

题号	一	二	三	四	五	六	七	八	总分
得分									

一 (10分) 求解定解问题

$$\begin{cases} u_{xx} + 3u_{xy} - 4u_{yy} = 0, \\ u(x, 0) = \sin 3x, \quad u_y(x, 0) = 2x. \end{cases}$$

二 (12分) 考虑如下形式的一阶线性方程; 考虑如下形式的一阶线性方程:

$$\frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = f(x, y)$$

1) 在 $f(x, y) = 0$ 时, 求方程通解.

2) 在 $f(x, y) = xy$ 时, 求满足 $u(0, y) = y$ 的特解.

三 (12分) 求以下固有值问题的固有值和固有函数.

$$\begin{cases} y'' + \lambda y = 0, \quad (0 < x < 2\pi) \\ y(0) = 0, \quad y(2\pi) = 0. \end{cases}$$

四 (14分) 求解定解问题:

$$\begin{cases} u_t = 4u_{xx} - 3u, \quad (t > 0, \quad 0 < x < 5) \\ u(t, 0) = u(t, 5) = 0, \\ u(0, x) = \varphi(x), \quad u_t(0, x) = 0. \end{cases}$$

五、(14分) 考虑如下定解问题:

$$\begin{cases} u_t = a^2 \Delta_3 u, \quad (t > 0, x^2 + y^2 < R^2, -\infty < z < +\infty) \\ u|_{x^2+y^2=R^2} = u_t, \\ u|_{t=0} = u_1 + R^2 - x^2 - y^2, \end{cases}$$

其中 a, R, u_1 均为给定的常数, 且 $a > 0, R > 0$.

(1) 当 $u_1 = 0$ 时, 求解上述定解问题.

(2) 当 $u_1 = 1$ 时, 求解上述定解问题.

六 (12分) (r, θ, φ) 为球坐标, 考虑如下定解问题:

$$\begin{cases} \Delta_3 u = 0, \quad r < 2, \\ u|_{r=2} = f(\theta). \end{cases}$$

(1) 当 $f(\theta) = 1 + \cos^2 \theta$ 时, 求解上述定解问题.

(2) 当 $f(\theta) = \begin{cases} 4, & 0 \leq \theta \leq \alpha, \\ 0, & \alpha < \theta \leq \pi \end{cases}$ 时, 求解上述定解问题.

七、(12分) 求解初值问题:

$$\begin{cases} u_t = u_{xx} + 20u_x + u, \quad (t > 0, -\infty < x < +\infty) \\ u|_{t=0} = \delta(x-2) + 3e^{-x^2}. \end{cases}$$

八 (14分) 已知空间区域 $V = \{(x, y, z) \mid x > 0, y > 0, -\infty < z < +\infty\}$, 求 V 内 Poisson 方程第一边值问题的格林函数, 并求解边值问题

$$\begin{cases} \Delta_3 u = 0, \quad (x, y, z) \in V \\ u = \begin{cases} \varphi(x, z), & \text{当 } y = 0, x \geq 0 \\ g(y, z), & \text{当 } x = 0, y > 0 \end{cases} \end{cases}$$

参考公式

1) 直角坐标系: $\Delta_3 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$, 柱坐标系: $\Delta_3 u = \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial u}{\partial r}) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2}$.

球坐标系: $\Delta_3 u = \frac{1}{r^2} (r^2 \frac{\partial}{\partial r} \frac{\partial u}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial u}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u}{\partial \varphi^2}$.

2) Bessel 函数在三类边界条件下的模方分别为: $N_{2n}^0 = \frac{\alpha^2}{2} J_{2n+1}(\omega_{1n} \alpha)$, $N_{2n}^1 = \frac{1}{2} [\alpha^2 - \frac{\nu^2}{\omega_{2n}^2}] J_{2n}(\omega_{2n} \alpha)$, $N_{2n}^2 = \frac{1}{2} [\alpha^2 - \frac{\nu^2}{\omega_{2n}^2} + \frac{\alpha^2 \alpha^2}{\beta^2 \omega_{2n}^2}] J_{2n}(\omega_{2n} \alpha)$.

