

电动力学作业题

20240322

答题不要求用英语，但应尽量做到逻辑缜密、清晰可读。提交作业不必通过BB网，截止时间请助教决定。

Compulsory:

1. 设粒子在惯性系 S 中以速度 $\mathbf{u} = c\boldsymbol{\beta}_u$ 运动，其4-速度表为 $U^\mu = \gamma_u c(1, \boldsymbol{\beta}_u)$ 。请证明： $U^\mu U_\mu = -c^2$ 。
2. 利用粒子的4-加速度 $\mathcal{A}^\mu = dU^\mu/d\tau$ 可以定义一个4-标量 $a = \sqrt{\mathcal{A}^\mu \mathcal{A}_\mu}$ 。请证明 a 恰为粒子瞬时自身系中物理加速度 $\hat{\mathbf{w}}$ 的大小。
3. Determine the space-like, null, or time-like character of the three 4-vectors given, in Cartesian coordinates, by

$$A^\mu = \left(1, 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \quad B^\mu = \left(1, 0, \frac{1}{2}, \frac{1}{2}\right),$$
$$C^\mu = (3, 12\lambda, 0, 0) \tag{1}$$

where λ is a real parameter.

4. If 4-vector v^μ is time-like and 4-vector s^μ is space-like, is it true that their scalar product $v_\mu s^\mu = 0$?

Optional :

5. An astronaut on an accelerated spaceship uses a coordinate system (T, X, Y, Z) related to an inertial system (t, x, y, z) as follows (we set $c = 1$):

$$t = X \sinh(aT), \quad x = X \cosh(aT), \quad y = Y, \quad z = Z \tag{2}$$

- Compute the metric tensor in the astronaut's coordinate system. (The metric tensor in the inertial system is, of course, given by the line element $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$.)
- Compute the duration of a trip of the spaceship on the astronaut's watch (i.e., the proper time) if the trajectory of his spaceship on this trip is, in the astronaut's coordinate system, $X(T) = X_0 = \text{constant}$, $Y(T) = vT$ for some constant $v > 0$, $Z(T) = 0$, and $0 \leq T \leq T_0$.