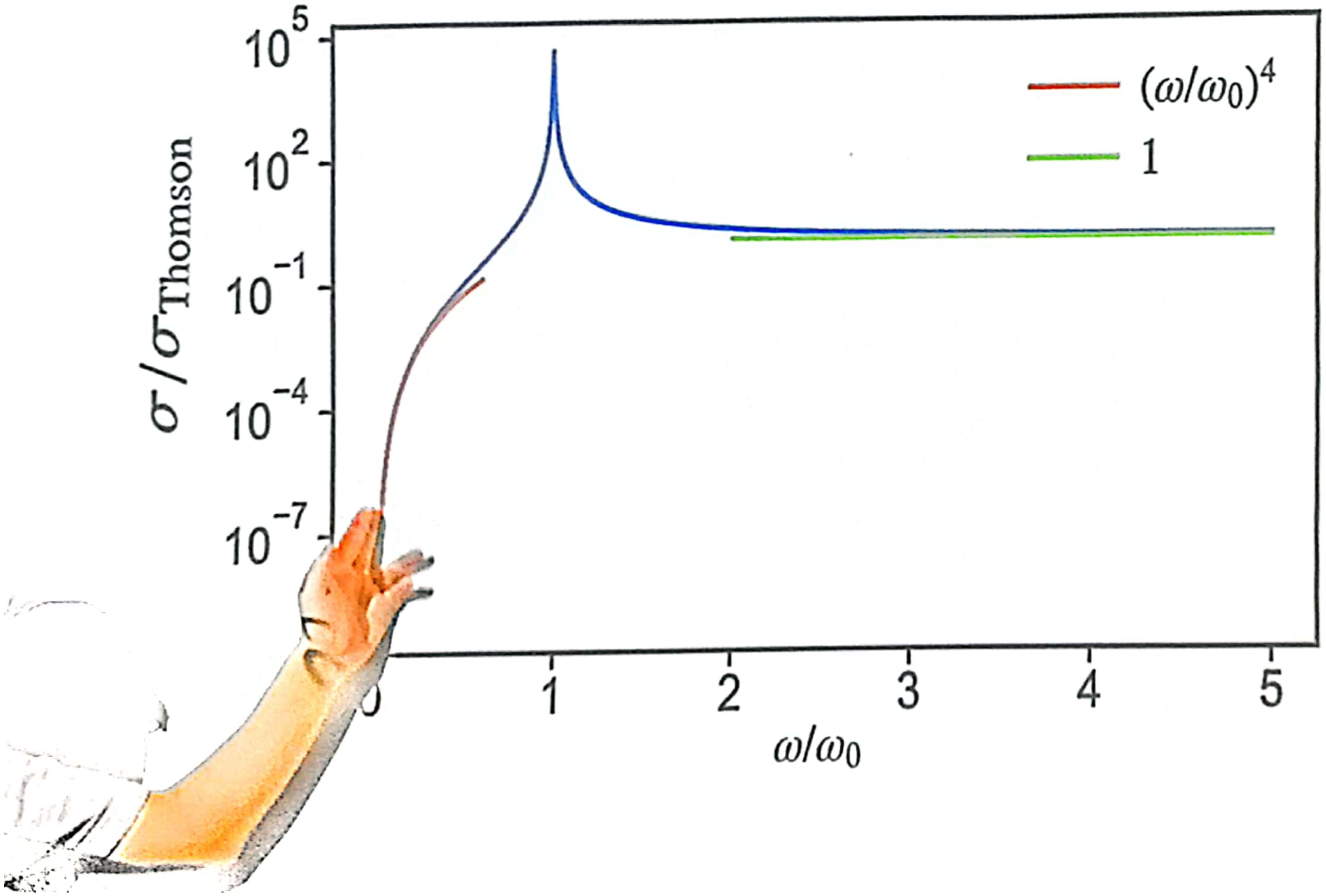
Scattering of Light by NR Electrons

$$\sigma = \frac{8}{3}\pi r_e^2 \frac{\omega^4}{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}$$



Scattering of Light by NR Electrons

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = -\frac{e}{m} E_0 e^{-i\omega t}$$