Bessel 函数有递推公式: $(x^\nu J_\nu)' = x^\nu J_{\nu-1}$, $(x^{-\nu} J_\nu)' = -x^{-\nu} J_{\nu+1}$, $2J_\nu' = J_{\nu-1} - J_{\nu+1}$, $2\nu x^{-1} J_\nu = J_{\nu-1} + J_{\nu+1}$.

3) n 阶 Legendre 多项式: $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$, $n = 0, 1, 2, 3, \dots$; 递推公式:

1. $(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0$, 2. $nP_n(x) - xP_n'(x) + P_{n-1}'(x) = 0$,
3. $nP_{n-1}(x) - P_n'(x) + xP_{n-1}'(x) = 0$, 4. $P_{n+1}'(x) - P_{n-1}'(x) = (2n+1)P_n(x)$.

4) $\frac{1}{\pi} \int_0^{+\infty} e^{-\alpha^2 \lambda^2} \cos \lambda x d\lambda = \frac{1}{2\alpha\sqrt{\pi}} \exp(-\frac{x^2}{4\alpha^2})$

5) 由 V 内 Poisson 方程第一边值问题的格林函数 $G(M; M_0)$, 求相应问题解 $u(M)$ 的公式:

$$u(M) = \iiint_V f(M_0) G(M; M_0) dM_0 - \iint_S \varphi(M_0) \frac{\partial G}{\partial n_0}(M; M_0) dS_0.$$



参考解答和参考评分标准

一 解: 特征线方程为:

$$\left(\frac{dy}{dx}\right)^2 - 3\frac{dy}{dx} - 4 = 0$$

得到两个首次积分:

$$x + y = c_1, \quad y - 4x = c_2.$$

作变换 $\xi = x + y, \eta = y - 4x$, 方程化为:

$$u_{\xi\eta} = 0 \implies u = f(\xi) + g(\eta) = f(x + y) + g(y - 4x)$$

.....(5分)

代入定解条件得到:

$$f(x) + g(-4x) = \sin 3x, \quad f'(x) + g'(-4x) = 2x$$

解得: $f(x) = \frac{1}{5}(\sin 3x + 4x^2 + 4c), \quad g(-4x) = \frac{1}{5}(4 \sin 3x - 4x^2 - 4c)$

即: $f(x) = \frac{1}{5}(\sin 3x + 4x^2 + 4c), \quad g(x) = \frac{1}{5}(-4 \sin \frac{3}{4}x - \frac{1}{4}x^2 - 4c)$

所以, 原定解问题解为:

$$\begin{aligned} u &= \frac{1}{5}(\sin 3(x + y) + 4(x + y)^2 + 4c) + \frac{1}{5}(-4 \sin \frac{3}{4}(y - 4x) - \frac{1}{4}(y - 4x)^2 - 4c) \\ &= \frac{1}{5} \sin(3x + 3y) + \frac{4}{5} \sin(3x - \frac{3}{4}y) + \frac{3}{4}y^2 + 2xy. \end{aligned}$$

.....(10分)

二 解: 1) 在 $f(x, y) = 0$ 时, 方程为:

$$\frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0.$$

其特征线方程为:

$$\frac{dx}{1} = \frac{dy}{-y} \implies ye^x = c$$

作变量替换: $\xi = e^xy, \eta = y$, 方程化为 $\frac{\partial u}{\partial \eta} = 0$. 解得通解:

$$\mathbf{u} = \mathbf{F}(\mathbf{e}^x\mathbf{y})$$

.....(6分)

2) 在 $f(x, y) = xy$ 时, 仍使用变换: $\xi = e^xy, \eta = y$

$$-\eta \frac{\partial u}{\partial \eta} = (\ln \xi - \ln \eta)\eta \implies \frac{\partial u}{\partial \eta} = \ln \eta - \ln \xi$$

