Solution 5 for 2019∼2020 USTC class 'Physics of Quantum Information'

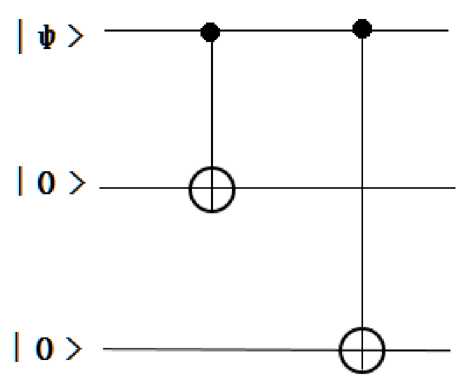
Qing Zhou, Xin-Yu Xu, Kai Chen

National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, 230026, P.R. China

1. Please draw the quantum circuit of the 3-qubit bit flip code, and certify that it can encode the qubit $a|0\rangle + b|1\rangle$ to $a|000\rangle + b|111\rangle$.

   Answer:

   The quantum circuit is as following:



   The encoding progress is as following:

$$\begin{aligned} |\psi\rangle|0\rangle|0\rangle &= (a|0\rangle + b|1\rangle)|0\rangle|0\rangle \\ &\xrightarrow{\text{C-NOT}} (a|00\rangle + b|11\rangle)|0\rangle \\ &\xrightarrow{\text{C-NOT}} (a|000\rangle + b|111\rangle) \end{aligned} \tag{1}$$

2. For 9-qubit Shor code, its logical bit code is
   $|0\rangle_L = (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)/2\sqrt{2}$,
   $|1\rangle_L = (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)/2\sqrt{2}$.

(1) Please give all the generators of the stabilizers;

(2) Please draw the encoding quantum circuit;

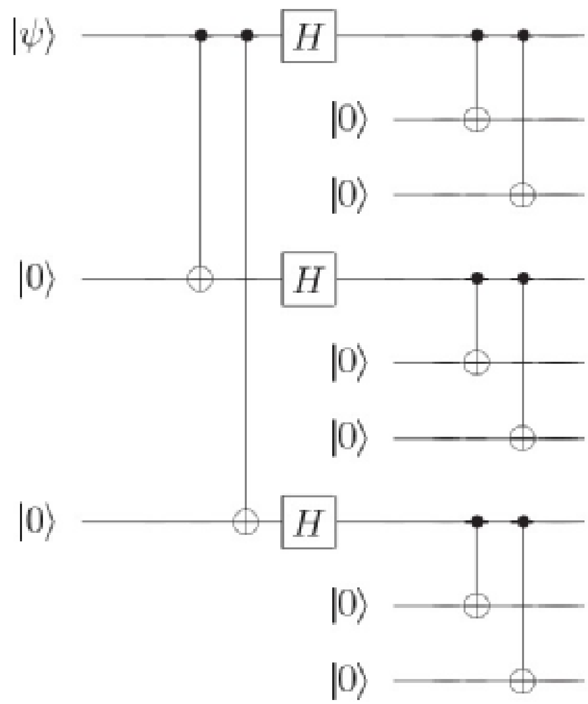
(3) For a bit/phase flip error of a certain bit, how to detect and correct it? Please take the bit flip error and phase flip error for example, write down the program of error detection and correction.

Answer:

(1) The generators of the stabilizers are as following:

Name	Operator
g_1	$Z\,Z\,I\,I\,I\,I\,I\,I\,I$
g_2	$I\,Z\,Z\,I\,I\,I\,I\,I\,I$
g_3	$I\,I\,I\,Z\,Z\,I\,I\,I\,I$
g_4	$I\,I\,I\,I\,Z\,Z\,I\,I\,I$
g_5	$I\,I\,I\,I\,I\,I\,Z\,Z\,I$
g_6	$I\,I\,I\,I\,I\,I\,I\,Z\,Z$
g_7	$X\,X\,X\,X\,X\,X\,I\,I\,I$
g_8	$I\,I\,I\,X\,X\,X\,X\,X\,X$
$\bar{Z}$	$X\,X\,X\,X\,X\,X\,X\,X\,X$
$\bar{X}$	$Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z$

(2) The encoding quantum circuit is as following:



(3) Please read page 433 of Nielsen's "Quantum Computation and Quantum Information" , or page 80 of the Chinese version translated by Qian-Chuan Zhao.

