\hat{A}$, $\hat{H}$ 在表象 I, II 中对应矩阵分别为 A^{I}, A^{II}, H^{I}, H^{II}

a. $$A^{I}H^{I} = \begin{pmatrix} -1 & 0 & -3i \\ 0 & 2 & 0 \\ 3i & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix} = \begin{pmatrix} -3 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & -3 \end{pmatrix}$$

$$H^{I}A^{I} = \begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}\begin{pmatrix} -1 & 0 & -3i \\ 0 & 2 & 0 \\ 3i & 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & -3 \end{pmatrix} = A^{I}H^{I}$$

即 $\hat{A}$, $\hat{H}$ 对易.

b. 首先求 H^{I} 的本征值.

令 $$\det\left(\begin{pmatrix} \lambda & 0 & -i \\ 0 & \lambda-1 & 0 \\ i & 0 & \lambda \end{pmatrix}\right) = 0 \Rightarrow \lambda_1 = \lambda_2 = 1 \quad \lambda_3 = -1.$$

$\lambda = 1$ 时

$$\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$ 得归一化正交本征矢 $|\alpha\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \\ 1 \end{pmatrix}$ $|\beta\rangle = \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix}$

$\lambda = -1$ 时 $$\begin{pmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$ 得归一化本征矢 $|\gamma\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 0 \\ -1 \end{pmatrix}$

$|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$ 对应的向量转置之后按 $|\gamma\rangle$, $|\alpha\rangle$, $|\beta\rangle$ 排列得到变换矩阵

$$S = \begin{pmatrix} \frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & i & 0 \end{pmatrix}$$ X

在该变换矩阵下 $H^{II} = diag\{-1, 1, 1\}$

则 $A^{II} = SA^{I}S^{+} = diag\{2, -4, 2\}$.

于是利用 A, H 的本征值, $|\alpha\rangle$, $|\beta\rangle$, $|\gamma\rangle$ 分别可标记为

$|1, -4\rangle$, $|1, 2\rangle$ 与 $|-1, 2\rangle$

C. $B^{II} = SB^{I}S^{+} = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \end{pmatrix}$

量子力学第五次作业答案

By 鸽子

1.

a. 矩阵元$A_{mn}=<m|A|n>$，则$trA=\sum_m <m|A|m>$.

b. 利用Einstein求和公式（对两个记号相同的下标求和），由基的完备性设

$$|\psi>=C_n|\psi>\rightarrow<\psi|=C_n^*<\psi_n| \tag{1}$$

由基的正交归一性：$<\psi_m|\psi_n>=\delta_{mn}$

则$<\psi|A|\psi>=C_m^*C_nA_{mn}=<\psi_n|\psi><\psi|\psi_m>A_{mn}=\rho_{mn}A_{mn}=tr(\rho A)$

且$tr(\rho A)=tr(A\rho)$

c.由基的正交完备归一性：

$trA=\sum_n <\psi_n|A|\psi_n>$

$=\sum_\beta\sum_\alpha\sum_n <\psi_n|\phi_\beta><\phi_\beta|A|\phi_\alpha><\phi_\alpha|\phi_n>$

$=\sum_\beta\sum_\alpha\sum_n <\phi_\alpha|\psi_n><\psi_n|\phi_\beta><\phi_\beta|A|\phi_\alpha>$

$=\sum_\beta\sum_\alpha <\phi_\beta|A|\phi_\alpha>\delta_{\alpha\beta}$

$=\sum_\alpha <\phi_\alpha|A|\phi_\alpha>$

2.

a.求解$B|\psi_n>=\lambda_n|\psi_n>$得

$\lambda_1=\frac{1}{2},|\psi_1>=\frac{1}{\sqrt{2}}(|1>+|2>)$

$\lambda_2=-\frac{1}{2},|\psi_2>=\frac{1}{\sqrt{2}}(|1>-|2>)$

b.正交归一：$<\psi_1|\psi_2>=0,<\psi_1|\psi_1>=1,<\psi_2|\psi_2>=1;$

完备性：$|\psi_1><\psi_1|+|\psi_1><\psi_1|=\begin{bmatrix}1&0\\0&1\end{bmatrix}$

c.变换矩阵

$S\begin{bmatrix}1&0\\0&1\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}\rightarrow S=\frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}=(|\psi_1>,|\psi_2>)$

d.在$\{|\psi_1>,|\psi_2>\}$中，

$|1>=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix},|2>=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$

e.矩阵表示：$A^{(B)}=S^\dagger AS=\frac{1}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}$

