第一章　事件的概率

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1.
(1) $\Omega = \{0, 1, 2, \cdots, 10\}$
(2) $\Omega = \{0, 1, 2, \cdots, n\}$, n为总字数
(3) $\Omega = \{0, 1, 2, \cdots, n\}$
(4) $\Omega = \{(x, y)|x^2 + y^2 < 1\}$

2.
(1) $B\,\overline{A}\,\overline{C}$
(2) $A\ B\,\overline{C}$
(3) $A \bigcup B \bigcup C$
(4) $AB \bigcup BC \bigcup AC$
(5) $AB\overline{C} + BC\overline{A} + AC\overline{B}$
(6) $\overline{A}\overline{B} \bigcup \overline{B}\overline{C} \bigcup \overline{A}\overline{C}$　至多一个事件发生$\Longleftrightarrow$至少两个事件不发生
(7) $\overline{A} \bigcup \overline{B} \bigcup \overline{C}$　至多两个事件发生$\Longleftrightarrow$至少一个事件不发生

3.
(1) $\overline{A}B = (\frac{1}{4}, \frac{1}{2}] \bigcup (1, \frac{3}{2}]$
(2) $\overline{A} \bigcup B = [0, 2]$
(3) $\overline{AB} = [0, \frac{1}{2}] \bigcup (1, 2]$
(4) $\overline{\overline{A}\overline{B}} = (\frac{1}{4}, \frac{3}{2}]$

4.
Easy 略

5.
$A_1 \bigcup A_2 \bigcup \cdots \bigcup A_n = A_1 + A_2 \setminus A_1 + A_3 \setminus (A_1 \bigcup A_2) + \cdots + A_n \setminus (A_1 \bigcup A_2 \bigcup \cdots \bigcup A_{n-1})$

6.
有误　那些比例加起来等于$101\% > 100\%$

7.
解法一:开奖号固定

$P_1 = \frac{1}{C_{33}^7} \qquad P_2 = \frac{C_7^6}{C_{33}^7} \qquad P_3 = \frac{C_7^6 \cdot 25}{C_{33}^7} \qquad P_4 = \frac{C_7^5 \cdot 25}{C_{33}^7}$

解法二:彩票号固定

$P_1 = \frac{26}{C_{33}^7 \cdot 26} \qquad P_2 = \frac{C_7^6 \cdot 26}{C_{33}^7 \cdot 26} \qquad P_3 = \frac{C_7^6 \cdot 26 \cdot 25}{C_{33}^7 \cdot 26} \qquad P_4 = \frac{C_7^5 \cdot C_{26}^2 \cdot C_2^1}{C_{33}^7 \cdot 26}$

8.
Easy 略

9.
$1 - \frac{A_{12}^6}{12^6}$

10.
$\frac{C_3^2 \cdot C_6^1 \cdot C_5^1}{6^3}$

11.
(1)
有放回: $\frac{7}{10} \times \frac{7}{10} \times \frac{7}{10}$

无放回: $\frac{7}{10} \times \frac{6}{9} \times \frac{5}{8}$

(2)
有放回: $C_3^1 \frac{1}{10} \times (\frac{6}{10})^2 + C_3^2 (\frac{1}{10})^2 \times \frac{6}{10} + C_3^3 (\frac{1}{10})^3$

无放回: $\frac{3 \times 6 \times 5}{10 \times 9 \times 8}$

12.
略

13.
枚举法

14.
由对称性，$\frac{1}{2}$

17.
儿子赢父亲p_1，儿子赢教练p_2，则$p_1 > p_2$

父亲-教练-父亲 : $p_1p_2 + (1-p_1)p_2p_1 = p_1p_2(2-p_1)$

教练-父亲-教练 : $p_2p_1 + (1-p_1)p_1p_2 = p_1p_2(2-p_2)$

选后一种

19.
$\frac{A_{19}^8}{19^8}$

21.
在某一次游戏后:
赢的概率:$\frac{2}{9}$，输的概率:$\frac{1}{9}$，继续的概率:$\frac{2}{3}$
∴最终赢的概率:$\frac{2}{9}\left[(\frac{2}{3})^0 + (\frac{2}{3})^1 + \cdots + (\frac{2}{3})^n + \cdots\right] = \frac{2}{3}$

24.
$A = \{$至少一个是6$\}$，$P(A) = \frac{36-25}{36} = \frac{11}{36}$

$B = \{$两个面不一样$\}$，$P(B) = \frac{36-6}{36} = \frac{5}{6}$

$AB = \{$两个中有且只有一个是6$\}$，$P(AB) = \frac{11-1}{36} = \frac{5}{18}$

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1}{3}$

27.
$P(AC) = P(A|C)P(C) = 0.45 \qquad P(BC) = P(B|C)P(C) = 0.45$

$P(A\overline{C}) = P(A|\overline{C})P(\overline{C}) = 0.1 \qquad P(B\overline{C}) = P(B|\overline{C})P(\overline{C}) = 0.05$

$P(A) = P(AC) + P(A\overline{C}) = 0.55 \qquad P(B) = P(BC) + P(B\overline{C}) = 0.5$

$P(AB|C) = P(A|C)P(B|C) = 0.81 \qquad P(AB|\overline{C}) = P(A|\overline{C})P(B|\overline{C}) = 0.02$ (条件独立)

$P(ABC) = P(AB|C)P(C) = 0.405 \qquad P(AB\overline{C}) = P(AB|\overline{C})P(\overline{C}) = 0.01$

$P(AB) = P(ABC) + P(AB\overline{C}) = 0.415 \neq 0.275 = P(A)P(B) \Longrightarrow A, B$不独立

30.