解得:

$$u = \eta \ln \eta - \eta - \eta \ln \xi + \varphi(\xi) = \varphi(e^xy) - y - xy$$

.....(10分)
 这样 $u(0, y) = \varphi(y) - y = y$, 即 $\varphi(y) = 2y$ 最后得到

$$u(x, y) = 2e^{xy} - y - xy.$$

.....(12分)

三 解: 直接讨论或根据Strum-Liouville 定理, 固有值 $\lambda > 0$, 故可令 $\lambda = \omega^2$, 代入方程得到

$$y(x) = A \cos \omega x + B \sin \omega x$$

.....(6分)

由 $y(0) = 0$, 得出 $A = 0$, 因此 $y(x) = B \sin \omega x$. 再由另一边界条件 $y'(20) = B\omega \cos 20\omega = 0$, 得出 $\cos 20\omega = 0$, 即

$$20\omega = n\pi + \frac{\pi}{2} \implies \omega_n = \frac{n\pi + \frac{\pi}{2}}{20}, \quad n = 0, 1, 2, \dots$$

$$\text{固有值: } \lambda_n = \left(\frac{n\pi + \frac{\pi}{2}}{20}\right)^2, \quad \text{固有函数: } \mathbf{y}_n(\mathbf{x}) = \sin \frac{n\pi + \frac{\pi}{2}}{20} \mathbf{x}$$

.....(12分)

四 解: 利用分离变量, 令 $u = T(t)X(x)$, 代入方程得到

$$\frac{X''(x)}{X(x)} - \frac{3}{4} = \frac{1}{4} \frac{T''(t)}{T(t)} = -\lambda$$

并利用边界条件, 得到固有值问题:

$$\begin{cases} X'' + (\lambda - \frac{3}{4})X = 0, & (0 < x < 5) \\ X(0) = 0, & X(5) = 0. \end{cases}$$

并相应 $T(t)$ 的方程: $T''(t) + 4\lambda T = 0$. 解固有值问题解得到:

$$\lambda_n = \left(\frac{n\pi}{5}\right)^2 + \frac{3}{4}, \quad X_n(x) = \sin \frac{n\pi}{5} x, \quad n = 1, 2, \dots$$

.....(7分)

相应地

$$T_n(t) = C_n \cos \left(\sqrt{\frac{4n^2\pi^2}{25} + 3} \right) t + D_n \sin \left(\sqrt{\frac{4n^2\pi^2}{25} + 3} \right) t$$

利用叠加原理, 设

$$u = \sum_{n=1}^{+\infty} \left[C_n \cos \left(\sqrt{\frac{4n^2\pi^2}{25} + 3} \right) t + D_n \sin \left(\sqrt{\frac{4n^2\pi^2}{25} + 3} \right) t \right] \sin \frac{n\pi}{5} x$$

最后利用初值条件:

$$u(0, x) = \sum_{n=1}^{+\infty} C_n \sin \frac{n\pi}{5} x = \varphi(x) \implies C_n = \frac{2}{5} \int_0^5 \varphi(\xi) \sin \frac{n\pi}{5} \xi d\xi$$

以及 $u_t(0, x) = 0$ 得出 $D_n = 0$. 最后求得

$$\mathbf{u} = \sum_{n=1}^{+\infty} \left(\frac{2}{5} \int_0^5 \varphi(\xi) \sin \frac{n\pi}{5} \xi d\xi \right) \cos \left(\sqrt{\frac{4n^2\pi^2}{25} + 3} \right) \mathbf{t} \cdot \sin \frac{n\pi}{5} \mathbf{x}$$

.....(14分)

五)

解: (1) 由于 z 方向无限长并且定解条件只与 r 有关, 故借助柱坐标方程化为:

$$u_t = a^2 \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right]$$

作分离变量, 令 $u = T(t)H(r)$, 得到

$$\frac{T'}{a^2 T} = \frac{H'' + \frac{1}{r} H'}{H} = -\lambda$$

结合边界条件, 得零阶 Bessel 方程固有值问题:

$$\begin{cases} r^2 H'' + r H' + \lambda r^2 H = 0 \\ H(0) \text{ 有界}, H(R) = 0 \end{cases}$$

