

有源场中磁介度:

$$F = qv \times B.$$


$$F = \int r \times dl \times B = \int r \times B \, ds = \int \vec{j} \times B \, dV.$$

$$L = r \times p$$

(1) 体电流元在其附近 $B=0$.

(2) 面电流元: $B = \frac{\mu_0 I}{2}$

载流线圈在均匀磁场中受力矩: $L = m \times B \quad m = I \cdot S$

无限长载流导线: $F = \frac{\mu_0 I_1 I_2}{2\pi r_0}$  $B = \frac{\mu_0 I}{2\pi r}$

电流强度 I , 单位长度匝数 n 的无限长螺线管:

$$\vec{i} = nI.$$

沿轴: $B_t = \mu_0 n I$ (内) $B_t = 0$ (外).

$$AS: B_t = \pm \frac{\mu_0 n I}{2}$$

$$\Rightarrow \text{内: } B = \frac{\mu_0 n I}{2} = \text{外}$$

如螺线管:

$$\left(\begin{matrix} H \\ H \end{matrix} \right) \quad H \cdot 2\pi r = NI.$$

$$H = nI \\ B = \mu H = \mu n I.$$

磁化强度: $\vec{M} = \frac{\sum m_{分子}}{\Delta V}$ - 磁矩矢量平均.

分子平均磁矩: $m_a = \frac{\sum m_{分子}}{n \Delta V}$ n : 分子数密度.

$$\Rightarrow M = n m_a$$

$$\int M \cdot dl = I'.$$

$$i' = M \times n.$$

磁矩守恒. 在变化的磁场中运动: $\mu = \frac{2m\omega \hbar}{B} = \text{const}$

$$R = \sqrt{\frac{2m\mu}{q^2 B}} \quad B \downarrow R \uparrow \quad v \downarrow v \uparrow.$$

磁矩: $\sin^2 \theta m$
 $= \frac{B_0}{B_m} = \frac{1}{\mu_m}$
 磁化比

磁介质中静磁场基本方程:

$$\oint B \cdot ds = 0.$$

$$H = \frac{B}{\mu_0} - M.$$

$$M = \chi_m H \quad \mu_r \equiv 1 + \chi_m \text{ 相对磁导率.}$$

$$\oint B \cdot dl = \mu_0 I' + \mu_0 I'.$$

$$\oint H \cdot dl = I'.$$

$$\Rightarrow B = \mu H = \mu_0 H + \mu_0 M = \mu_0 (1 + \chi_m) H.$$

磁化分类: $M = \frac{\mu_0 n m_0^2}{3kT} H \Rightarrow \chi_m = \frac{\mu_0 n m_0^2}{3kT}$

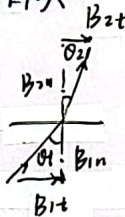
① 顺磁物质 $\chi_m > 0 \quad \mu > \mu_0 \quad \mu = \mu_0 \left(1 + \frac{C}{T} \right)$

② 抗磁物质 $\chi_m < 0 \quad \mu < \mu_0$

$$m_0 = 0 \quad \chi_m = - \frac{\mu_0 n_0 z e^2 \hbar^2}{6 m_e} \bar{r}^2$$

- 一个分子中电子数.

与 I' 反



边界关系: $n_1 \cdot (B_2 - B_1) = 0.$

$$i' = n \times (M_2 - M_1)$$

$$\vec{\omega} = n \times (H_2 - H_1).$$

③ 铁磁物质: $\uparrow \uparrow \uparrow$

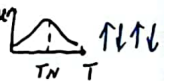
硬磁材料: 磁滞回线宽: 永磁体.

软磁材料: ... 窄: 高磁导.

$$T_c \uparrow \Rightarrow \text{顺磁性 } \chi_m = \frac{C}{T - T_c}$$

④ 亚铁磁物质 $\uparrow \downarrow \uparrow$

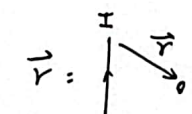
⑤ 反铁磁物质 $\uparrow \downarrow \uparrow \downarrow$



重合就 H, E .