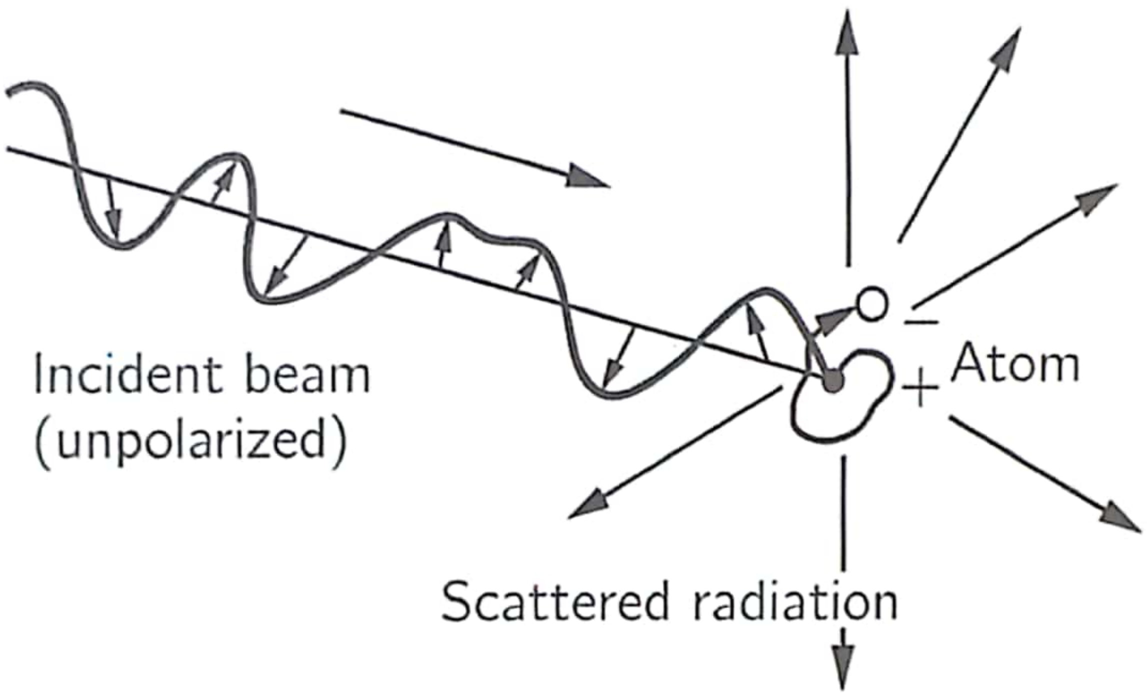
$$x = -\frac{e}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma} E_0 e^{-i\omega t}$$

$$p = -ex = \frac{e^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma} E_0 e^{-i\omega t} = p_0 e^{-i\omega t}$$

$$\omega_0^2 - \omega^2 - i\omega\gamma = \sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2} e^{-i\delta}$$

$$p_0 = \frac{e^2}{m} \frac{e^{-i\delta}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} E_0$$

Scattering of Light by NR Electrons



Scattering by both bounded and free electrons will be discussed.

The Electric Dipole Radiation

$$\left\langle \frac{\mathrm{d}P}{\mathrm{d}\Omega} \right\rangle = \frac{\mu_0 \omega^4}{32\pi^2 c} p_0^2 \sin^2\theta \qquad \langle P \rangle = \frac{\mu_0 \omega^4}{12\pi c} p_0^2$$

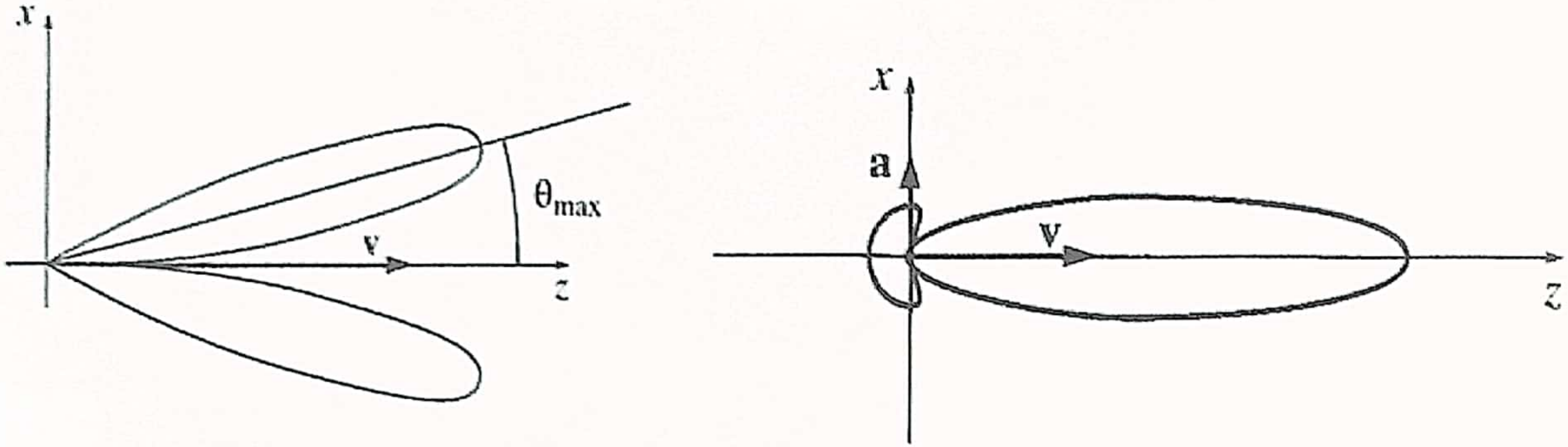
Compare this with radiation by a NR particle.

