$$\nabla(fg) = f\nabla g + g\nabla f$$
$$\nabla\cdot(f\boldsymbol{A}) = f\nabla\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\nabla f$$
$$\nabla\times(f\boldsymbol{A}) = f\nabla\times\boldsymbol{A} + \nabla f\times\boldsymbol{A}$$
$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot\nabla\times\boldsymbol{A} - \boldsymbol{A}\cdot\nabla\times\boldsymbol{B}$$
$$\nabla(\boldsymbol{A}\cdot\boldsymbol{B}) = \boldsymbol{A}\times(\nabla\times\boldsymbol{B}) + \boldsymbol{B}\times(\nabla\times\boldsymbol{A}) + (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + (\boldsymbol{B}\cdot\nabla)\boldsymbol{A}$$
$$\nabla\times(\nabla\times\boldsymbol{A}) = \nabla(\nabla\cdot\boldsymbol{A}) - \nabla^2\boldsymbol{A}$$
$$\nabla\cdot\nabla\times\boldsymbol{A} = 0$$
$$\nabla\times(A\times B) = B\cdot\nabla A + (\nabla\cdot B)A - A\cdot\nabla B - (\nabla\cdot A)B$$
$$\nabla\times\nabla f = 0$$
$$I\times A = A\times I$$
$$A\times(\nabla\times B) = (\nabla B)\cdot A - (A\cdot\nabla)B$$
$$(A\times B)\times I = I\times(A\times B) = BA - AB$$
$$\nabla\cdot\boldsymbol{F} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial x^i}(\sqrt{g}f^i) = \frac{1}{D}\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\left(\frac{DF_i}{h_i}\right),$$
$$\nabla\times\boldsymbol{F} = \frac{1}{\sqrt{g}}\frac{\partial f_i}{\partial x_j}\varepsilon_{kji}\boldsymbol{e}_k = \sum_{i,j,k=1}^{3}\frac{h_k}{D}\frac{\partial(h_iF_i)}{\partial x_j}\varepsilon_{kji}\hat{\boldsymbol{e}}_k.$$
$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0. \qquad (7.45)$$
$$\varphi(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta).$$
$$\int_{-1}^{1}P_l(x)P_{l'}(x)\mathrm{d}x = \int_0^{\pi}P_l(\cos\theta)P_{l'}(\cos\theta)\sin\theta\mathrm{d}\theta = \begin{cases}0, & \text{if } l'\neq l,\\ \frac{2}{2l+1}, & \text{if } l' = l.\end{cases}$$
$$\boldsymbol{E}'_\parallel = \boldsymbol{E}_\parallel$$
$$\boldsymbol{E}'_\perp = \gamma(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})_\perp$$
$$\boldsymbol{B}'_\parallel = \boldsymbol{B}_\parallel$$
$$\boldsymbol{B}'_\perp = \gamma\left(\boldsymbol{B} - \frac{\boldsymbol{v}}{c^2}\times\boldsymbol{E}\right)_\perp$$
$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int(r')^nP_n(\cos\alpha)\rho(\mathbf{r}')\,d\tau',$$
$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial\phi^2} = 0.$$
$$V(s,\phi) = a_0 + b_0\ln s + \sum_{k=1}^{\infty}\left[s^k(a_k\cos k\phi + b_k\sin k\phi) + s^{-k}(c_k\cos k\phi + d_k\sin k\phi)\right]$$
$I = \int(\nabla V_1)\cdot(\nabla V_2)\,d\tau$. But $\nabla\cdot(V_1\nabla V_2) = (\nabla V_1)\cdot(\nabla V_2) + V_1(\nabla^2 V_2)$, so
$$I = \int\nabla\cdot(V_1\nabla V_2)\,d\tau - \int V_1(\nabla^2V_2) = \oint_S V_1(\nabla V_2)\cdot d\mathbf{a} + \frac{1}{\epsilon_0}\int V_1\rho_2\,d\tau.$$
$$\int\nabla\cdot\mathsf{T}\mathrm{d}V = \oint\mathrm{d}\boldsymbol{S}\cdot\mathsf{T}' = \oint\mathsf{T}\cdot\mathrm{d}\boldsymbol{S}$$