(1) $P_A P_B P_C$

(2) $1-(1-P_A)(1-P_B)(1-P_C)$

(3) $1-(1-P_A^2)(1-P_B^2)(1-P_C^2)$

(4) $P_D^2\left[1-(1-P_A)(1-P_B)(1-P_C)\right]$

(5)

由全概率公式:

$$\begin{aligned} P(\text{正常}) &= P(\text{正常}|C\text{不正常})P(C\text{不正常}) + P(\text{正常}|C\text{正常})P(C\text{正常}) \\ &= (1-P_C)\left[1-(1-P_A P_B)^2\right] \\ &\quad +P_C\left[1-(1-P_A)^2\right]\left[1-(1-P_B)^2\right] \end{aligned}$$

33.

(1) $0.5\times0.4\times0.2+0.5\times0.6\times0.2+0.5\times0.4\times0.8=0.04+0.06+0.16=0.26$

(2) $1-0.5\times0.4\times0.2=1-0.04=0.96$

36.

A_i : 3件中有i件音色不纯，$i=0,1,2,3$

B : 乐器被接收

$$\begin{aligned} P(B) &= P(B|A_0)P(A_0)+P(B|A_1)P(A_1)+P(B|A_2)P(A_2)+P(B|A_3)P(A_3) \\ &= \frac{C_4^0C_{96}^3}{C_{100}^3}\times(0.99)^3+\frac{C_4^1C_{96}^2}{C_{100}^3}\times0.05\times(0.99)^2 \\ &\quad +\frac{C_4^2C_{96}^1}{C_{100}^3}\times(0.05)^2\times0.99+\frac{C_4^3C_{96}^0}{C_{100}^3}\times(0.05)^3 \end{aligned}$$

38.

(1) 全概率公式: $\frac{1}{2}\times1\%+\frac{1}{3}\times1\%+\frac{1}{6}\times2\%=\frac{7}{600}$

(2) A : 产品是次品　　A_i : 产品由一号车间生产, $i=1,2,3$

Bayes公式: $P(A_1|A)=\frac{P(A|A_1)P(A_1)}{P(A|A_1)P(A_1)+P(A|A_2)P(A_2)+P(A|A_3)P(A_3)}=\frac{3}{7}$

40.

(1) $P(\text{先抽女}) = \frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{7}{15} + \frac{1}{3} \times \frac{5}{25} = \frac{29}{90}$

(2) $P(\text{后抽男}) = \frac{1}{3} \times \left(\frac{3}{10} \times \frac{7}{9} + \frac{7}{10} \times \frac{6}{9}\right) + \frac{1}{3} \times \left(\frac{7}{15} \times \frac{8}{14} + \frac{8}{15} \times \frac{7}{14}\right) + \frac{1}{3} \times \left(\frac{5}{25} \times \frac{20}{24} + \frac{20}{25} \times \frac{19}{24}\right)$

$= \frac{61}{90}$

$P(\text{先抽女，后抽男}) = \frac{1}{3} \times \frac{3}{10} \times \frac{7}{9} + \frac{1}{3} \times \frac{7}{15} \times \frac{8}{14} + \frac{1}{3} \times \frac{5}{25} \times \frac{20}{24} = \frac{2}{9}$

$P(\text{先抽女}|\text{后抽男}) = \frac{20}{61}$

42.

A : 确有乙肝

B : 诊断有乙肝

Bayes公式:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.98 \times 0.05}{0.98 \times 0.05 + 0.05 \times 0.95}$$

43.

$$P(\text{双正面}|\text{正面}) = \frac{P(\text{正面}|\text{双正面})P(\text{双正面})}{P(\text{正面}|\text{双正面})P(\text{双正面}) + P(\text{正面}|\text{均匀})P(\text{均匀}) + P(\text{正面}|\text{不均匀})P(\text{不均匀})}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{3}} = \frac{4}{9}$$

44.

(a)

$$P(\text{带菌}|\text{阳性}) = \frac{P(\text{阳性}|\text{带菌})P(\text{带菌})}{P(\text{阳性}|\text{带菌})P(\text{带菌}) + P(\text{阳性}|\text{不带菌})P(\text{不带菌})} = \frac{0.95 \times 10\%}{0.95 \times 10\% + 0.01 \times 90\%}$$

$$= \frac{95}{104}$$

(b)

$$\frac{0.95^2 \times 10\%}{0.95^2 \times 10\% + 0.01^2 \times 90\%} = \frac{9025}{9034}$$