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi^2 c} a^2 \sin^2\theta = \frac{\mu_0 \omega^4}{16\pi^2 c} p^2 \sin^2\theta$$

$$P = \frac{\mu_0 q^2}{6\pi c} a^2 = \frac{\mu_0 \omega^4}{6\pi c} p^2$$

$$qa = \ddot{p} = -\omega^2 p$$

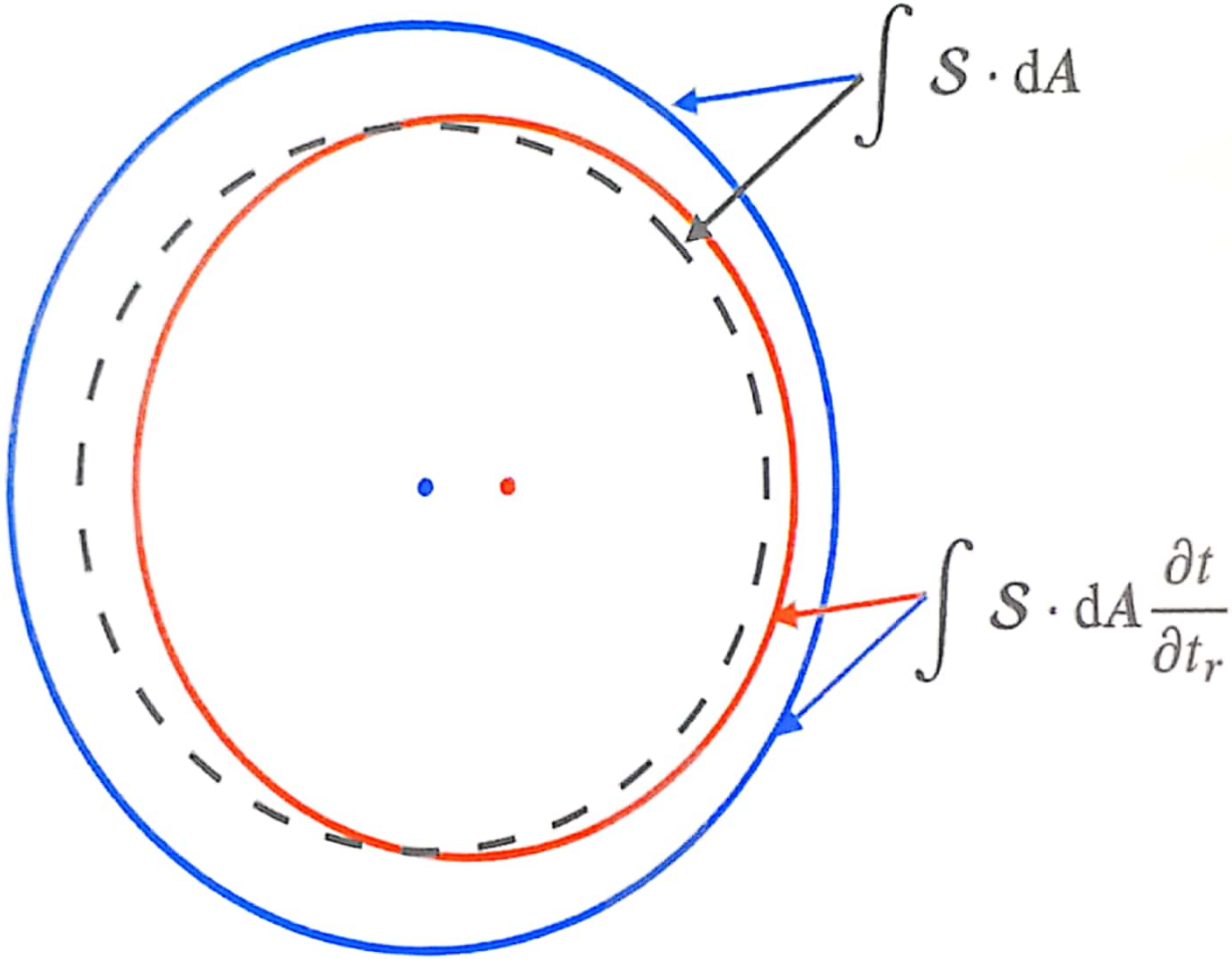
Comparison of the radiated power from the two cases



$$P_{\|} = \frac{\mu_0 q^2}{6\pi c} a^2 \gamma^6 \qquad\qquad P_{\perp} = \frac{\mu_0 q^2}{6\pi c} a^2 \gamma^4$$