The *total* electromagnetic force on the charges in $\mathcal{V}$ (Eq. 13) is
$$\mathbf{F} = \oint_S \overleftrightarrow{\mathbf{T}}\cdot d\mathbf{a} - \epsilon_0\mu_0\frac{d}{dt}\int_{\mathcal{V}}\mathbf{S}\,d\tau.$$
$$P_0(x) = 1,$$
$$P_1(x) = x,$$
$$P_2(x) = (3x^2-1)/2,$$
$$P_3(x) = (5x^3-3x)/2,$$
$$P_4(x) = (35x^4-30x^2+3)/8.$$
$$P_l(x) \equiv \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l(x^2-1)^l$$
$$\eta^{\alpha\beta} = diag(-1,1,1,1)$$
$$\int_V \mathrm{d}^3\boldsymbol{x}(\nabla\delta\phi\cdot\nabla\delta\phi + \delta\phi\nabla^2\delta\phi) = \oint_\Sigma\mathrm{d}\boldsymbol{S}\cdot(\delta\phi\nabla\delta\phi)$$
$$T^{\alpha\beta} = \frac{1}{\mu_o}[-F^{\alpha\gamma}F_\gamma{}^\beta - \frac{1}{4}g^{\alpha\beta}F_{\gamma\delta}F^{\gamma\delta}].$$
$$\sigma_{ij} = \epsilon_o E_iE_j + \frac{1}{\mu_0}B_iB_j - \frac{1}{2}\left(\epsilon_o E^2 + \frac{1}{\mu_0}B^2\right)\delta_{ij}.$$
$$F^{\alpha\beta} = \begin{bmatrix}0 & -E_x/c & -E_y/c & -E_z/c\\ E_x/c & 0 & -B_z & B_y\\ E_y/c & B_z & 0 & -B_x\\ E_z/c & -B_y & B_x & 0\end{bmatrix}$$
$$\begin{bmatrix}\frac{1}{2}(\epsilon_o E^2 + \frac{1}{\mu_0}B^2) & S_x & S_y & S_z\\ S_x & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz}\\ S_y & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz}\\ S_z & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz}\end{bmatrix}$$
$$c^2B^2 - E^2 = \text{inv} \qquad F_{\alpha\beta}F^{\alpha\beta}$$
$$E_x = E'_x, \qquad E_y = \frac{E'_y + VB'_z}{\sqrt{1-V^2/c^2}} \qquad E_z = \frac{E'_z - VB'_y}{\sqrt{1-V^2/c^2}}$$
$$B_x = B'_x, \qquad B_y = \frac{B'_y - (V/c^2)E'_z}{\sqrt{1-V^2/c^2}} \qquad B_z = \frac{B'_z + (V/c^2)E'_y}{\sqrt{1-V^2/c^2}}.$$
$$E = \frac{qR}{4\pi\epsilon_0 R^3}\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}}.$$
$$\phi/c = \gamma(\phi'/c + \beta A'_x) = \frac{\phi'(r')/c}{\sqrt{1-v^2/c^2}},$$
$$A_x = \gamma(A'_x + \beta\phi'/c) = \frac{(v/c^2)\phi'(r')}{\sqrt{1-v^2/c^2}},$$
$$\nabla\cdot\boldsymbol{E} = \rho/\epsilon_0,$$
$$\nabla\cdot\boldsymbol{B} = 0, \qquad \boldsymbol{B} = \frac{\boldsymbol{v}}{c^2}\times\boldsymbol{E}. \qquad \equiv\frac{q}{4\pi\epsilon_0 r^*}$$
$$A_x = \frac{v}{c^2}\phi,$$
$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \qquad \mathbf{E}\cdot\mathbf{B} = \text{inv}$$
$$\boldsymbol{E}_\parallel = \frac{1}{\gamma^2}\boldsymbol{E}_{0,\parallel}$$
$$\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{j} + \mu_0\epsilon_0\frac{\partial\boldsymbol{E}}{\partial t}. \qquad r^* = \sqrt{(x-vt)^2 + (1-v^2/c^2)(y^2+z^2)}$$
$$E\times(\nabla\times E) = (\nabla\cdot E)E - \nabla\cdot\left(EE - \frac{1}{2}E^2\mathsf{I}\right) \qquad E_\perp = \gamma E_{0,\perp},$$
$$\hat{T}^{00} = \frac{1}{2}\left(\epsilon_0E^2 + \frac{1}{\mu_0}B^2\right),$$
$$\hat{T}^{0i} = \frac{1}{c}\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B},$$
$$\hat{T}^{ij} = -\left[\frac{1}{\mu_0}\boldsymbol{BB} + \epsilon_0\boldsymbol{EE} - \left(\frac{1}{2}\mu_0B^2 + \frac{1}{2}\epsilon_0E^2\right)\mathsf{I}\right].$$
$$\boldsymbol{S} = \frac{1}{\epsilon_0\mu_0}\boldsymbol{g} = c^2\boldsymbol{g}.$$
$$\frac{\partial g}{\partial t} + \nabla\cdot\mathsf{G} = -\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{p}_m \text{ per unit volume})$$
Field momentum flux tensor

$$\phi(\boldsymbol{x}) = \frac{1}{4\pi\epsilon_0}\left(Q - \boldsymbol{p}\cdot\nabla + \frac{1}{6}\boldsymbol{\mathcal{D}}:\nabla\nabla + \dots\right)\frac{1}{|\boldsymbol{x}|}.$$

$$D_{ij} = \sum q(3x'_i x'_j - r'^2\delta_{ij})$$

$$\nabla\nabla\frac{1}{r} = \frac{3\hat{r}\hat{r} - \overleftrightarrow{I}}{r^3} - \frac{4\pi\delta(\vec{r})\,\overleftrightarrow{I}}{3}$$

$$\boldsymbol{E}^{(1)} = \frac{3(\boldsymbol{n}\cdot\boldsymbol{p})\boldsymbol{n} - \boldsymbol{p}}{4\pi\epsilon_0 r^3}.$$

$$\phi(\boldsymbol{x}) = \frac{1}{4\pi\epsilon_0}\left(Q - \boldsymbol{p}\cdot\nabla + \frac{1}{6}\boldsymbol{\mathcal{D}}:\nabla\nabla + \dots\right)\frac{1}{|\boldsymbol{x}|}.$$

$$= Q\phi_{ex}(0) + \boldsymbol{p}\cdot\nabla\phi_{ex}(0) + \frac{1}{6}\boldsymbol{\mathcal{D}}:\nabla\nabla\phi_{ex}(0) + \dots$$

$$\nabla\cdot\boldsymbol{f} = \frac{1}{r}\frac{\partial}{\partial r}(rf_r) + \frac{1}{r}\frac{\partial f_\theta}{\partial\theta} + \frac{\partial f_z}{\partial z}$$

$$\nabla\times\boldsymbol{f} = \left(\frac{1}{r}\frac{\partial f_z}{\partial\theta} - \frac{\partial f_\theta}{\partial z}\right)\boldsymbol{e}_r + \left(\frac{\partial f_r}{\partial z} - \frac{\partial f_z}{\partial r}\right)\boldsymbol{e}_\theta + \left[\frac{1}{r}\frac{\partial}{\partial r}(rf_\theta) - \frac{1}{r}\frac{\partial f_r}{\partial\theta}\right]\boldsymbol{e}_z,$$

$$\nabla\cdot\boldsymbol{f} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\theta) + \frac{1}{r\sin\theta}\frac{\partial f_\phi}{\partial\phi}. \quad (\text{I}$$

$$\nabla\times\boldsymbol{f} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta f_\phi) - \frac{\partial f_\theta}{\partial\phi}\right]\boldsymbol{e}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial f_r}{\partial\phi} - \frac{\partial}{\partial r}(rf_\phi)\right]\boldsymbol{e}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rf_\theta) - \frac{\partial f_r}{\partial\theta}\right]\boldsymbol{e}_\phi, \quad (\text{I}$$

$$\boldsymbol{\nabla}(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{\nabla}\boldsymbol{A}\times\boldsymbol{B} - \boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A}$$

$$\boldsymbol{\nabla}\times(\boldsymbol{A}\boldsymbol{B}) = (\boldsymbol{\nabla}\times\boldsymbol{A})\boldsymbol{B} - \boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B}$$

$$\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B} - (\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B})^{\mathrm{T}} = [(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A}]\times\boldsymbol{I}$$