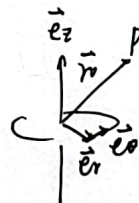
垂直就 D, B .

$$dB = \frac{\mu_0}{4\pi} I \, dl \times \frac{\vec{r}}{r^3}$$



半径 R 圆形电流:

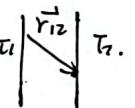
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{m}}{(r^2 + z^2)^{3/2}}$$



$$\vec{B} = -\frac{\mu_0 \vec{m}}{4\pi r^3} + \frac{3\mu_0 (m \cdot \vec{r})}{4\pi r^5} \vec{r}$$

安培定律: $dF_{12} = k \frac{I_2 \, dz \times (I_1 \, dl \times \vec{r}_{12})}{r_{12}^3}$

$$k = \frac{\mu_0}{4\pi} \quad \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$$



磁通定律: $\oint H \cdot dl = \Sigma I_0 = \epsilon_m$ 磁通势, $R_m = \oint \frac{1}{\mu_0} \cdot dl$ 磁阻.

$\Sigma m = \Phi_B \cdot R_m \Rightarrow NI_0 = \Phi_B \cdot (R_{m1} + R_{m2} + \dots + R_{mi})$

互感: $\psi_{12} = M_{12} I_1$

$\psi_{21} = M_{21} I_2$

$M_{12} = M_{21} = M$

自感: $\psi = LI, \quad \epsilon = -L \frac{dI}{dt}$

串联: 顺接: $\epsilon = \epsilon_1 + \epsilon_2 = -(L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + 2M \frac{dI}{dt}) = -(L_1 + L_2 + 2M) \frac{dI}{dt}$

逆接: $L' = L_1 + L_2 - 2M \quad M < \frac{1}{2}(L_1 + L_2)$

并联: 同号端: $\epsilon = \epsilon_1 = \epsilon_2 \quad I = I_1 + I_2$

$\epsilon = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \frac{dI}{dt} \quad L' = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

$M < \sqrt{L_1 L_2}$ 令 $M = k \sqrt{L_1 L_2}$

异号端: $\epsilon = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \frac{dI}{dt} \quad L' = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

k耦合系数, 1: 理想 0: 无耦

不能将变压器原副线圈并联 L=0 短路

电路大.

概念: 电源周期 T 电场 E 一周用时 t. $\lambda = cT$ 电磁波条件.

$j = \sigma E = \sigma (E_{源} + E_{阻} + E_{容} + E_{感})$

$\frac{c}{\lambda} = c \cdot T =$ 电源变化周期时间里电场传播的距离.

$E_{总} = E_{源} + E_{阻} + E_{容} + E_{感}$

① 电源:

$E_{感} = 0, \quad \sigma \rightarrow \infty, \quad \frac{1}{\sigma} = 0 \Rightarrow E_{总} = -K$

$\int E_{总} \cdot dl = -\int K \cdot dl = -\epsilon$

② 电阻:

$E_{阻} = \frac{j}{\sigma} \quad U_R = IR$

$K = 0 \quad E_{感} = 0$

③ 电容:

$u_c = \frac{Q}{C} \quad K = 0 \quad E_{感} = 0$

$\int \frac{1}{C} i dt$

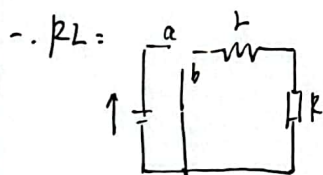
④ 电感:

$K = 0, \quad \frac{1}{\sigma} = 0 \Rightarrow E_{总} = -E_{感}$

$u_L = \int E_{总} \cdot dl = -\int E_{感} \cdot dl = -\epsilon_L$

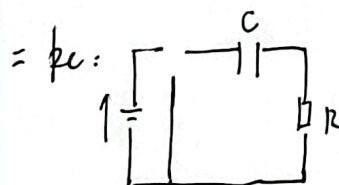
$= L \frac{di}{dt} + M \frac{di'}{dt}$

似稳方程: $\epsilon = iR + \frac{1}{C} \int i dt + (L \frac{di}{dt} + M \frac{di'}{dt})$