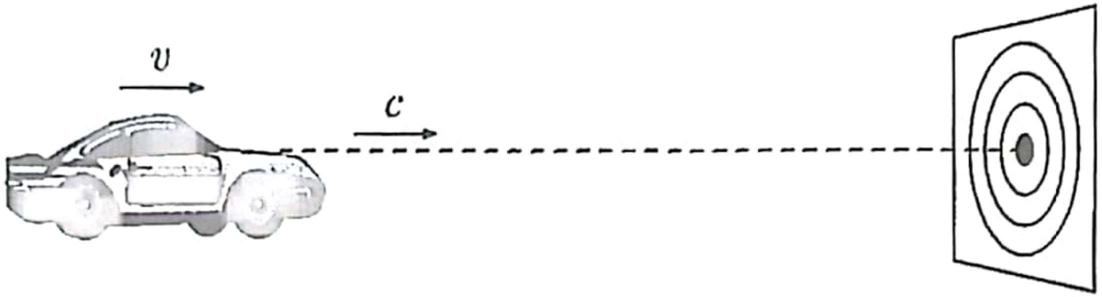
For a given a and γ, $P_{\|} > P_{\perp}$.

Power radiated by a point charge



Wave surface radiated between: t_r and $t_r + 1$

Power radiated by a point charge



$$N_t = \frac{\mathbf{S} \cdot \mathrm{d}\mathbf{A}\Delta t}{\Delta t} = \mathbf{S} \cdot \mathrm{d}\mathbf{A}$$

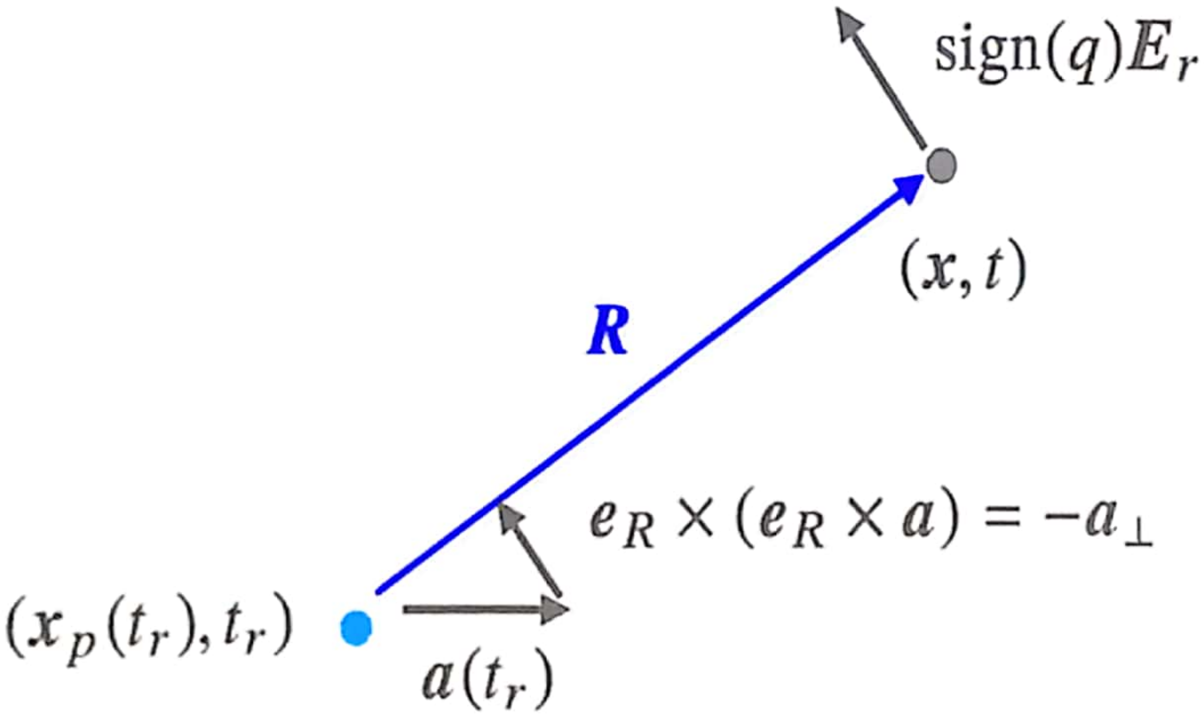
These bullets were fired during Δt_r.

$$N_g = \frac{\mathbf{S} \cdot \mathrm{d}\mathbf{A}\Delta t}{\Delta t_r} = \frac{\mathbf{S} \cdot \mathrm{d}\mathbf{A}(\partial t/\partial t_r)\Delta t_r}{\Delta t_r} = \mathbf{S} \cdot \mathrm{d}\mathbf{A}\frac{\partial t}{\partial t_r}$$