$$\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A} - (\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A})^{\mathrm{T}} = [(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B}]\times\boldsymbol{I}$$

$$(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A} = -(\boldsymbol{A}\times\boldsymbol{\nabla})\times\boldsymbol{B}$$

$$(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B} = \boldsymbol{A}\times(\boldsymbol{\nabla}\times\boldsymbol{B}) - (\boldsymbol{A}\times\boldsymbol{\nabla})\times\boldsymbol{B}$$

$$(\boldsymbol{A}\times\boldsymbol{\nabla})\cdot\boldsymbol{B} = \boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

$$\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B} + (\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A})^{\mathrm{T}} = [\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})]\boldsymbol{I} - (\boldsymbol{\nabla}\times\boldsymbol{B})\boldsymbol{A}$$

$$\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A} + (\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B})^{\mathrm{T}} = [\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})]\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{\nabla}\times\boldsymbol{B})$$

$$\begin{aligned}\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B} + \boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A} &= \boldsymbol{I}\times[(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A}] \\ &\quad + [\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})]\boldsymbol{I} - \boldsymbol{A}(\boldsymbol{\nabla}\times\boldsymbol{B}) \\ &= \boldsymbol{I}\times[(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{A} - \boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B}] \\ &\quad + [\boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})]\boldsymbol{I} - (\boldsymbol{\nabla}\times\boldsymbol{B})\boldsymbol{A}\end{aligned}$$

$$(\boldsymbol{A}\times\boldsymbol{C})\cdot\boldsymbol{F} = \boldsymbol{A}\cdot(\boldsymbol{C}\times\boldsymbol{F}) = -\boldsymbol{C}\cdot(\boldsymbol{A}\times\boldsymbol{F})$$

$$\boldsymbol{F}\cdot(\boldsymbol{A}\times\boldsymbol{C}) = (\boldsymbol{F}\times\boldsymbol{A})\cdot\boldsymbol{C} = -(\boldsymbol{F}\times\boldsymbol{C})\cdot\boldsymbol{A}$$

$$\boldsymbol{A}\cdot\boldsymbol{F}\times\boldsymbol{C} = (\boldsymbol{A}\cdot\boldsymbol{F})\times\boldsymbol{C} = \boldsymbol{A}\cdot(\boldsymbol{F}\times\boldsymbol{C}) = -\boldsymbol{C}\times(\boldsymbol{A}\cdot\boldsymbol{F})$$

$$\boldsymbol{A}\times\boldsymbol{F}\cdot\boldsymbol{C} = \boldsymbol{A}\times(\boldsymbol{F}\cdot\boldsymbol{C}) = (\boldsymbol{A}\times\boldsymbol{F})\cdot\boldsymbol{C} = -(\boldsymbol{F}\cdot\boldsymbol{C})\times\boldsymbol{A}$$

$$\boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{C} + \boldsymbol{C}\times\boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A} = \boldsymbol{C}\times[\boldsymbol{A}\times(\boldsymbol{\nabla}\times\boldsymbol{B})]$$

$$\boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{C} - \boldsymbol{C}\cdot\boldsymbol{\nabla}\boldsymbol{B}\times\boldsymbol{A} = [(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{I} - \boldsymbol{\nabla}\boldsymbol{B}]\cdot(\boldsymbol{A}\times\boldsymbol{C})$$

$$\boldsymbol{A}\times\boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{C} - \boldsymbol{C}\times\boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A} = (\boldsymbol{A}\times\boldsymbol{C})\cdot[(\boldsymbol{\nabla}\cdot\boldsymbol{B})\boldsymbol{I} - \boldsymbol{\nabla}\boldsymbol{B}]$$

$$\boldsymbol{A}\cdot\boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{C} - \boldsymbol{C}\cdot\boldsymbol{\nabla}\boldsymbol{B}\cdot\boldsymbol{A} = (\boldsymbol{A}\times\boldsymbol{C})\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$$

$$\boldsymbol{\nabla}\boldsymbol{R} = \boldsymbol{I}\,, \qquad \boldsymbol{R} = \text{position vector}\,.$$

$$\mathcal{S} = \int dt d^3\boldsymbol{x}\left(\frac{1}{2}\epsilon_0 E^2 - \frac{1}{2\mu_0}B^2 - \rho\phi + \boldsymbol{j}\cdot\boldsymbol{A}\right)$$

$$\delta_A\mathcal{S} = \int d^3\boldsymbol{x}dt\left[\delta\boldsymbol{A}\cdot\left(\boldsymbol{j} - \frac{1}{\mu_0}\nabla\times\boldsymbol{B} + \epsilon_0\partial_t\boldsymbol{E}\right) - \partial_t(\epsilon_0\boldsymbol{E}\cdot\delta\boldsymbol{A}) + \nabla\cdot\left(\frac{1}{\mu_0}\boldsymbol{B}\times\delta\boldsymbol{A}\right)\right]$$

$$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}} + \frac{e}{c}\boldsymbol{A}\cdot\boldsymbol{v} - e\varphi.$$

$$U = \frac{1}{2}\int\rho\phi\,\mathrm{d}V = \frac{1}{2}\sum_a q_a\Phi_a$$

$$p_x = \frac{p'_x + \frac{V}{c^2}\mathscr{E}'}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad p_y = p'_y, \quad p_z = p'_z, \quad \mathscr{E} = \frac{\mathscr{E}' + Vp'_x}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\nabla^2\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\mu_0\boldsymbol{J}$$

$$E^2 - p^2c^2 = m^2c^4.$$

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

$$\boldsymbol{\nabla}^2\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} - \boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\boldsymbol{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right) = -\mu_0\boldsymbol{J}$$

$$\boldsymbol{\nabla}^2\varphi + \frac{\partial}{\partial t}(\boldsymbol{\nabla}\cdot\boldsymbol{A}) = -\frac{\rho}{\varepsilon_0}$$

$$\boldsymbol{A} = \gamma\frac{\boldsymbol{v}_e}{c^2}\frac{1}{4\pi\epsilon_0}\frac{e}{\tilde{r}},$$

$$\begin{cases}\nabla\cdot D = \rho_f, \\ \nabla\times E = -\partial_t B, \\ \nabla\cdot B = 0, \\ \nabla\times H = j_f + \partial_t D.\end{cases} \qquad \begin{cases}\boldsymbol{D} = \epsilon\boldsymbol{E}, \\ \boldsymbol{B} = \mu\boldsymbol{H}, \\ \boldsymbol{j}_f = \sigma\boldsymbol{E}.\end{cases}$$

$$\phi = \gamma\frac{1}{4\pi\epsilon_0}\frac{e}{\tilde{r}},$$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\frac{e\boldsymbol{v}_e}{r - \frac{\boldsymbol{v}_e}{c}\cdot\boldsymbol{r}},$$