3.

a. 由于 $S^\dagger S=1$，所以

$$A^{(F)}=SA^{(G)}S^\dagger$$

所以有

$$(e^{\hat{A}})^{(F)}=\sum_{n=0}^{\infty}\frac{A^{(F)}}{n!}=S\frac{A^{(G)}}{n!}S^\dagger=S(e^{\hat{A}})^{(G)}S^\dagger$$

b.

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}^n=\begin{pmatrix} a^n & 0 \\ 0 & b^n \end{pmatrix}$$

故

$$exp(\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix})=\sum\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}^n/n!=\sum\begin{pmatrix} \frac{1}{n!}\frac{1}{2^n} & 0 \\ 0 & \frac{1}{n!}\frac{1}{(-2)^n} \end{pmatrix}=\begin{pmatrix} e^{\frac{1}{2}} & 0 \\ 0 & e^{-\frac{1}{2}} \end{pmatrix}$$

c. 首先将A对角化，然后同b相同操作。结果为：

$$\frac{1}{2}\begin{bmatrix} e^{1/2}+e^{-1/2} & e^{1/2}-e^{-1/2} \\ e^{1/2}-e^{-1/2} & e^{1/2}+e^{-1/2} \end{bmatrix}$$

4.

a. 直接计算$\hat{H}\hat{A}$和$\hat{A}\hat{H}$即可说明两者相等，从而对易。

b. $\hat{H}$的本征值：

$$\begin{vmatrix} \lambda & 0 & -i \\ 0 & \lambda-1 & 0 \\ i & 0 & \lambda \end{vmatrix}=0\rightarrow\lambda_1=\lambda_2=1,\lambda_3=-1$$

$\lambda_1=\lambda_2=1$:

$\begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\begin{bmatrix} a \\ b \\ c \end{bmatrix}$，归一化后得到本征态：$|\alpha>=\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 0 \\ 1 \end{bmatrix}$，$|\beta>=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_3=-1$:

$\begin{bmatrix} 0 & 0 & i \\ 0 & 1 & 0 \\ -i & 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=-\begin{bmatrix} a \\ b \\ c \end{bmatrix}$，归一化后得到本征态：$|\gamma>=\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 0 \\ -1 \end{bmatrix}$

得到变换矩阵：$S=\begin{bmatrix}\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ \frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ 0 & i & 0\end{bmatrix}$

此处S似乎有误

$$S=\begin{bmatrix}-\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ -\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ 0 & -i & 0\end{bmatrix}$$

从而得到：$\hat{H}_{\mathrm{II}}=\begin{bmatrix}-1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$，$\hat{A}_{\mathrm{II}}=SA_IS^{\dagger}=\begin{bmatrix}2 & 0 & 0\\ 0 & -4 & 0\\ 0 & 0 & 2\end{bmatrix}$

从而，$|\alpha>\to|1,-4>,|\beta>\to|1,2>,|\gamma>\to|-1,2>$

c.$\hat{B}_{\mathrm{II}}=SB_IS^{\dagger}=\begin{bmatrix}0 & 0 & \sqrt{2}\\ 0 & 0 & 0\\ \sqrt{2} & 0 & 0\end{bmatrix}$

HW5. 4.

b.

$$S = \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{matrix} |\gamma\rangle^T \longrightarrow -1 \\ |\alpha\rangle^T \longrightarrow 1 \\ |\beta\rangle^T \longrightarrow 1 \end{matrix}$$

$$A^{(II)} = SAS^{+}$$

$$= \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & -3i \\ 0 & 2 & 0 \\ 3i & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & & \\ & -4 & \\ & & 2 \end{pmatrix}$$

$$H^{(II)} = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

c.

$$B^{(II)} = SB^{(I)}S^{+}$$

$$= \begin{pmatrix} -\frac{i}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -\sqrt{2}\,i \\ 0 & 0 & 0 \\ \sqrt{2}\,i & 0 & 0 \end{pmatrix}$$