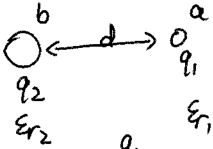
两带电介质球之间的相互作用



q_1 p_1

电荷分布 点 + 偶极子 + ⋯

0 0.5 1

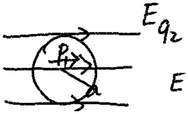
点 + 偶极子 + ⋯

q_2 p_2

0级：

$$F=\frac{q_1q_2}{4\pi\varepsilon_0}\frac{1}{d^2}$$ 斥力为正

0.5级



$$E'=-\frac{P}{3\varepsilon_0}$$

$$E_{q_2}+E'=E$$

$$P=\varepsilon_0(\varepsilon_r-1)E$$

$$E_{q_2}=E+\frac{1}{3\varepsilon_0}\varepsilon_0(\varepsilon_r-1)E$$

$$E=\frac{3}{\varepsilon_r+2}E_{q_2}$$

$$p_1=\cancel{\tfrac{4}{3}}\cancel{\pi}a^3\varepsilon_0(\varepsilon_{r1}-1)\frac{\cancel{3}}{\varepsilon_{r1}+2}\frac{q_2}{\cancel{4\pi\varepsilon_0}}\frac{1}{d^2}=\frac{\varepsilon_{r1}-1}{\varepsilon_{r1}+2}\frac{q_2a^3}{d^2}$$ 引力

$$F_{q_2对p_1}=(p_1\cdot\nabla)E_{q_2}=\frac{\varepsilon_{r1}-1}{\varepsilon_{r1}+2}\frac{q_2a^3}{d^2}\frac{q_2}{\underset{2}{\cancel{4}}\pi\varepsilon_0}\frac{-\cancel{2}}{d^3}=-\frac{\varepsilon_{r1}-1}{\varepsilon_{r1}+2}\frac{q_2^2}{2\pi\varepsilon_0}\frac{a^3}{d^5}$$

$$F_{p_2对q_1}=F_{q_1对p_2}$$

$$=-\frac{\varepsilon_{r2}-1}{\varepsilon_{r2}+2}\frac{q_1^2}{2\pi\varepsilon_0}\frac{b^3}{d^5}$$

$\varepsilon_{r1},\varepsilon_{r2}\to\infty$

$$F=-\frac{q_1^2b^3+q_2^2a^3}{2\pi\varepsilon_0d^5}$$

1级 $\overleftarrow{p_2}\ \xrightarrow{\ \vec r\ }\ \overrightarrow{p_1}$

$$F_{p_2对p_1}=\quad -p_1p_2\hat r\quad -p_1p_2\hat r\quad -p_1p_2\hat r$$

$$\frac{3}{4\pi\varepsilon_0d^4}\left[(\vec p_1\cdot\hat r)\vec p_2+(\vec p_2\cdot\hat r)\vec p_1+(\vec p_1\cdot\vec p_2)\hat r-5(\vec p_1\cdot\hat r)(\vec p_2\cdot\hat r)\hat r\right]$$

$$-5\,p_1\cdot(-p_2)\hat r$$

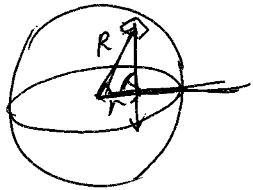
$$=\frac{3p_1p_2\hat r}{2\pi\varepsilon_0d^4}=\cdots$$

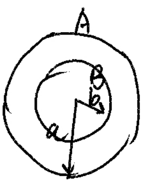
$F \propto \frac{1}{r^{2+\delta}}$

Maxwell验证 $\delta=0$.