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{e}{r - \frac{\boldsymbol{v}_e}{c}\cdot\boldsymbol{r}}.$$

$$\iint_D(u\nabla^2 v - v\nabla^2 u)\,\mathrm{d}\sigma = \int_C\left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right)\mathrm{d}l$$

$$u(\boldsymbol{r}) = \iiint_\Omega G(\boldsymbol{r},\boldsymbol{r}_0)f(\boldsymbol{r}_0)\mathrm{d}V_0 + \iint_\Gamma\left[G(\boldsymbol{r},\boldsymbol{r}_0)\frac{\partial u(\boldsymbol{r}_0)}{\partial n_0} - u(\boldsymbol{r}_0)\frac{\partial G(\boldsymbol{r},\boldsymbol{r}_0)}{\partial n_0}\right]\mathrm{d}S_0$$

$$\nabla^2 u(\boldsymbol{r}) = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = -f(\boldsymbol{r})$$

$$u(\boldsymbol{r})|_\Gamma = g(\boldsymbol{r}) \quad \left.\frac{\partial u}{\partial n}\right|_\Gamma = h(\boldsymbol{r})$$

$$\nabla^2 G = -\delta(\boldsymbol{r} - \boldsymbol{r}_0) \qquad \left.\frac{\partial G}{\partial n}\right|_\Gamma = 0$$

$$\begin{cases}\nabla^2 G = -\delta(\boldsymbol{r} - \boldsymbol{r}_0) \\ G|_\Gamma = 0\end{cases}$$

$$\oiint_{\mathbb{S}}\mathbf{E}\cdot\mathrm{d}\mathbf{s} = \frac{Q}{\varepsilon_0}$$

$$\oiint_{\mathbb{S}}\mathbf{B}\cdot\mathrm{d}\mathbf{s} = 0$$

$$\oint_{\mathbb{L}}\mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}\Phi_{\mathbf{B}}}{\mathrm{d}t}$$

$$\oint_{\mathbb{L}}\mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \mu_0 I + \mu_0\varepsilon_0\frac{\mathrm{d}\Phi_{\mathbf{E}}}{\mathrm{d}t}$$

$$\partial_\beta F^{\alpha\beta} + \Gamma^\alpha{}_{\omega\beta}F^{\omega\beta} + \Gamma^\beta{}_{\omega\beta}F^{\alpha\omega} = \mu_0 j^\alpha$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\gamma F_{\alpha\beta} + \partial_\beta F_{\gamma\alpha} = 0.$$

$$k^\mu = \begin{bmatrix}\boldsymbol{k} \\ \frac{\omega}{c}\end{bmatrix}$$

$$\frac{\partial F^{\alpha\beta}}{\partial x^\beta} = \mu_0 J^\alpha$$

$$-\nabla\cdot\left(\boldsymbol{E}\times\frac{\boldsymbol{B}}{\mu_0}\right) = \partial_t\left(\frac{B^2}{2\mu_0} + \frac{1}{2}\epsilon_0 E^2\right) + (\boldsymbol{j}_f + \boldsymbol{j}_p + \boldsymbol{j}_M)\cdot\boldsymbol{E}$$

$$-\oint\boldsymbol{S}\cdot\mathrm{d}\boldsymbol{\sigma} = \int\boldsymbol{f}\cdot\boldsymbol{v}\,\mathrm{d}V + \frac{\mathrm{d}}{\mathrm{d}t}\int w\,\mathrm{d}V$$

$$w = \frac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{H}\cdot\boldsymbol{B})$$

$$\boldsymbol{J}\cdot\boldsymbol{E} = -\nabla\cdot(\boldsymbol{E}\times\boldsymbol{H}) - \boldsymbol{E}\cdot\frac{\partial\boldsymbol{D}}{\partial t} - \boldsymbol{H}\cdot\frac{\partial\boldsymbol{B}}{\partial t}$$

$$A^\mu \overset{def}{=} (\varphi, \mathbf{A}). \qquad \Box A^\mu = -\mu_0 j^\mu$$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{j}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}V' \qquad \boldsymbol{B} = \frac{\mu_0}{4\pi}\int \frac{\boldsymbol{j}\times\boldsymbol{R}}{R^3}\mathrm{d}V'$$

$$A^{(1)} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{x}}{r^3}.$$

$$\boldsymbol{m} = \frac{1}{2}\int \boldsymbol{x}'\times\boldsymbol{j}\,\mathrm{d}V'.$$

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{3(\boldsymbol{n}\cdot\boldsymbol{m})\boldsymbol{n}-\boldsymbol{m}}{r^3} \qquad \boldsymbol{F}^{(1)} = \nabla(\boldsymbol{m}\cdot\boldsymbol{B}).$$

$$\frac{\mathrm{d}\boldsymbol{L}}{\mathrm{d}t} = \frac{q}{2m}\boldsymbol{L}\times\boldsymbol{B}_0 = \boldsymbol{L}\times\boldsymbol{\Omega}_L$$

$$\boldsymbol{E} = \boldsymbol{E}_0 e^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)},$$

$$\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp},$$

$$\boldsymbol{B} = \boldsymbol{n}\times\frac{\boldsymbol{E}}{c} = \sqrt{\mu\epsilon}\frac{\boldsymbol{k}}{k}\times\boldsymbol{E}.$$

$$\boldsymbol{E}_1^{\parallel} = \boldsymbol{E}_2^{\parallel},$$

$$B_1^{\perp} = B_2^{\perp},$$

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k'}{k} = \frac{n'}{n}.$$

$$\frac{\boldsymbol{B}_1^{\parallel}}{\mu_1} = \frac{\boldsymbol{B}_2^{\parallel}}{\mu_2}. \qquad k\sin\theta_i = k''\sin\theta_{r'} = k'\sin\theta_r.$$