和方程

$$T' + \lambda a^2 T = 0.$$

解固有值问题得到: 固有值: $\lambda_n = \omega_n^2$, 固有函数 $J_0(\omega_n r)$, 而 ω_n 是 $J_0(\omega R) = 0$ 的第 n 个正根. 相应地: $T_n(t) = e^{-a^2 \omega_n^2 t}$. 设

$$u(t, r) = \sum_{n=1}^{+\infty} C_n e^{-a^2 \omega_n^2 t} J_0(\omega_n r)$$

.....(7分)

再由初值条件:

$$u|_{t=0} = \sum_{n=1}^{+\infty} C_n J_0(\omega_n r) = R^2 - r^2$$

根据 Bessel 函数系数确定公式

$$C_n = \frac{\int_0^R r(R^2 - r^2) J_0(\omega_n r) dr}{N_{01n}^2} = \frac{1}{N_{01n}^2} \frac{1}{\omega_n^2} \int_0^{\omega_n R} t \left(R^2 - \frac{t^2}{\omega_n^2} \right) J_0(t) dt$$

$$\begin{aligned}
&= \frac{1}{N_{01n}^2 \omega_n^2} \left[\left(R^2 - \frac{t^2}{\omega_n^2} \right) t J_1(t) \Big|_0^{\omega_n R} + \frac{2}{\omega_n^2} \int_0^{\omega_n R} t^2 J_1(t) dt \right] \\
&= \frac{2}{R^2 J_1^2(\omega_n R) \omega_n^2} \cdot \frac{2}{\omega_n^2} \cdot \omega_n^2 R^2 J_2(\omega_n R) = \frac{4 J_2(\omega_n R)}{\omega_n^2 J_1^2(\omega_n R)} = \frac{8}{\omega_n^3 R J_1(\omega_n R)}
\end{aligned}$$

所以得到解

$$u = \sum_{n=1}^{+\infty} \frac{8}{\omega_n^3 R J_1(\omega_n R)} e^{-a^2 \omega_n^2 t} J_0(\omega_n r) \dots\dots\dots(12分)$$

2) 当 $u_1 = 1$ 时, 作变换 $V = u - u_1$, 则 V 的定解问题同情形(1), 因此这时

$$u = u_1 + V = 1 + \sum_{n=1}^{+\infty} \frac{8}{\omega_n^3 R J_1(\omega_n R)} e^{-a^2 \omega_n^2 t} J_0(\omega_n r) \dots\dots\dots(14分)$$

六)

解: (1) 由于是球内问题, 因此

$$u(r, \theta) = \sum_{n=0}^{+\infty} A_n r^n P_n(\cos \theta). \dots\dots\dots(4分)$$

而

$$u|_{r=2} = \sum_{n=0}^{+\infty} A_n 2^n P_n(\cos \theta) = 1 + \cos^2 \theta \implies \sum_{n=0}^{+\infty} A_n 2^n P_n(x) = 1 + x^2.$$

比较得到: $A_0 P_0(x) + 4A_2 P_2(x) = 1 + x^2 \implies A_0 + 4A_2 \times \frac{1}{2}(3x^2 - 1) = 1 + x^2$ 比较系数, 易得: $A_0 = \frac{4}{3}, A_2 = \frac{1}{6}$. 这样

$$\mathbf{u}(\mathbf{r}, \theta) = \frac{4}{3} + \frac{1}{6} \mathbf{r}^2 \mathbf{P}_2(\cos \theta), \quad \left(\text{或 } \mathbf{u}(\mathbf{r}, \theta) = \frac{4}{3} + \mathbf{r}^2 \left(\frac{1}{4} \cos^2 \theta - \frac{1}{12} \right) \right). \dots\dots\dots(8分)$$

