Topology (H) Revision.

I. Counterexamples

Condition	Counterexample.
Hausdorff space	$(X, \mathcal{T}_{cofinite})$
Seq. cnt. $\not\Rightarrow$ cnt.	$Id: (\mathbb{R}, \mathcal{T}_{cocountable}) \to (\mathbb{R}, \mathcal{T}_{discrete})$.
Cnt. map $\not\Rightarrow$ closed map	$\pi: \mathbb{R}^2 \to \mathbb{R}$ $\pi(\{(x, \frac{1}{x}) \mid x > 0\})$
box topology universal property?	$f: \mathbb{R} \to \mathbb{R}^{\mathbb{N}}$, $t \mapsto (t, \dots)$ $f^{-1}\left(\prod_{n=1}^{\infty} (-\frac{1}{n}, \frac{1}{n})\right) = \{0\}$ ~~open~~
Set containing all seq. l.p $\not\Rightarrow$ closed.	$X = \mathcal{M}([0,1], \mathbb{R})$, $\mathcal{T}_{p.c.}$ $A = \{f \in X \mid f(x) \neq 0 \text{ for finite many } x\text{'s}\}$
topology not metrizable	• $(\mathbb{R}^{[0,1]}, \mathcal{T}_{prod})$ • $(\mathbb{R}, \mathcal{T}_{sorgenfrey})$
topology <u>NOT</u> (A1)	$(\mathbb{R}^{[0,1]}, \mathcal{T}_{prod})$.

seq. l.p. $\not\Rightarrow$ l.p.	$A = (0,1) \cup (1,3) \cup \{5\} \subseteq \mathbb{R}$ 5 seq. l.p. $\not\Rightarrow$ l.p.
l.p $\not\Rightarrow$ seq. l.p	$(X, \mathcal{T}_{cocountable})$ X uncountable. $x_0 \in X$, $x_0 \in (X - \{x_0\})'$ x_0 is not a l.p. of $X - \{x_0\}$
$\partial A \neq \partial\partial A$ $\partial \overline{A} \neq \partial A$	$A = \mathbb{Q}$ $A = \mathbb{Q}$
$X = \bigcup_{n=1}^{\infty} A_n$, $A_n \subset X$ closed $f\|_{A_n}$ cnt. $\not\Rightarrow$ f cnt.	$\iota : (\mathbb{Z}, \mathcal{T}_{cofinite}) \to \mathbb{R}$. $n \longmapsto n$
$\mathrm{Int}(\prod_\alpha A_\alpha) \neq$ $\prod_\alpha \mathrm{Int}(A_\alpha)$ for $\mathcal{T}_{prod}$.	$X_\alpha = \mathbb{R}$. $A_\alpha = (0,1)$ $\mathrm{Int}(\prod_\alpha A_\alpha) = \phi$ $\prod_\alpha \mathrm{Int}(A_\alpha) = \prod_\alpha (0,1)$
$\bigcup_\alpha \overline{A_\alpha} \neq \overline{\bigcup_\alpha A_\alpha}$	$X = \mathbb{R}$ $A_n = \{\frac{1}{n}\}$ $\overline{\bigcup_\alpha A_\alpha} = \{0, 1, \frac{1}{2}, \cdots\}$

l.p. cpt $\Rightarrow$ cpt. or. seq. cpt.	$(\mathbb{N}, \mathcal{T}_{discrete}) \times (\mathbb{N}, \mathcal{T}_{trivial})$
★ f(l.p. cpt.) $\neq$ l.p. cpt.	$(\mathbb{N}, \mathcal{T}_{discrete}) \times (\mathbb{N}, \mathcal{T}_{trivial}) \twoheadrightarrow (\mathbb{N}, \mathcal{T}_{discrete})$
★ cpt. $\not\Rightarrow$ seq. cpt.	$([0,1]^{[0,1]}, \mathcal{T}_{pwd})$.
★ seq. cpt. $\not\Rightarrow$ cpt.	$X = (\mathcal{M}([0,1],[0,1]), \mathcal{T}_{p.c.})$ $A = \{ f \mid f(x) \neq 0$ for countable many x's$\}$
(A2) – space seq. cpt. $\not\Rightarrow$ closed.	$(X, \mathcal{T}_{trivial})$ any subset is seq. cpt.
metric space bounded closed $\not\Rightarrow$ cpt.	$(\mathbb{N}, d_{discrete})$
$(X, \mathcal{T})$ not locally compact	• $(\mathbb{R}, \mathcal{T}_{sorgenfrey})$ • $(\mathbb{Q}, d)$

uni bounded $\not\Rightarrow$ equicontinuous	$f_n(x) = x^n$ in $[0,1]$
equicontinuous $\not\Rightarrow$ uni. bounded	$f_n(x) = n$.
X nowhere vanishing ✗	$\left\{ \sum_{k=1}^{n} a_k x^k \mid n, a_k \in \mathbb{R} \right\}$ $f(0) = 0$.
X point separable ✗	$\left\{ \sum_{k=0}^{n} a_k \cos kx + b_k \sin kx \right\}$ $f(0) = f(2\pi)$
NOT (A2)	$(\mathbb{R}, \mathcal{T}_{Sorgenfrey})$
Lindelöf $\not\Rightarrow$ (A2)	$([0,1]^{[0,1]}, \mathcal{T}_{prod})$
Lindelöf $\not\Rightarrow$ σ-cpt.	$(\mathbb{R}, \mathcal{T}_{cocountable})$
NOT σ-cpt.	$(\mathbb{R}, \mathcal{T}_{Sorgenfrey})$

NOT Hereditary Properties:

- (T4)
- locally cpt. $(0,1) \subseteq [0,1]$
- compact $(0,1) \subseteq [0,1]$
- locally compact: $\mathbb{Q} \subseteq \mathbb{R}$
- separable.
- Lindelöf. one-point compactification
  cpt. $\Rightarrow$ Lindelöf.

NOT Closed Hereditary

- Separable.

CLOWN!