$$\frac{E_0'}{E_0} = \frac{2n\cos\theta_i}{n\cos\theta_i + (\mu/\mu')\sqrt{n'^2 - n^2\sin^2\theta_i}},$$

$$E_0 + E_0'' - E_0' = 0,$$

$$\sqrt{\frac{\epsilon}{\mu}}(E_0 - E_0'')\cos\theta_i - \sqrt{\frac{\epsilon'}{\mu'}}E_0'\cos\theta_r = 0.$$

$$\frac{E_0''}{E_0} = \frac{n\cos\theta_i - (\mu/\mu')\sqrt{n'^2 - n^2\sin^2\theta_i}}{n\cos\theta_i + (\mu/\mu')\sqrt{n'^2 - n^2\sin^2\theta_i}},$$

$$\cos\theta_i(E_0 - E_0'') - \cos\theta_r E_0' = 0, \qquad \frac{E_0'}{E_0} = \frac{2nn'\cos\theta_i}{(\mu/\mu')n'^2\cos\theta_i + n\sqrt{n'^2 - n^2\sin^2\theta_i}},$$

$$\sqrt{\frac{\epsilon}{\mu}}(E_0 + E_0'') - \sqrt{\frac{\epsilon'}{\mu'}}E_0' = 0.$$

$$\theta_B = \tan^{-1}\left(\frac{n'}{n}\right). \qquad \frac{E_0''}{E_0} = \frac{(\mu/\mu')n'^2\cos\theta_i - n\sqrt{n'^2 - n^2\sin^2\theta_i}}{(\mu/\mu')n'^2\cos\theta_i + n\sqrt{n'^2 - n^2\sin^2\theta_i}}.$$

$$\boldsymbol{k}' = k'\sin\theta_r\boldsymbol{e}_x + \mathrm{i}k'\sqrt{\left(\frac{\sin\theta_i}{\sin\theta_0}\right)^2 - 1}\,\boldsymbol{e}_z = k_x'\boldsymbol{e}_x + \mathrm{i}k_{zi}'\boldsymbol{e}_z.$$

$$e^{\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{x}} = e^{-k_{zi}'z}e^{\mathrm{i}k_x'x}. \qquad \langle\boldsymbol{S}\rangle\cdot\boldsymbol{e}_z = 0.$$

$$n = 1 + A\left(1 + \frac{B}{\lambda^2}\right). \qquad \boldsymbol{P} = \frac{Ne^2}{m}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - \mathrm{i}\omega\gamma_j}\boldsymbol{E} = \chi_e\epsilon_0\boldsymbol{E}.$$

$$\epsilon_r = 1 + \chi_e = 1 + \frac{Ne^2}{m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - \mathrm{i}\omega\gamma_j}.$$

$$n_r = 1 + \frac{Ne^2}{2m\epsilon_0}\sum_j \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + (\omega\gamma_j)^2}$$

$$\sigma = \frac{Ne^2 f_0}{m(\gamma_0 - \mathrm{i}\omega)}. \qquad n_i = \frac{Ne^2\omega}{2m\epsilon_0}\sum_j \frac{f_j\gamma_j}{(\omega_j^2 - \omega^2)^2 + (\omega\gamma_j)^2}$$

$$\epsilon_r(\omega) = 1 + \frac{Ne^2}{m\epsilon_0}\sum_{j\neq 0}\frac{f_j}{\omega_j^2 - \omega^2 - \mathrm{i}\omega\gamma_j} + \frac{Ne^2}{m\epsilon_0}\frac{f_0}{\omega_0^2 - \omega^2 - \mathrm{i}\omega\gamma_0} \approx \epsilon_e + \mathrm{i}\frac{Ne^2 f_0}{m\epsilon_0\omega(\gamma_0 - \mathrm{i}\omega)}$$

$$\nabla\times H = \frac{\partial D}{\partial t} = -\mathrm{i}\omega\epsilon_r(\omega)\epsilon_0 E. \qquad \nabla\times H = j_f + \frac{\partial D}{\partial t} = -\mathrm{i}\omega\left(\epsilon_e\epsilon_0 + \mathrm{i}\frac{\sigma}{\omega}\right)E.$$

$$1 - \frac{NZe^2}{m\epsilon_0}\frac{1}{\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}. \quad \omega^2 = \omega_p^2 + c^2k^2. \quad \omega \gg \omega_j$$

$$\rho_f = \rho_{f0}e^{-t/\tau}, \qquad \nabla^2\boldsymbol{E} = \mu\epsilon\frac{\partial^2\boldsymbol{E}}{\partial t^2} + \mu\sigma\frac{\partial\boldsymbol{E}}{\partial t},$$

$$k^2 = \mu\epsilon\omega^2 + \mathrm{i}\mu\sigma\omega. \qquad \nabla^2\boldsymbol{B} = \mu\epsilon\frac{\partial^2\boldsymbol{B}}{\partial t^2} + \mu\sigma\frac{\partial\boldsymbol{B}}{\partial t}.$$

$$k = \beta + \mathrm{i}\alpha/2$$

$\sigma/\epsilon\omega \ll 1$, then

$$\beta = \sqrt{\frac{\mu\epsilon}{2}}\omega\left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} + 1\right]^{1/2} \qquad k \approx \sqrt{\mu\epsilon}\omega + \mathrm{i}\frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}.$$

$$\frac{\alpha}{2} = \sqrt{\frac{\mu\epsilon}{2}}\omega\left[\sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right]^{1/2} \qquad k = (1+\mathrm{i})\sqrt{\mu\sigma\omega/2} \qquad \delta = \frac{2}{\alpha} \approx \sqrt{\frac{2}{\mu\sigma\omega}}.$$

$$\boldsymbol{B} = \sqrt{\frac{\mu\sigma}{\omega}}e^{\mathrm{i}\pi/4}\boldsymbol{e}_k\times\boldsymbol{E}. \qquad \frac{w_B}{w_E} = \frac{B^2/\mu}{\epsilon E^2} = \frac{\sigma}{\epsilon\omega} \gg 1.$$

$$\frac{E_0''}{E_0} = -\frac{1 + \mathrm{i} - \sqrt{2\omega/\mu_0\sigma}/c}{1 + \mathrm{i} + \sqrt{2\omega/\mu_0\sigma}/c}. \qquad R = \left|\frac{E_0''}{E_0}\right|^2 = \frac{\left(1 - \sqrt{2\omega/\mu_0\sigma}/c\right)^2 + 1}{\left(1 + \sqrt{2\omega/\mu_0\sigma}/c\right)^2 + 1} \approx 1 - \frac{2}{c}\sqrt{\frac{2\omega}{\mu_0\sigma}} = 1 - 2\sqrt{\frac{2\omega\epsilon_0}{\sigma}}.$$