(2) 同样

$$u(r, \theta) = \sum_{n=0}^{+\infty} A_n r^n P_n(\cos \theta).$$

而

$$u|_{r=2} = \sum_{n=0}^{+\infty} A_n 2^n P_n(\cos \theta) = f(\theta) = \begin{cases} 4, & 0 \leq \theta \leq \alpha, \\ 0, & \alpha < \theta \leq \pi. \end{cases}$$

因此

$$A_0 = \frac{1}{2} \int_0^\pi f(\theta) P_0(\cos \theta) \sin \theta d\theta = \frac{4}{2} \int_0^\alpha P_0(\cos \theta) \sin \theta d\theta = 2 \int_0^\alpha \sin \theta d\theta = 2(1 - \cos \alpha).$$

$n \geq 1$ 时,

$$\begin{aligned} A_n 2^n &= \frac{2n+1}{2} \int_0^\pi f(\theta) P_n(\cos \theta) \sin \theta d\theta = \frac{2n+1}{2} \int_0^\alpha 4P_n(\cos \theta) \sin \theta d\theta \\ &= 2 \int_{\cos \alpha}^1 (2n+1)P_n(x) dx = 2 \int_{\cos \alpha}^1 (P'_{n+1}(x) - P'_{n-1}(x)) dx \\ &= 2(P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)), \end{aligned}$$

即

$$A_n = \frac{P_{n-1}(\cos \alpha) - P_{n+1}(\cos \alpha)}{2^{n-1}}, \quad n \geq 1$$

因此

$$\mathbf{u}(\mathbf{r}, \theta) = \mathbf{2}(\mathbf{1} - \cos \alpha) + \sum_{\mathbf{n}=1}^{+\infty} \left[\frac{\mathbf{P}_{\mathbf{n}-1}(\cos \alpha) - \mathbf{P}_{\mathbf{n}+1}(\cos \alpha)}{\mathbf{2}^{\mathbf{n}-1}} \right] \mathbf{r}^{\mathbf{n}} \mathbf{P}_{\mathbf{n}}(\cos \theta).$$

.....(14分)

七 解: 记 $\delta(x-2) + 3e^{-x^2} = \varphi(x)$, 并令 $\hat{u}(t, \lambda) = \int_{-\infty}^{+\infty} u(t, x) e^{-i\lambda x} dx$, 再作Fourier变换得:

$$\begin{cases} \hat{u}_t = -\lambda^2 \hat{u} + 20i\lambda \hat{u} + \hat{u}, \\ \hat{u}|_{t=0} = \hat{\varphi}(\lambda) \end{cases}$$

解得:

$$\hat{u} = \hat{\varphi}(\lambda) e^{(-\lambda^2 + 20i\lambda + 1)t}.$$

.....(5分)

因此

$$u(t, x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{\varphi}(\lambda) e^{(-\lambda^2 + 20i\lambda + 1)t} e^{i\lambda x} d\lambda = e^t \times \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{\varphi}(\lambda) e^{-\lambda^2 t} e^{i\lambda(x+20t)} d\lambda$$

记 $h(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{\varphi}(\lambda) e^{-\lambda^2 t} e^{i\lambda x} dx$, 则由上式知: $u(t, x) = e^t h(x + 20t)$, 而

$$h(x) = F^{-1}[\hat{\varphi}(\lambda) e^{-\lambda^2 t}] = \varphi(x) * \frac{1}{2\sqrt{\pi t}} \exp\left\{-\frac{x^2}{4t}\right\} = \frac{1}{2\sqrt{\pi t}} \int_{-\infty}^{+\infty} \varphi(\xi) \exp\left\{-\frac{(x-\xi)^2}{4t}\right\} d\xi$$

而

$$\delta(x-2) * \frac{1}{2\sqrt{\pi t}} \exp\left(-\frac{x^2}{4t}\right) = \frac{1}{2\sqrt{\pi t}} \exp\left(-\frac{(x-2)^2}{4t}\right)$$

$$e^{-x^2} * \frac{1}{2\sqrt{\pi t}} \exp\left\{-\frac{x^2}{4t}\right\} = \frac{1}{\sqrt{1+4t}} e^{-\frac{1}{1+4t}x^2}$$