3. Please write down a group of universial quantum logical gate set.

Answer: A group of universial quantum logical gate set is as following: T(45-degree rotation of Z), H(Hadamard gate), C-NOT.

4. Please write down the DiVincenzo criterion that quantum computer implementation must satisfy.

Answer:

(1) **Scalability:** A scalable physical system with well characterized parts, usually qubits.

(2) **Initialization:** The ability to initialize the system in a simple fiducial state.

(3) **Control:** The ability to control the state of the computer using sequences of elementary universal gates.

(4) **Stability:** Decoherence times much longer than gate times, together with the

ability to suppress decoherence through error correction and fault-tolerant computation.

(5) **Measurement:**The ability to read out the state of the computer in a convenient product basis.

5. Please write down the matrix form of the C-NOT gate, and the state resulting from the application of this gate to four Bell states.

Answer: The matrix form of the C-NOT gate is as following:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Denote the four Bell states as

$$|\beta_{xy}\rangle = \frac{|0, y\rangle + (-1)^x |1, \bar{y}\rangle}{\sqrt{2}}$$

, then

$$\text{C-NOT}(|\beta_{00}\rangle) = (|0\rangle + |1\rangle) |0\rangle / \sqrt{2}$$
$$\text{C-NOT}(|\beta_{01}\rangle) = (|0\rangle + |1\rangle) |1\rangle / \sqrt{2}$$
$$\text{C-NOT}(|\beta_{10}\rangle) = (|0\rangle - |1\rangle) |0\rangle / \sqrt{2}$$
$$\text{C-NOT}(|\beta_{11}\rangle) = (|0\rangle - |1\rangle) |1\rangle / \sqrt{2}$$

6. Please construct the quantum SWAP gate to swap two qubits using the C-NOT gate.

Answer: The circuit swapping two qubits is as following:

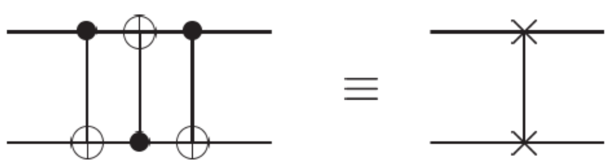


Figure 1.7. Circuit swapping two qubits, and an equivalent schematic symbol notation for this common and useful circuit.

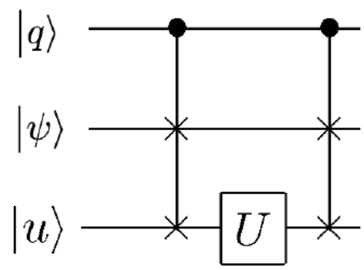
To see that this circuit accomplishes the swap operation, note that the sequence of gates has the following sequence of effects on a computational basis state $|a, b\rangle$,

$$
\begin{aligned}
|a, b\rangle &\longrightarrow |a, a \oplus b\rangle \\
&\longrightarrow |a \oplus (a \oplus b), a \oplus b\rangle = |b, a \oplus b\rangle \\
&\longrightarrow |b, (a \oplus b) \oplus b\rangle = |b, a\rangle \,,
\end{aligned}
$$

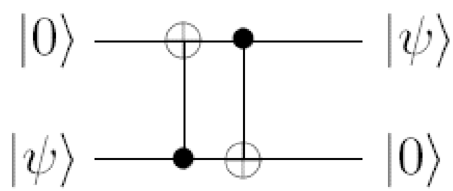
where all additions are done modulo 2.

7. Suppose you are given a box which performs a unitary gate U on a one-qubit input state. In addition, you are given $|u\rangle$, and eigenstate of U with eigenvalue one($U|u\rangle = |u\rangle$). Please design a quantum circuit which peforms a controlled-U gate(control qubit $|q\rangle$ and target qubit $|\psi\rangle$), using this box, $|u\rangle$ and quantum Fredkin (i.e. controlled-swap) gates.