$\sigma \to \infty$, and $R \to 1$.

$$E_x = \frac{\mathrm{i}}{\mu\epsilon\omega^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right), \qquad B_z = B_0\cos(m\pi x/a)\cos(n\pi y/b)$$

$$E_y = \frac{\mathrm{i}}{\mu\epsilon\omega^2 - k^2}\left(k\frac{\partial E_z}{\partial y} - \omega\frac{\partial B_z}{\partial x}\right), \qquad k = \sqrt{\mu\epsilon\omega^2 - \pi^2[(m/a)^2 + (n/b)^2]}.$$

$$B_x = \frac{\mathrm{i}}{\mu\epsilon\omega^2 - k^2}\left(k\frac{\partial B_z}{\partial x} - \mu\epsilon\omega\frac{\partial E_z}{\partial y}\right) \qquad v_g = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{1}{\mathrm{d}k/\mathrm{d}\omega} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1 - (\omega_{mn}/\omega)^2}.$$

$$B_y = \frac{\mathrm{i}}{\mu\epsilon\omega^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \mu\epsilon\omega\frac{\partial E_z}{\partial x}\right) \qquad S(\boldsymbol{x}, t') = R - \boldsymbol{\beta}\cdot\boldsymbol{R}$$

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{4\pi\epsilon_0}\int\left[\left(\frac{\rho(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|^3} + \frac{1}{|\mathbf{r}-\mathbf{r}'|^2 c}\frac{\partial\rho(\mathbf{r}',t')}{\partial t}\right)(\mathbf{r}-\mathbf{r}') - \frac{1}{|\mathbf{r}-\mathbf{r}'|c^2}\frac{\partial\mathbf{j}(\mathbf{r}',t')}{\partial t}\right]\mathrm{d}^3\mathbf{r}$$

$$\phi(x,t) = \int\frac{1}{4\pi\epsilon_0 R}\rho\left(x', t - \frac{R}{c}\right)\mathrm{d}V'$$

$$A(x,t) = \int\frac{\mu_0}{4\pi R}j\left(x', t - \frac{R}{c}\right)\mathrm{d}V'. \qquad \mathbf{B}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\mathbf{j}(\mathbf{r}',t')}{|\mathbf{r}-\mathbf{r}'|^3} + \frac{1}{|\mathbf{r}-\mathbf{r}'|^2 c}\frac{\partial\mathbf{j}(\mathbf{r}',t')}{\partial t}\right]\times(\mathbf{r}-\mathbf{r}')\mathrm{d}^3\mathbf{r}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0\gamma^2}\frac{\boldsymbol{R} - R\boldsymbol{\beta}}{S^3} + \frac{q}{4\pi\epsilon_0 c}\frac{\boldsymbol{R}\times\left[(\boldsymbol{R} - R\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]}{S^3},$$

$$\boldsymbol{B} = \boldsymbol{e}_R\times\boldsymbol{E}/c. \qquad \boldsymbol{\mathcal{S}}(\boldsymbol{r},t) = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B} = \frac{1}{\mu_0 c}E^2\boldsymbol{e}_R,$$

$$\frac{\partial t'}{\partial t} = \frac{c}{c - \boldsymbol{R}\cdot\boldsymbol{v}/R} \qquad = \frac{1}{\mu_0 c}\left(\frac{q}{4\pi\epsilon_0 c}\right)^2\frac{\left|\boldsymbol{e}_R\times\left[(\boldsymbol{e}_R - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right|^2}{(1 - \boldsymbol{\beta}\cdot\boldsymbol{e}_R)^6 R^2}\boldsymbol{e}_R$$

$$\nabla t' = -\frac{1}{c}\frac{\boldsymbol{R}}{R}\frac{1}{1 - \boldsymbol{R}\cdot\boldsymbol{v}/Rc} = -\frac{\boldsymbol{R}}{cS} \qquad \frac{\partial S}{\partial t} = -\frac{\boldsymbol{R}\cdot\boldsymbol{v}}{S} - \frac{R}{S}\frac{\boldsymbol{a}\cdot\boldsymbol{R}}{c} + \frac{v^2}{c}\frac{R}{S}.$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2\mu_0 c}{16\pi^2}\frac{\left|\boldsymbol{e}_R\times\left[(\boldsymbol{e}_R - \boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right]\right|^2}{(1 - \boldsymbol{\beta}\cdot\boldsymbol{e}_R)^5}. \qquad \frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0|\ddot{\boldsymbol{p}}|^2}{16\pi^2 c}\sin^2\Theta.$$

$$P = \int\mathrm{d}P = \int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}\mathrm{d}\Theta\frac{\mu_0|\ddot{\boldsymbol{p}}|^2}{16\pi^2 c}\sin^3\Theta = \frac{\mu_0|\ddot{\boldsymbol{p}}|^2}{6\pi c} = \frac{\mu_0 q^2 a^2}{6\pi c}.$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2\mu_0 c}{16\pi^2}\frac{\dot{\beta}^2\sin^2\theta}{(1 - \beta\cos\theta)^5}, \qquad \boldsymbol{P}_{\parallel}\ \frac{q^2\mu_0}{6\pi c}\gamma^6 a^2$$