The radiation field of a moving non-relativistic charge

$$E_r = \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{R\times[(R-R\beta)\times a]}{(R-R\cdot\beta)^3} \approx \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{R\times(R\times a)}{R^3}$$

$$= \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{e_R\times(e_R\times a)}{R} = -\frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{a_\perp}{R}$$



Fields of a moving charge: E and B.

$$\mathbf{B} = \mathbf{e}_R \times \mathbf{E}/c \qquad \mathbf{E} = \mathbf{E}_v + \mathbf{E}_r$$

$$\mathbf{E}_v = \frac{q}{4\pi\epsilon_0}(1-\beta^2)\frac{\mathbf{R} - R\boldsymbol{\beta}}{(R - \mathbf{R}\cdot\boldsymbol{\beta})^3} \sim \frac{1}{R^2}$$

velocity field

$$\mathbf{E}_r = \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{\mathbf{R}\times[(\mathbf{R} - R\boldsymbol{\beta})\times\mathbf{a}]}{(R - \mathbf{R}\cdot\boldsymbol{\beta})^3} \sim \frac{1}{R}$$

radiation field

$$\mathbf{a} = \frac{\partial \mathbf{v}}{\partial t_r} \quad \text{acceleration at } t_r.$$

$$P_v \sim \frac{1}{R^4}R^2 \rightarrow 0$$

$$P \sim \oint E^2 \mathrm{d}S \qquad \text{As } R \rightarrow \infty,$$

Fields of a moving charge: The Lienard-Wiechert potentials

$$\phi(x,t) = \frac{q}{4\pi\epsilon_0}\int \frac{\delta[x' - x_p(t_r)]}{|x - x'|} dV'$$

$$= \frac{q}{4\pi\epsilon_0}\int d\tau \frac{\delta(\tau - t_r)}{|x - x_p(\tau)|} \qquad t_r = t - |x - x_p(\tau)|/c$$

$$g(\tau) = \tau - \left[t - |x - x_p(\tau)|/c\right] = \tau - t + R/c$$

$$g'(t_{r0}) = \left.\frac{\partial g(\tau)}{\partial \tau}\right|_{t_{r0}} = 1 + \frac{1}{c}\frac{\partial R}{\partial \tau} = 1 - \frac{\mathbf{R}}{R}\cdot\frac{\mathbf{v}_p(t_{r0})}{c}$$

$$R^2 = \mathbf{R}\cdot\mathbf{R} \;\Rightarrow\; R\frac{\partial R}{\partial \tau} = \mathbf{R}\cdot\frac{\partial \mathbf{R}}{\partial \tau} \Rightarrow \frac{\partial R}{\partial \tau} = (\mathbf{R}/R)\cdot(-\frac{\partial \mathbf{x}_p}{\partial \tau})$$

$$= -(\mathbf{R}/R)\cdot \mathbf{v}_p(\tau)$$

Propagation of EM waves in a wave guide

$E_z, B_z = 0$: TEM mode, transverse electric and magnetic mode

$$\boldsymbol{E}(x, y, z, t) = E_x \boldsymbol{e}_x + E_y \boldsymbol{e}_y = \boldsymbol{E}_0(x, y) e^{i(kz-\omega t)}$$

$$\boldsymbol{E}_0(x, y) = E_{0x} \boldsymbol{e}_x + E_{0y} \boldsymbol{e}_y$$

$$\nabla \cdot \boldsymbol{E} = 0 \Rightarrow \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0 \Rightarrow \frac{\partial E_{0x}}{\partial x} + \frac{\partial E_{0y}}{\partial y} = 0$$

$$\Rightarrow \nabla \cdot \boldsymbol{E}_0(x, y) = 0$$

$$\boldsymbol{E}_0(x, y) = -\nabla \varphi(x, y) \quad \Rightarrow \nabla^2 \varphi(x, y) = 0$$

b/c: $E_{\parallel} = 0 \Rightarrow \varphi =$ const at boundary $\Rightarrow \varphi =$ const inside

$$\Rightarrow \boldsymbol{E}_0(x, y) = 0$$

Propagation of EM waves in a wave guide

$E_z, B_z = 0$: TEM mode, transverse electric and magnetic mode

$$E(x,y,z,t) = E_x e_x + E_y e_y = E_0(x,y)e^{i(kz-\omega t)}$$

$$E_0(x,y) = E_{0x} e_x + E_{0y} e_y$$

$$\nabla \times E_0 = \left(e_x \frac{\partial}{\partial x} + e_y \frac{\partial}{\partial y}\right) \times (E_{0x} e_x + E_{0y} e_y)$$

$$= \left(\frac{\partial E_{0y}}{\partial x} - \frac{\partial E_{0x}}{\partial y}\right) e_z = 0 \Rightarrow E_0(x,y) = -\nabla \varphi(x,y)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z \Rightarrow \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \Rightarrow \frac{\partial E_{0y}}{\partial x} - \frac{\partial E_{0x}}{\partial y} = 0$$