$E = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^{2+\delta}}$ 仍保守 有势

$\left(V = \frac{q}{4\pi\varepsilon_0}\frac{1}{1+\delta}\frac{1}{r^{1+\delta}} \qquad V(\infty)=0\right)$





$$\frac{1}{1+\delta}\frac{R^2\sigma}{4\pi\varepsilon_0}\iint_S \frac{\sin\theta \, d\theta \, d\varphi}{(R^2+r^2-2Rr\sin\theta\cos\varphi)^{\frac{1+\delta}{2}}}$$

$$= \int_{-1}^{1} du \, \frac{2\pi}{(R^2+r^2-2Rru)^{\frac{1+\delta}{2}}}$$

① AB接通 加高压V_0

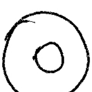
② 拆去导线 假设$\delta\neq0$则B带电.

③ A接地

④ 测V_B

$$2\pi \frac{2}{1-\delta}\frac{1}{-2Rr}(R^2+r^2-2Rru)^{\frac{1-\delta}{2}}\Big|_{-1}^{1}$$

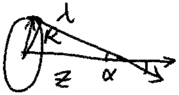
$$|R-r|^{1-\delta} - |R+r|^{1-\delta}$$



$$V(r) = \frac{\sigma R^2}{4\pi\varepsilon_0}\frac{1}{1-\delta^2}\frac{2\pi}{Rr}\left(|R+r|^{1-\delta}-|R-r|^{1-\delta}\right)$$

$\Rightarrow$ 导体电荷分布

圆环在轴线上的电场

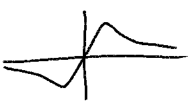


$$\int \frac{\lambda R\,d\theta}{4\pi\varepsilon_0}\frac{\cos\alpha}{R^2+z^2}$$

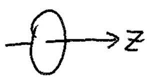
$$E=\frac{\lambda 2\pi R}{4\pi\varepsilon_0}\frac{z}{(R^2+z^2)^{3/2}}$$

$$\frac{dE}{dz}=k\left(-\frac{3}{2}\frac{z\cdot 2z}{(R^2+z^2)^{5/2}}+\frac{1}{(R^2+z^2)^{3/2}}\right)$$

$$=k\,\frac{R^2-2z^2}{(R^2+z^2)^{5/2}}=0 \Rightarrow z=\pm\sqrt{2}R$$

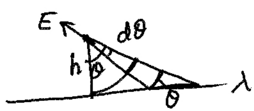


圆盘.

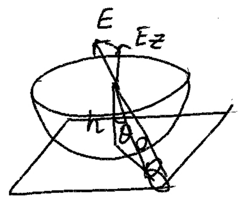


$$\int_0^R \frac{\sigma\cdot 2\pi r\,dr}{4\pi\varepsilon_0}\frac{z}{(r^2+z^2)^{3/2}}=\frac{\sigma z}{2\varepsilon_0}\left(\frac{1}{|z|}-\frac{1}{\sqrt{R^2+z^2}}\right)$$

等效



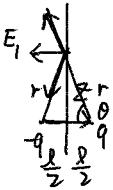
$$E=\frac{\lambda}{4\pi\varepsilon_0}\frac{\frac{h}{\cos\theta}d\theta\frac{1}{\cos\theta}}{\left(\frac{h}{\cos\theta}\right)^2}=\frac{\lambda h\,d\theta}{4\pi\varepsilon_0\,h^2}$$



$$E_{平面}=\frac{\sigma}{4\pi\varepsilon_0}\frac{\left(\frac{h}{\cos\theta}\right)^2 d\Omega\frac{1}{\cos\theta}}{\left(\frac{h}{\cos\theta}\right)^2}=\frac{\sigma\,d\Omega}{4\pi\varepsilon_0}\frac{1}{\cos\theta}$$

$$|E_{z平面}|=|E_{球}|$$

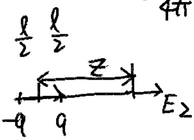
偶极子的电场



$$E_1 = \frac{q}{4\pi\varepsilon_0} \frac{\frac{l}{2}\times 2}{\left(z^2+\left(\frac{l}{2}\right)^2\right)^{3/2}}$$

$$= \frac{ql}{4\pi\varepsilon_0} \frac{1}{z^3} \left(1+\left(\frac{l}{2z}\right)^2\right)^{-\frac{3}{2}}$$