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \gamma^3 m\dot{\boldsymbol{v}}. \quad P_\parallel = \frac{q^2\mu_0}{6\pi c m^2}\left(\frac{d\boldsymbol{p}}{dt}\right)^2.$$

$$\frac{dP}{d\Omega} = \frac{q^2\mu_0 c}{16\pi^2}\frac{\dot\beta^2[(1-\beta\cos\theta)^2 + (\sin\theta\cos\phi)^2(\beta^2-1)]}{(1-\beta\cos\theta)^5} \qquad \frac{d\boldsymbol{p}}{dt} = \gamma m\dot{\boldsymbol{v}}$$

$$P_\perp = \frac{q^2\mu_0 c\dot\beta^2}{16\pi^2}2\pi\frac{4}{3(1-\beta^2)^2} = \frac{q^2\mu_0}{6\pi c}\dot v^2\gamma^4. \quad P_\perp = \frac{q^2\mu_0}{6\pi c}\frac{\gamma^2}{m^2}\left(\frac{d\boldsymbol{p}}{dt}\right)^2.$$

$$P = P_\parallel + P_\perp = \frac{q^2\mu_0}{6\pi c}\gamma^6 a_\parallel^2 + \frac{q^2\mu_0}{6\pi c}\gamma^4 a_\perp^2$$

$$\boldsymbol{F}_{\text{rad}} = \frac{\mu_0 q^2}{6\pi c}\dot{\boldsymbol{a}}. \quad \phi(\boldsymbol{r},t) = \frac{1}{4\pi\epsilon_0}\int\frac{\rho_0(\boldsymbol{r}')e^{i(kR-\omega t)}}{R}dV',$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int\frac{\boldsymbol{j}_0(\boldsymbol{r}')e^{i(kR-\omega t)}}{R}dV'.$$

$$\nabla\times\boldsymbol{B} = \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} = -i\frac{\omega}{c^2}\boldsymbol{E} = -ik\boldsymbol{E}/c, \qquad E = i\frac{c}{k}\nabla\times B, \text{ and } B = \nabla\times A.$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \boldsymbol{A}(\boldsymbol{r})e^{-i\omega t}$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int\frac{\boldsymbol{j}_0(\boldsymbol{r}')e^{ikR}}{R}dV'. \quad A(r) \approx \frac{\mu_0 e^{ikr}}{4\pi r}\int j_0(r')dV'.$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0 e^{ikr}}{4\pi r}\dot{\boldsymbol{p}}.$$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 e^{ikr}}{4\pi c r}\ddot{p}\sin\theta\boldsymbol{e}_\phi, \quad \ddot{\boldsymbol{p}} = -\omega^2\boldsymbol{p}.$$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\mu_0 e^{ikr}}{4\pi r}\ddot{p}\sin\theta\boldsymbol{e}_\theta. \quad \boldsymbol{B} = \frac{\mu_0 e^{ikr}}{4\pi c r}\ddot{\boldsymbol{p}}\times\boldsymbol{n}.$$

$$\boldsymbol{E} = i\frac{c}{k}\nabla\times\boldsymbol{B} = i\frac{c}{k}ik\boldsymbol{n}\times\boldsymbol{B} = c\boldsymbol{B}\times\boldsymbol{n} = \frac{\mu_0 e^{ikr}}{4\pi r}(\ddot{\boldsymbol{p}}\times\boldsymbol{n})\times\boldsymbol{n}.$$

$$\langle\boldsymbol{S}\rangle = \frac{1}{4\mu_0}(\boldsymbol{E}\times\boldsymbol{B}^* + \boldsymbol{E}^*\times\boldsymbol{B}) = \frac{1}{2\mu_0}\text{Re}(\boldsymbol{E}\times\boldsymbol{B}^*).$$

$$\boldsymbol{E}\times\boldsymbol{B}^* = (c\boldsymbol{B}\times\boldsymbol{n})\times\boldsymbol{B}^* = c\boldsymbol{n}(\boldsymbol{B}\cdot\boldsymbol{B}^*) = c|\boldsymbol{B}|^2\boldsymbol{n}.$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle = \frac{\mu_0}{32\pi^2 c}|\ddot{\boldsymbol{p}}|^2\sin^2\theta = \frac{\mu_0\omega^4}{32\pi^2 c}|\boldsymbol{p}|^2\sin^2\theta \quad \frac{\mu_0\omega^4|p_0|^2}{12\pi c},$$

$$\boldsymbol{m} = \frac{1}{2}\int(\boldsymbol{r}'\times\boldsymbol{j})dV' \quad \left\langle\frac{dP}{d\Omega}\right\rangle = \langle\boldsymbol{S}\rangle\cdot\boldsymbol{n}r^2 = \frac{\mu_0}{32\pi^2 c}|\ddot{\boldsymbol{p}}\times\boldsymbol{n}|^2$$

$$A = -\frac{ik\mu_0 e^{ikr}}{4\pi r}\left(\frac{1}{6}\dot{D} - n\times m\right) \qquad \frac{\boldsymbol{n}}{6}\cdot\frac{d}{dt}\sum q\left(3\boldsymbol{r}'\boldsymbol{r}' - r'^2\mathsf{I}\right)$$

$$\boldsymbol{n}\cdot\frac{1}{6}\dot{\mathsf{D}} \equiv \frac{1}{6}\dot{\boldsymbol{D}},$$

$$B = \nabla\times A = ikn\times\frac{-ik\mu_0 e^{ikr}}{4\pi r}\left(\frac{1}{6}\dot{D} - n\times m\right) \qquad E_m = cB_m\times n = (n\times\ddot{m})\frac{\mu_0 e^{ikr}}{4\pi c r},$$

$$= \frac{k^2\mu_0 e^{ikr}n}{4\pi r}\times\left(\frac{1}{6}\dot{D} - n\times m\right). \qquad E_D = cB_D\times n = (\dddot{D}\times n)\times n\frac{\mu_0 e^{ikr}}{24\pi c r}.$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle_m = \left\langle\frac{c}{\mu_0}|\boldsymbol{B}_m|^2 r^2\right\rangle = \left\langle\frac{c}{\mu_0}\left|\frac{\mu_0(\ddot{\boldsymbol{m}}\times\boldsymbol{n})\times\boldsymbol{n}}{4\pi c^2}\right|^2\right\rangle = \frac{\mu_0|\ddot{\boldsymbol{m}}|^2}{32\pi^2 c^3}\sin^2\theta. \tag{11.151}$$

$$\left\langle\frac{dP}{d\Omega}\right\rangle_D = \left\langle\frac{c}{\mu_0}|\boldsymbol{B}_D|^2 r^2\right\rangle = \left\langle\frac{c}{\mu_0}\left|\frac{\mu_0\dddot{\boldsymbol{D}}\times\boldsymbol{n}}{24\pi c^2}\right|^2\right\rangle = \frac{\mu_0}{1152\pi^2 c^3}|\dddot{\boldsymbol{D}}\times\boldsymbol{n}|^2.$$