Propagation of EM waves in a wave guide

We now show that if $E_z = 0, B_z = 0 \Rightarrow \omega^2/c^2 = k^2$

$$\frac{\partial B_z}{\partial y} - ikB_y = -i\frac{\omega}{c^2}E_x \qquad \frac{\omega}{c^2}E_x - kB_y = 0$$

$$\Rightarrow$$

$$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y \qquad kE_x - \omega B_y = 0$$

$$\Rightarrow \begin{bmatrix} \omega/c^2 & k \\ k & \omega \end{bmatrix} \begin{pmatrix} E_x \\ B_y \end{pmatrix} = 0 \quad \Rightarrow \quad \begin{vmatrix} \omega/c^2 & k \\ k & \omega \end{vmatrix} = 0$$

$$\Rightarrow \quad \omega^2/c^2 = k^2$$

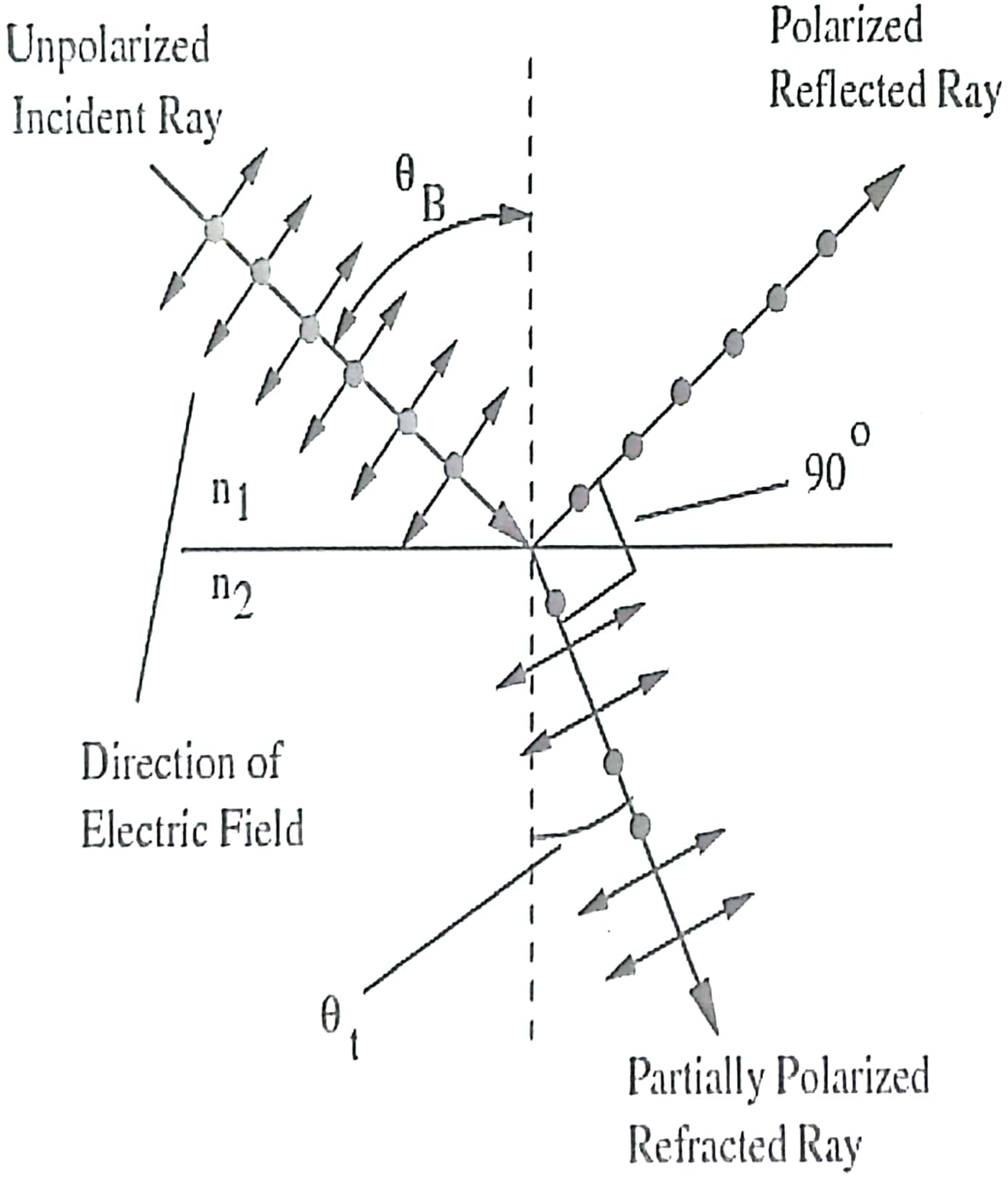
Electromagnetic Waves in Conductors

$$\nabla^2 E = \mu\epsilon \frac{\partial^2 E}{\partial t^2} + \mu\sigma \frac{\partial E}{\partial t}$$

