

电动力学作业题

20240301

答题不要求用英语，但应尽量做到逻辑缜密、清晰可读。提交作业不必通过BB网，截止时间请助教决定。

Compulsory:

1. Can a rest frame be chosen for a photon? Explain why !
2. A rod with length of $1m$ is inclined in the xy -plane with respect to the x -axis. An observer with the speed $v = \sqrt{2/3} c$ approaches the rod in the positive direction along the x -axis. How long does the observer measure the rod to be and at which angle does he observe it to be inclined relative to its x -axis?
3. A rod moves with velocity v along the positive x -axis in an inertial frame Σ . An observer Alice at rest in Σ measures the length of the rod to be L . Another observer Bob moves with the velocity $-v$ along the x -axis. What length, expressed as a function of L and v , will Bob measure for the rod? The measurement is done as usual with the endpoints being measured simultaneously for each observer in their respective frames.

Optional:

4. In the following exercises, \mathbf{a} , \mathbf{b} and \mathbf{c} are vector fields.

- Simplify

$$\nabla \times \mathbf{a}(\nabla \cdot \mathbf{a}) + \mathbf{a} \times [\nabla \times (\nabla \times \mathbf{a})] + \mathbf{a} \times \nabla^2 \mathbf{a} \quad (1)$$

- By explicitly writing out the terms in Cartesian coordinates, prove that

$$[\mathbf{c} \cdot (\mathbf{b} \cdot \nabla) - \mathbf{b} \cdot (\mathbf{c} \cdot \nabla)] \mathbf{a} = (\nabla \times \mathbf{a}) \cdot (\mathbf{b} \times \mathbf{c}) \quad (2)$$

- Prove that $\mathbf{a} \times (\nabla \times \mathbf{a}) = \nabla \left(\frac{1}{2} a^2 \right) - (\mathbf{a} \cdot \nabla) \mathbf{a}$.