Midterm Exam

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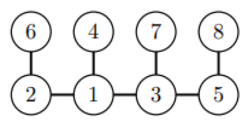
Graph Theory, Winter 23/24

This is a closed book exam. Duration: 90 minutes.

Problem 1.

(a) (10 points) Draw the tree whose Prüfer code is $(7, 5, 5, 3, 1)$.

(b) (10 points) Compute the Prüfer code of the following labeled tree:



Problem 2. (20 points) Let $k \geq 2$ be an integer. Show that in a k-connected graph any k vertices lie on a common cycle.

Problem 3. (20 points) Let $k \geq 2$ be an integer. Prove that every $(k-1)$-edge-connected k-regular graph on an even number of vertices has a perfect matching.

Problem 4. On a chessboard, a knight can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate. Show that:

(i) (10 points) The 4×4 chessboard does not have a knight's tour: a traversal by knight's moves that visits each square once and returns to the start.

(ii) (10 points) For every $n > 4$, the $4 \times n$ chessboard does not have a knight's tour.

Hint: If G is Hamiltonian then for any set $S \subseteq V(G)$ the graph $G \setminus S$ has at most $|S|$ connected components.

Problem 5. (20 points) An $n \times n$ *Latin square* is an $n \times n$ matrix with entries in $\{1, \ldots, n\}$ such that no row contains a number twice and no column contains a number twice. If the first r rows of the matrix have been filled with integers in $\{1, \ldots, n\}$ such that these conditions hold, we speak of an $r \times n$ *Latin square.*
Show that any $r \times n$ Latin square can be completed to an $n \times n$ Latin square by filling in the remaining rows.