Re-organize it as: $\mu\epsilon \frac{\partial^2 E}{\partial t^2} - \nabla^2 E = -\mu\sigma \frac{\partial E}{\partial t}$

Compare it with $m \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + m\omega_0^2 x = -m\gamma v$

Reflection and Refraction of Electromagnetic Waves



Fresnel's Equations: Amplitudes of Reflected and Transmitted Waves

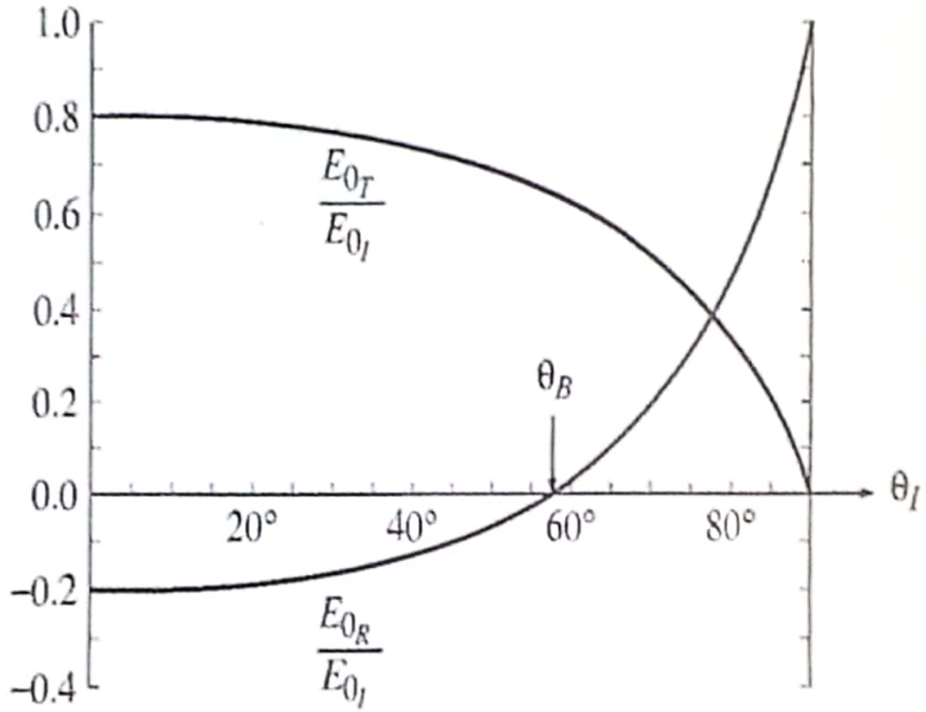
E_I in the plane of incidence, so are E_R and E_T

$$E_{R0} = \frac{\alpha - \beta}{\alpha + \beta} E_{I0}$$

$$E_{T0} = \frac{2}{\alpha + \beta} E_{I0}$$

$$\beta \approx n_2 / n_1$$

$$\alpha \equiv \cos\theta_T / \cos\theta_I$$



$n_1 = 1$, (air) $\quad n_2 = 1.5$, (glass)

Fresnel's Equations: Amplitudes of Reflected and Transmitted Waves

E_I in the plane of incidence, so are E_R and E_T

$$E_{I0} - E_{R0} = \beta E_{T0} \qquad \beta \approx n_2 / n_1$$

$$E_{I0} + E_{R0} = \alpha E_{T0} \qquad \alpha \equiv \cos\theta_T / \cos\theta_I$$

$$\Rightarrow \quad E_{R0} = \frac{\alpha - \beta}{\alpha + \beta} E_{I0}$$

$$E_{T0} = \frac{2}{\alpha + \beta} E_{I0}$$

Fresnel's Equations: Amplitudes of Reflected and Transmitted Waves

$\mathbf{E}_I$ in the plane of incidence, so are $\mathbf{E}_R$ and $\mathbf{E}_T$

$$\epsilon_1(-E_{I0}\sin\theta_I + E_{R0}\sin\theta_R) = \epsilon_2(-E_{T0}\sin\theta_T)$$