这样

$$u(\mathbf{t}, \mathbf{x}) = \mathbf{e}^t \mathbf{h}(\mathbf{x} + 20\mathbf{t}) = \frac{\mathbf{e}^t}{2\sqrt{\pi t}} \exp\left(-\frac{(\mathbf{x} - 2 + 20\mathbf{t})^2}{4t}\right) + \frac{3\mathbf{e}^t}{\sqrt{1+4t}} e^{-\frac{1}{1+4t}(\mathbf{x}+20\mathbf{t})^2}$$

.....(12分)

八) 解 1) 记 $M_0 = (\xi, \eta, \zeta)$, 利用镜像法, 在 $M_0 = (\xi, \eta, \zeta)$ 放 $+\epsilon$ 点电荷, 在 $M_1 = (-\xi, \eta, \zeta)$ 放 $-\epsilon$ 点电荷, 在 $M_2 = (\xi, -\eta, \zeta)$ 放 $-\epsilon$ 点电荷, $M_3 = (-\xi, -\eta, \zeta)$ 放 $+\epsilon$ 点电荷. 产生的电场的势函数叠加即为格林函数:

$$G = \frac{1}{4\pi r(M, M_0)} - \frac{1}{4\pi r(M, M_1)} - \frac{1}{4\pi r(M, M_2)} + \frac{1}{4\pi r(M, M_3)}$$

其中

$$r(M, M_0) = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}, \quad r(M, M_1) = \sqrt{(x + \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}$$

$$r(M, M_2) = \sqrt{(x - \xi)^2 + (y + \eta)^2 + (z - \zeta)^2}, \quad r(M, M_3) = \sqrt{(x + \xi)^2 + (y + \eta)^2 + (z - \zeta)^2}$$

.....(7分)

2) 对于区域在 $y = 0$ 的边界, 其外法方向 $\vec{n}_0 = (0, -1, 0)$, 则

$$\frac{\partial G}{\partial \vec{n}_0} \Big|_{\eta=0} = -\frac{\partial G}{\partial \eta} \Big|_{\eta=0}$$

$$\frac{\partial G}{\partial \eta} \Big|_{\eta=0} = \frac{1}{4\pi} \left(\frac{y - \eta}{[(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} + \frac{y + \eta}{[(x - \xi)^2 + (y + \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right) \Big|_{\eta=0}$$

$$+ \frac{1}{4\pi} \left(-\frac{y - \eta}{[(x + \xi)^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} - \frac{y + \eta}{[(x + \xi)^2 + (y + \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right) \Big|_{\eta=0}$$

$$= \frac{1}{2\pi} \left(\frac{y}{[(x - \xi)^2 + y^2 + (z - \zeta)^2]^{\frac{3}{2}}} - \frac{y}{[(x + \xi)^2 + y^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right)$$

同理

$$\frac{\partial G}{\partial \xi} \Big|_{\xi=0} = \frac{1}{2\pi} \left(\frac{x}{[x^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} - \frac{x}{[x^2 + (y + \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right).$$

最后

$$u(x, y, z) = \int_{\substack{\xi=0 \\ (\eta>0)}} g(\eta, \zeta) \frac{\partial G}{\partial \xi} dS_0 + \int_{\substack{\eta=0 \\ (\xi>0)}} \varphi(\xi, \zeta) \frac{\partial G}{\partial \eta} dS_0$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\zeta \int_0^{+\infty} \left(\frac{y}{[(x - \xi)^2 + y^2 + (z - \zeta)^2]^{\frac{3}{2}}} - \frac{y}{[(x + \xi)^2 + y^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right) \varphi(\xi, \zeta) d\xi$$

$$+ \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\zeta \int_0^{+\infty} \left(\frac{x}{[x^2 + (y - \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} - \frac{x}{[x^2 + (y + \eta)^2 + (z - \zeta)^2]^{\frac{3}{2}}} \right) g(\eta, \zeta) d\eta$$

.....(14分)