$$\langle P\rangle_m = \int\left\langle\frac{dP}{d\Omega}\right\rangle d\Omega = \iint\frac{\mu_0|\ddot{\boldsymbol{m}}|^2}{16\pi^2 c^3}\sin^3\theta d\theta d\varphi = \frac{\mu_0|\ddot{\boldsymbol{m}}|^2}{12\pi c^3}.$$

$$\langle P\rangle = \frac{\mu_0 q^4}{12\pi m^2 c}\frac{\omega^4 E_0^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}.$$

$$\sigma = \frac{\langle P\rangle}{\langle S\rangle} = \frac{\langle P\rangle}{(1/2\mu_0 c)E_0^2} = \frac{\mu_0^2 q^4}{6\pi m^2}\frac{\omega^4}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}, \qquad \sigma = \frac{8\pi}{3}r_e^2\frac{\omega^4}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}.$$

$$\boxed{\sigma \approx \frac{8\pi}{3}r_e^2\frac{\omega^4}{\omega_0^4}.} \quad \boxed{\sigma \approx \frac{8\pi}{3}r_e^2,} \quad \boxed{\sigma = \frac{8\pi}{3}r_e^2\left(\frac{\omega}{\gamma}\right)^2,}$$

$$B = \nabla\times A = \frac{\mu_0 e^{ikr}}{4\pi}\left(\frac{ik}{r} - \frac{1}{r^2}\right)\nabla r\times\dot{p} \qquad \varphi(\boldsymbol{r},t) = \int\frac{1}{R}\rho\left(\boldsymbol{r}', t - \frac{R}{c}\right)dV' + \varphi_0,$$

$$= \frac{ik\mu_0 e^{ikr}}{4\pi r}\left(1 - \frac{1}{ikr}\right)\boldsymbol{n}\times\dot{\boldsymbol{p}}. \qquad \boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}', \quad dV' = dx'dy'dz',$$

解：我们将 (62.8) 写为形式

$$\varphi(\boldsymbol{r},t) = \iint\frac{\rho(\boldsymbol{r}',\tau)}{|\boldsymbol{r}-\boldsymbol{r}'|}\delta\left(\tau - t + \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}'|\right)d\tau dV'$$

（对 $\boldsymbol{A}(\boldsymbol{r},t)$ 作同样处理），引入附加的 δ 函数从而消去函数 ρ 中的隐宗量. 对于沿轨道 $\boldsymbol{r} = \boldsymbol{r}_0(t)$ 运动的点电荷我们有：

$$\rho(\boldsymbol{r}',\tau) = e\delta[\boldsymbol{r}' - \boldsymbol{r}_0(\tau)].$$

将这个表达式代入并对 dV' 积分，我们得到：

$$\varphi(\boldsymbol{r},t) = e\int\frac{d\tau}{|\boldsymbol{r}-\boldsymbol{r}_0(\tau)|}\delta\left[\tau - t + \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r}_0(\tau)|\right],$$

$$\boldsymbol{p} = 4\pi\epsilon_0 a^3\boldsymbol{E}, \text{ and } \alpha = 4\pi\epsilon_0 a^3$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \left(\mu\epsilon\omega^2 - k^2\right)\right]\begin{pmatrix}E_z\\B_z\end{pmatrix} = 0.$$

$$\boldsymbol{R}_{t'} - \frac{1}{c}\boldsymbol{R}_{t'}\cdot\boldsymbol{v} = \sqrt{R_t^2 - \frac{1}{c^2}(\boldsymbol{v}\times\boldsymbol{R}_t)^2} = R_t\sqrt{1 - \frac{v^2}{c^2}\sin^2\theta_t},$$

$$\int\nabla\times\vec{C}\,dv = -\oint\vec{C}\times d\vec{s} \qquad E^z_{mn} = E_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{i(kz-wt)}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{\vec{M}\times\vec{e}_R}{R^2}dv' = \frac{\mu_0}{4\pi}\int\vec{M}\times\vec\nabla\frac{1}{R}dv' \qquad \Delta f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\theta^2} + \frac{\partial^2 f}{\partial z^2}.$$

$$= \frac{\mu_0}{4\pi}\int\left(\frac{1}{R}(\vec\nabla\times\vec{M}) - \vec\nabla\times\frac{\vec{M}}{R}\right)dv \qquad \nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{\partial^2 f}{\partial\varphi^2}\frac{1}{r^2\sin^2\theta}$$

$$= \frac{\mu_0}{4\pi}\int\frac{1}{R}\vec\nabla\times\vec{M}\,dv' + \frac{\mu_0}{4\pi}\oint\frac{1}{R}\vec{M}\times d\vec{s}'$$

$$= \frac{\mu_0}{4\pi}\left(\int\frac{\vec{j}_b}{R}dv' + \oint\frac{\vec{k}_b}{R}ds\right) \qquad \vec{j}_b = \qquad \nabla\cdot\mathbf{A} = -\mu_0\epsilon_0\frac{\partial V}{\partial t}.$$

$$\nabla\cdot\left(\frac{\mathbf{J}}{\imath}\right) = \frac{1}{\imath}\left[-\frac{1}{c}\frac{\partial\mathbf{J}}{\partial t_r}\cdot(\nabla'\imath)\right] + \frac{1}{\imath}\left[-\frac{\partial\rho}{\partial t} - \frac{1}{c}\frac{\partial\mathbf{J}}{\partial t_r}\cdot(\nabla'\imath)\right] - \nabla\cdot\left(\frac{\mathbf{J}}{\imath}\right) = -\frac{1}{\imath}\frac{\partial\rho}{\partial t} - \nabla'\cdot\left(\frac{\mathbf{J}}{\imath}\right)$$

$$\nabla\cdot\mathbf{A} = \frac{\mu_0}{4\pi}\left[-\frac{\partial}{\partial t}\int\frac{\rho}{\imath}d\tau - \int\nabla'\cdot\left(\frac{\mathbf{J}}{\imath}\right)d\tau\right] = -\mu_0\epsilon_0\frac{\partial}{\partial t}\left[\frac{1}{4\pi\epsilon_0}\int\frac{\rho}{\imath}d\tau\right] - \frac{\mu_0}{4\pi}\oint\frac{\mathbf{J}}{\imath}\cdot d\mathbf{a}.$$