Solution 4 for 2019∼2020 USTC class 'Physics of Quantum Information'

Qing Zhou, Xin-Yu Xu, Kai Chen

National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, P.R. China

1. $U = |0\rangle\langle 0| \otimes I_2 + |1\rangle\langle 1| \otimes \sigma_x$ is a C-NOT gate, calculate the following in terms of $I_2, \sigma_x, \sigma_y, \sigma_z$,

$$(1)\ U(\sigma_x \otimes I_2)U; (2)\ U(\sigma_z \otimes I_2)U; (3)\ U(\sigma_x \otimes \sigma_x)U; (4)\ U(\sigma_z \otimes \sigma_z)U.$$

Answer:

$$(1)\ \sigma_x \otimes \sigma_x;\ \ (2)\ \sigma_z \otimes I_2;\ \ (3)\ \sigma_x \otimes I_2;\ \ (4)\ I_2 \otimes \sigma_z.$$

2. Find a parity check matrix H for the $[6, 2]$ repetition code defined by the generator matrix G. Then verify that $HG = 0$.

$$G = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Answer: For example,

$$H = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

(Other answers are welcome.)

3. Please write down the difference between quantum error correction and classical error correction.

   Answer: Read page 3-6 in the lecture "QIP2019chapt_5_Kai Chen.pdf" for reference.

4. (1) Please write down the qunatum error-correction condition.

   (2) Consider the three qubit bit filp code, with corresponding projector $P = |000\rangle\langle 000| + |111\rangle\langle 111|$. The noise process that this code protects against has operation elements

   $$\{\sqrt{(1-p)^3}I, \sqrt{p(1-p)^2}X_1, \sqrt{p(1-p)^2}X_2, \sqrt{p(1-p)^2}X_3\},$$

   where p is the probability that one bit filps. Note that this quantum operation is not trace-preserving, since we have omitted operation elements corresponding to bit flips on two and three qubits. Verify the quantum error-correction conditions for this code and noise process.

   Answer:

   (1) Read page 20-22 in the lecture "QIP2019chapt_5_Kai Chen.pdf" for reference.

   (2) The operation elements of the noise process is $\{E_0, E_1, E_2, E_3\}$, where $E_0 = \sqrt{(1-p)^3}I$ and $E_i = \sqrt{p(1-p)^2}X_i$ for $i = 1, 2, 3$. If $i \neq j$, $PX_i^\dagger X_j P = 0$, if $i = j$, $PX_i^\dagger X_j P = P^2 = P$. So $PE_i^\dagger E_j P = \alpha_{ij} P$, in which $\alpha_{00} = (1-p)^3, \alpha_{ii} = p(1-p)^2$ for $i = 1, 2, 3$ and $\alpha_{ij} = 0 (i \neq j)$. α is an Hermitian matrix, so the three qubit bit filp code and noise process satisfies the quantum error-correction conditions.

5. For 4-qubit GHZ state $|\psi\rangle = (|0011\rangle + |1100\rangle)/\sqrt{2}$, please write down its linearly independent stabilizers.

   Answer: An example is as following: $g_1 = ZZII$, $g_2 = -IZZI$, $g_3 = IIZZ$, $g_4 = XXXX$. (Other answers are welcome.)

6. Given a set of independent stabilizers $g_1 = XZII$, $g_2 = ZXZI$, $g_3 = IZXZ$, $g_4 = IIZX$, please write down the common eigenstate of them with eigenvalue $+1$.

   Answer:

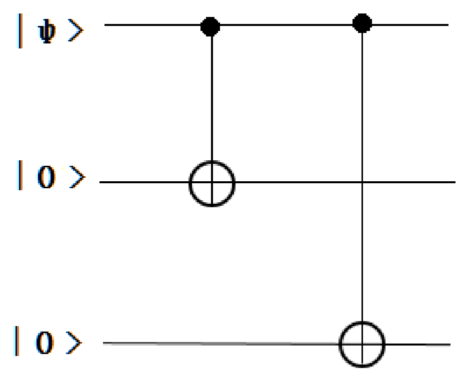
   $$|\psi\rangle\langle\psi| = (\frac{I+g_1}{2})(\frac{I+g_2}{2})(\frac{I+g_3}{2})(\frac{I+g_4}{2})$$

$$|\psi\rangle = (|+\rangle|0\rangle|+\rangle|0\rangle + |+\rangle|0\rangle|-\rangle|1\rangle + |-\rangle|1\rangle|-\rangle|0\rangle + |-\rangle|1\rangle|+\rangle|1\rangle)/2$$