$$\frac{1}{\mu_1 v_1}(E_{I0} - E_{R0}) = \frac{1}{\mu_2 v_2}E_{T0}$$

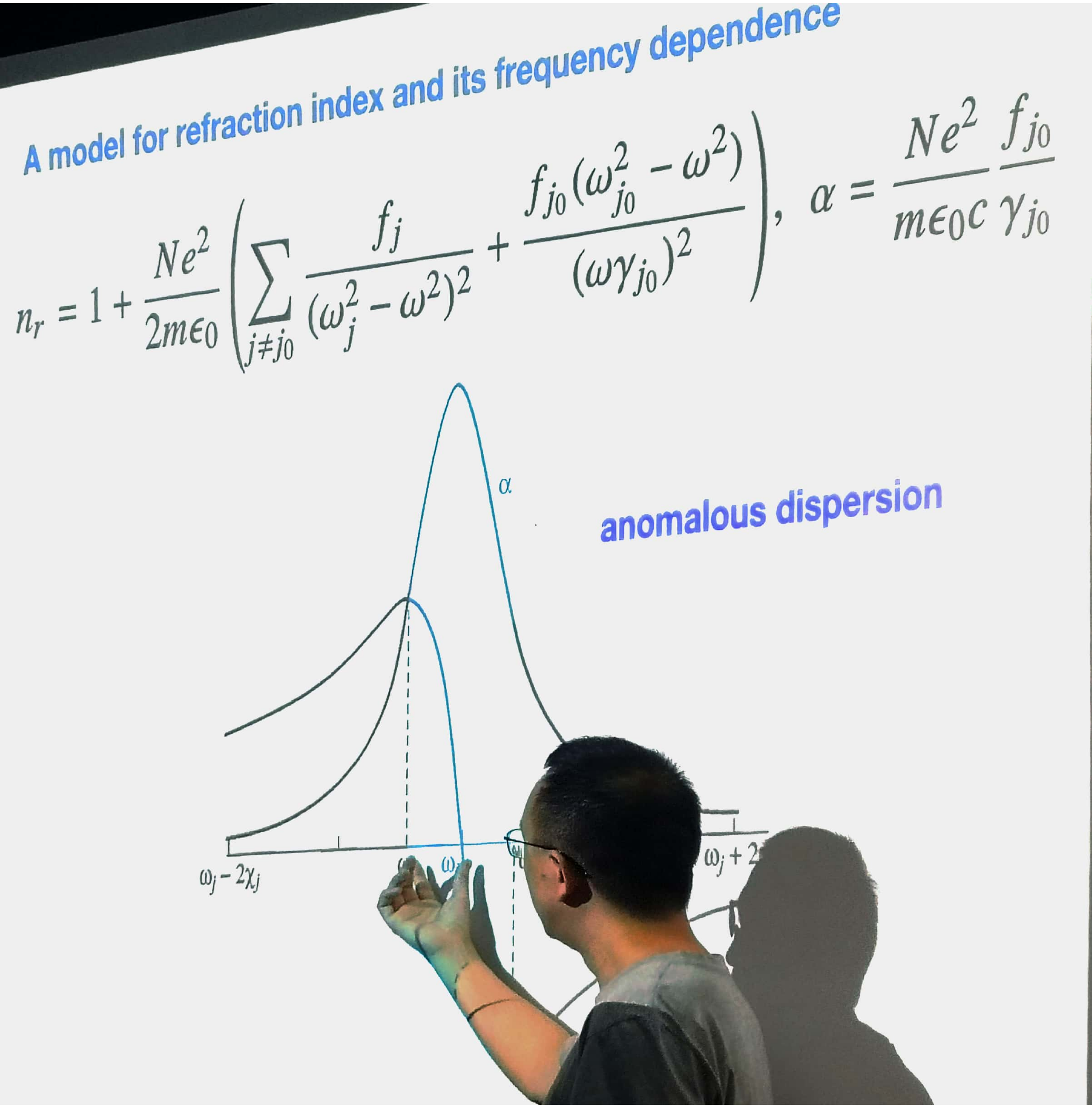
both $\Rightarrow$

$$E_{I0} - E_{R0} = \beta E_{T0}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} \approx n_2/n_1$$

$$E_{I0}\cos\theta_I + E_{R0}\cos\theta_R = E_{T0}\cos\theta_T \Rightarrow E_{I0} + E_{R0} = \alpha E_{T0}$$

$$\alpha \equiv \cos\theta_T/\cos\theta_I$$


A model for refraction index and its frequency dependence
$n_r = 1 + \frac{Ne^2}{2m\epsilon_0}\left(\sum_{j\neq j_0}\frac{f_j}{(\omega_j^2-\omega^2)^2} + \frac{f_{j_0}(\omega_{j_0}^2-\omega^2)}{(\omega\gamma_{j_0})^2}\right), \quad \alpha = \frac{Ne^2}{m\epsilon_0 c}\frac{f_{j_0}}{\gamma_{j_0}}$
α
anomalous dispersion
$\omega_j - 2\chi_j$