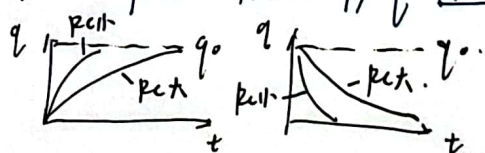
充电: $\epsilon = iR + L \frac{di}{dt} \quad i|_{t=0} = 0 \Rightarrow i = \frac{\epsilon}{R} (1 - e^{-\frac{t}{\tau}})$ 令 $\frac{L}{R} = \tau \Rightarrow i = I_0 (1 - e^{-\frac{t}{\tau}})$

放电: $L \frac{di}{dt} + iR = 0 \quad i|_{t=0} = I_0 \Rightarrow i = I_0 e^{-\frac{t}{\tau}}$ 经过 τ 时间, 达到 37% I_0 .

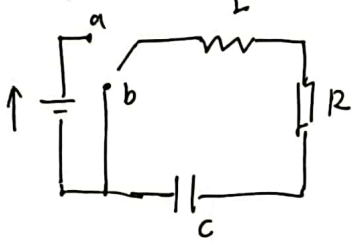


充电: $\epsilon = \frac{dq}{dt} R + \frac{q}{C} \quad q|_{t=0} = 0 \Rightarrow q = C\epsilon (1 - e^{-\frac{t}{RC}}) = q_0 (1 - e^{-\frac{t}{\tau}})$ 经过 τ 达到 63% q_0

放电: $\frac{dq}{dt} R + \frac{q}{C} = 0 \quad q|_{t=0} = q_0 \Rightarrow q = C\epsilon e^{-\frac{t}{RC}} = q_0 e^{-\frac{t}{\tau}}$ 经过 τ 时达到 37% q_0 ($RC = \tau$)



三. $R < C$:



充电: $L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = \varepsilon$

同除L, 令 $\beta = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$, $q_0 = C\varepsilon$:

$\int \frac{d^2q}{dt^2} + 2\beta \frac{dq}{dt} + \omega_0^2 q = \omega_0^2 q_0$ β : 阻尼系数, ω_0 : 固有频率.

$\left\{ \begin{aligned} q|_{t=0} &= 0 \\ \frac{dq}{dt}|_{t=0} &= 0 \end{aligned} \right.$

(1). 欠阻尼 $\beta < \omega_0$

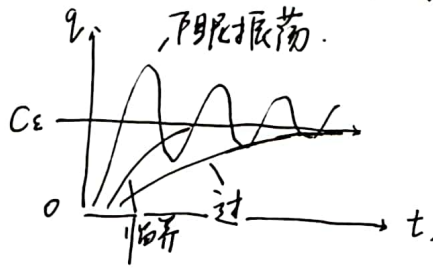
$q = q_0 - q_0 e^{-\beta t} (\cos \omega t + \frac{\beta}{\omega} \sin \omega t)$
 $\omega = \sqrt{\omega_0^2 - \beta^2}$

(2). 过阻尼 (q 随 t 而 \uparrow , β 个上开越慢) $\beta > \omega_0$ 当 $\beta \rightarrow \omega_0$, $R_{PL} \rightarrow C$

$q = q_0 - \frac{1}{2\gamma} q_0 e^{-\beta t} ((\beta + \gamma)e^{\gamma t} - (\beta - \gamma)e^{-\gamma t})$ 回到RC电路
 $\gamma = \sqrt{\beta^2 - \omega_0^2}$

(3). 临界阻尼 $\beta = \omega_0$

$q = q_0 - q_0 (1 + \beta t) e^{-\beta t}$
 q 也随 t 而 \uparrow , 比过阻快



放电: $\frac{d^2q'}{dt^2} + 2\beta \frac{dq'}{dt} + \omega_0^2 q' = 0$

$\left\{ \begin{aligned} q'|_{t=0} &= q_0 \\ \frac{dq'}{dt}|_{t=0} &= 0 \end{aligned} \right.$

$\Rightarrow q' = q_0 - q$

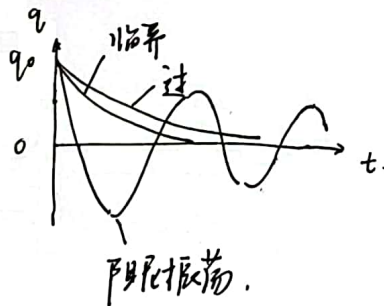
充电情况的解.