7. Please draw the quantum circuit of the 3-qubit bit flip code, and certify that it can encode the qubit $a|0\rangle + b|1\rangle$ to $a|000\rangle + b|111\rangle$.

Answer:

The quantum circuit is as following:



The encoding progress is as following:

$$\begin{aligned} |\psi\rangle|0\rangle|0\rangle &= (a|0\rangle + b|1\rangle)|0\rangle|0\rangle \\ &\xrightarrow{\text{C-NOT}} (a|00\rangle + b|11\rangle)|0\rangle \\ &\xrightarrow{\text{C-NOT}} (a|000\rangle + b|111\rangle) \end{aligned} \tag{1}$$

8. For 9-qubit Shor code, its logical bit code is
$|0\rangle_L = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)/2\sqrt{2}$,
$|1\rangle_L = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)/2\sqrt{2}$.

(1) Please give all the generators of the stabilizers;

(2) Please draw the encoding quantum circuit;

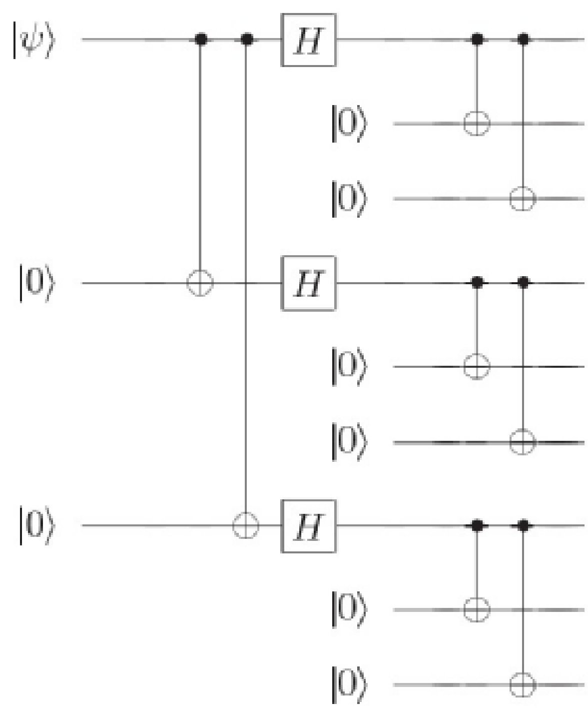
(3) For a bit/phase flip error of a certain bit, how to detect and correct it? Please take the bit flip error and phase flip error for example, write down the program of error detection and correction.

Answer:

(1) The generators of the stabilizers are as following:

Name	Operator
g_1	$Z\,Z\,I\,I\,I\,I\,I\,I\,I$
g_2	$I\,Z\,Z\,I\,I\,I\,I\,I\,I$
g_3	$I\,I\,I\,Z\,Z\,I\,I\,I\,I$
g_4	$I\,I\,I\,I\,Z\,Z\,I\,I\,I$
g_5	$I\,I\,I\,I\,I\,I\,Z\,Z\,I$
g_6	$I\,I\,I\,I\,I\,I\,I\,Z\,Z$
g_7	$X\,X\,X\,X\,X\,X\,I\,I\,I$
g_8	$I\,I\,I\,X\,X\,X\,X\,X\,X$
$\bar{Z}$	$X\,X\,X\,X\,X\,X\,X\,X\,X$
$\bar{X}$	$Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z$

(2) The encoding quantum circuit is as following:



(3) Please read page 433 of Nielsen's "Quantum Computation and Quantum Information" , or page 80 of the Chinese version translated by Qian-Chuan Zhao.