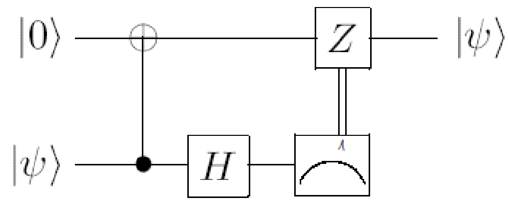
   Answer:



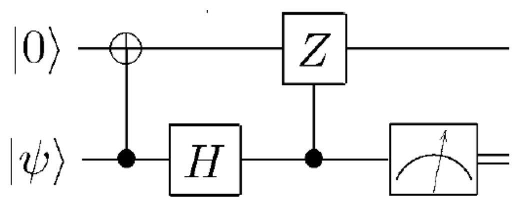
8. An unknown qubit in the state $|\psi\rangle$ can be swapped with a second qubit which is prepared in the state $|0\rangle$ using only two C-NOT gates, with the circuit



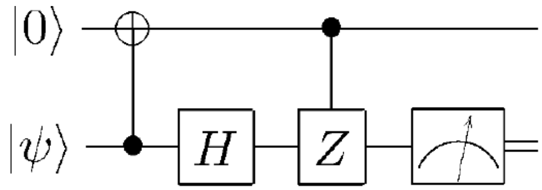
   Show that the circuit below, which use only a single cnot gate, with measurement and a classically controlled single qubit operation, also accomplish the same task(use circuit equivalences):



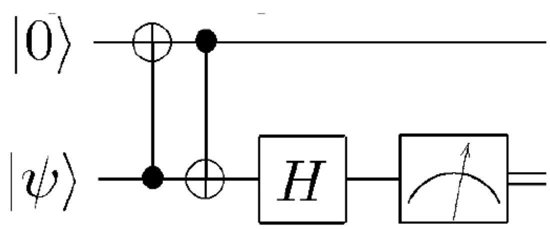
Answer: Classical control operation after measurement is equivalent to quantum control operation before measurement, hence the circuit is equivalent to



Controlled-Z operation is symmetric between control bit and target bit, hence the circuit is equivalent to



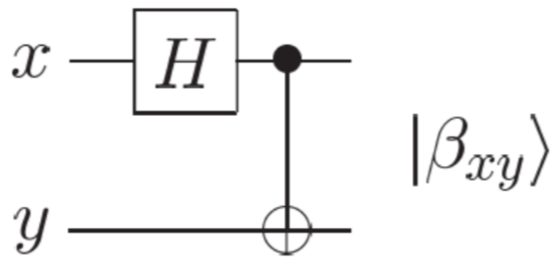
Commuting controlled-Z through Hadamard we get C-NOT



The two C-NOT gates swaps the two qubits. Obviously, $|\psi\rangle$ is output on the first line.

9. Please design a quantum circuit which converts the state $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ into four Bell states.

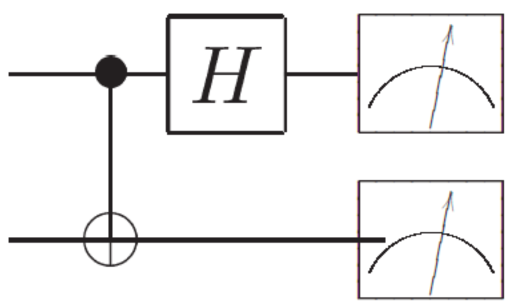
Answer: The quantum circuit to create Bell state is as following:



The proof is omitted.

10. Please design a quantum circuit to perform full Bell state measurement, i.e. to distinguish four Bell states by projective measurement at $|0\rangle, |1\rangle$ basis.

    Answer: The quantum circuit to perform full Bell state measurement is as following:



The proof is omitted.

11. Please draw the quantum circuit of Deutsch algorithm, and analyse how it works.

    Answer: Please read page 47-50 of lecture "QIP2019chapt_6_Kai Chen.pdf" for reference.