(1): 电阻 $R \neq 0$ ($\beta \neq 0$).

$\beta < \omega_0$: 阻尼振荡.

$\beta = \omega_0$: 临界阻尼.

$\beta > \omega_0$: 过阻尼.



(2): 电阻 $R = 0$.

电容能量: $\frac{1}{2} C U^2 = \frac{1}{2} \frac{Q^2}{C}$

线圈: 无B能量. \checkmark ω 中.

无阻自由振荡: $\beta = 0$. $q = q_0 \cos \omega_0 t$



介质中静态场, 静态场:

$$\int_S \rho \cdot dS = \int_V \rho_0 dV, \quad \nabla \cdot D = \rho_0, \quad \int_S B \cdot dS = 0, \quad \nabla \cdot B = 0$$

$$\int_C E \cdot dl = 0, \quad \nabla \times E = 0, \quad \int_C H \cdot dl = \int_S J_0 \cdot dS, \quad \nabla \times H = J_0$$

$$\boxed{-\int \nabla \times = -\nabla \times}$$

麦克斯韦理论基础:

①. 四大方程

②. 两假设

1): $\nabla \cdot D = \rho_0$

1): 涡旋 $E = -\int_S \frac{\partial B}{\partial t} dS = \int_C E \cdot dl$

2): $\nabla \cdot B = 0$ 无自由磁荷

2): 位移电流:

$$j_d \equiv \frac{\partial P}{\partial t} = \epsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} \text{ - 极化电流}$$

$$\int_C H \cdot dl = \int_S (j_0 + j_d) \cdot dS = \int_S (j_0 + \frac{\partial P}{\partial t}) dS$$

介质中, 非稳态:

$$\int_S D \cdot dS = \int_V \rho_0 dV$$

$$\nabla \cdot D = \rho_0$$

$$\int_C E \cdot dl = -\int_S \frac{\partial B}{\partial t} dS$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{ (涡旋)}$$

$$\int_S B \cdot dS = 0$$

$$\nabla \cdot B = 0$$

$$\int_C H \cdot dl = \int_S (j_0 + \frac{\partial P}{\partial t}) dS$$

$$\nabla \times H = j_0 + \frac{\partial P}{\partial t}$$

均匀线性各向同性:

$$D = \epsilon E, \quad B = \mu H, \quad j_0 = \sigma E$$

真空:

$$D = \epsilon_0 E, \quad B = \mu_0 H, \quad j_0 = 0, \quad \sigma = 0$$

边界:

$$n \cdot (D_2 - D_1) = \sigma_0$$

$$n \cdot (B_2 - B_1) = 0$$

$$n \times (E_2 - E_1) = 0$$

$$n \times (H_2 - H_1) = j_0$$

$$\nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - \nabla^2 E = -\nabla^2 E$$

平面电磁波:

自由空间: $j_0 = 0, \rho_0 = 0, \sigma = 0$

有: $\nabla \cdot E = 0$

$\nabla \cdot H = 0$

$$\frac{\nabla \times E}{\nabla \times H} = \frac{-\mu \frac{\partial H}{\partial t}}{\epsilon \frac{\partial E}{\partial t}}$$

$$\Rightarrow \frac{\partial^2 E}{\partial t^2} - \frac{1}{\mu \epsilon} \nabla^2 E = 0$$

$$\frac{\partial^2 H}{\partial t^2} - \frac{1}{\mu \epsilon} \nabla^2 H = 0$$

$$v = \frac{1}{\sqrt{\mu \epsilon}} = \sqrt{\frac{\mu_0 \epsilon_0}{\mu \epsilon}} c$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

定态电磁波的解:

平面电磁波, 即 E, H 只与 z, t 有关:

$$E(z) = E_0 e^{ikz}, \quad H(z) = H_0 e^{ikz}$$

E_0, H_0 由激励源确定

$$E(z, t) = E_0 e^{j(kz - \omega t)}$$

$$H(z, t) = H_0 e^{j(kz - \omega t)}$$

性质: $k \perp E, k \perp H$ 横波
 $E \perp H$, 构成右手系
 $E \times H = \vec{e}_k$

由 $k_1 \cdot k_2 = k^2 = \mu \epsilon \omega^2$

$$\frac{\omega}{k} = \frac{1}{\sqrt{\mu \epsilon}} = v \text{ 同线 } k_2 = \sqrt{\epsilon} E_0 = \sqrt{\mu} H_0$$

$$\Rightarrow \frac{1}{2} \epsilon E^2 = \frac{1}{2} \mu H^2$$

能量密度

$$\frac{c}{v} = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}} = n \approx \sqrt{\frac{\epsilon}{\epsilon_0}}$$



电磁能量、动量、角动量

$$w = \frac{1}{2} \vec{D} \cdot \vec{E} + \frac{1}{2} \vec{B} \cdot \vec{H} \quad \text{总能量 } w = \int_V w dV \quad \text{非电磁能量}$$

$$\vec{S} = \vec{E} \times \vec{H} \quad \text{波印廷矢量} \quad \int_S \vec{S} \cdot d\vec{A} = -\frac{d}{dt}(W + W_m)$$

$$\vec{g} = \vec{D} \times \vec{B} \quad \text{总动量: } \vec{G} = \int_V \vec{g} dV$$

$$\vec{l} = \vec{r} \times \vec{g} \quad \text{总角动量: } \vec{L} = \int_V \vec{l} \cdot dV = I w$$

某一体积 \$V\$ 内电磁场的单位时间损失:

$$-\frac{\partial W}{\partial t} = \int_A \vec{S} \cdot d\vec{A} + \int_V (\vec{j} \cdot \vec{E}) dV$$

对真空:

$$-\frac{\partial W}{\partial t} = \int_A \vec{S} \cdot d\vec{A} \leftarrow -\frac{\partial W}{\partial t} = \int_A \vec{j} \cdot d\vec{A}$$

平面电磁波:

$$w = \epsilon E^2 = \mu H^2$$

$$S = \vec{E} \times \vec{H} = \sqrt{\frac{\epsilon}{\mu}} E^2 \cdot \frac{\vec{v}}{v} = \frac{\epsilon E^2}{\sqrt{\mu \epsilon}} \cdot \frac{\vec{v}}{v} = w \vec{v}$$

$$\vec{g} = \vec{D} \times \vec{B} = \mu \epsilon \vec{E} \times \vec{H} = \vec{S} / v^2 = \frac{w \vec{v}}{v^2}$$

真空中:

$$v = c$$

$$S = w c \Rightarrow w = S / c \quad (w = S / v)$$

$$\vec{g} = w / c = \frac{\vec{S}}{c^2} \Rightarrow \vec{S} = \vec{g} \cdot c^2 \quad (\vec{S} = \vec{g} \cdot v^2)$$

反射:

$$R \equiv \frac{S_R}{S_\lambda} \quad (\text{反射系数}) = \begin{cases} 1 & \text{全反射} \\ 0 & \text{全吸收} \end{cases}$$

$$(\vec{g}_\lambda - \vec{g}_R) \cdot A \cdot c \cdot \Delta t = \frac{AA \cdot \Delta t}{c} \cdot c^2 \cdot (g_\lambda + g_R) \cdot \hat{z} = \frac{AA \cdot \Delta t}{c} (S_\lambda + S_R) \hat{z} = \frac{AA \cdot \Delta t}{c} S_\lambda (1+R) = AA \cdot \Delta t \cdot w (1+R) \hat{z}$$

$$\therefore AA \Delta t w (1+R) = p \cdot AA \cdot \Delta t \cdot \hat{z} \quad p \cdot \text{压强} \Rightarrow p = w(1+R) \quad \text{平均光压: } \bar{p} = (1+R) \bar{w}$$