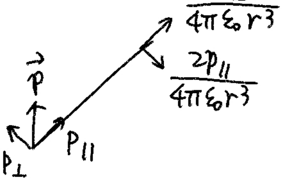
$$\sim \frac{ql}{4\pi\varepsilon_0 z^3} \sim \frac{p}{4\pi\varepsilon_0} \frac{1}{r^3}$$



$$E_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\left(z-\frac{l}{2}\right)^2} - \frac{1}{\left(z+\frac{l}{2}\right)^2}\right)$$

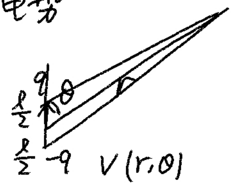
$$\sim \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{z^2}\left(1+\cancel{2}\frac{l}{\cancel{2}z}\right) - \frac{1}{z^2}\left(1-\cancel{2}\frac{l}{\cancel{2}z}\right)\right]$$

$$= \frac{ql}{4\pi\varepsilon_0} \frac{2}{z^3} = \frac{2p}{4\pi\varepsilon_0 r^3}$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}}{r^3}$$

电势



$$= \frac{q}{4\pi\varepsilon_0}\left[\left(r^2+\left(\frac{l}{2}\right)^2 - rl\cos\theta\right)^{-\frac{1}{2}} - \left(r^2+\left(\frac{l}{2}\right)^2 + rl\cos\theta\right)^{-\frac{1}{2}}\right]$$

$$\sim \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

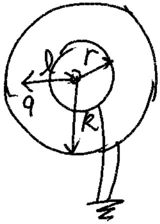
$$Q = \int_V \rho(r')\,dV'$$

$$\vec{p} = \int_V \rho(r')\,r'\,dV'$$

$$\varphi(\vec{R}) \doteq \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \vec{p}\cdot\nabla\frac{1}{R} + \cdots\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{\vec{p}\cdot\vec{R}}{R^3} + \cdots\right)$$

电场线方程



感应电荷

Q_1(内) Q_2(外)

$$\underset{=0}{(q+Q_1+Q_2)}\underset{\forall}{U_0}+kQ\underset{=0}{\left(\frac{q}{l}+\frac{Q_1}{r}+\frac{Q_2}{R}\right)}=0$$

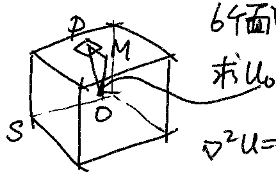
$$\Rightarrow\begin{cases}Q_1\\ Q_2\end{cases} \checkmark$$

O点电势为0 (叠加原理)

$$k\left(\frac{Q_1}{r}+\frac{q}{l}+\frac{Q_2}{R}\right)=0$$

又 $Q_1+Q_2+q=0$

$$\Rightarrow\begin{cases}Q_2=\dfrac{l-r}{r-R}\dfrac{R}{l}q\\[2ex] Q_1=\dfrac{l-R}{R-r}\dfrac{r}{l}q\end{cases}$$



6个面电势为U_i

求U_0

$\nabla^2U=0$

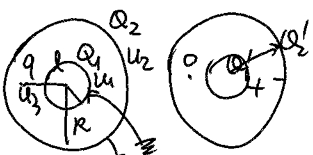
Poisson方程的线性性 $\Rightarrow U_0=\sum_i k_iU_i$

对称性：$k_i\equiv k$

$U_i\equiv U$时 唯一性 $U_0=U\Rightarrow k=\frac{1}{6}$

综上 $U_0=\frac{1}{6}\sum_{i=1}^{6}U_i$

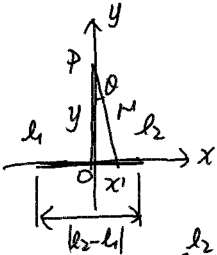
Green倒易定理



$$U'=U_0+kQ\frac{1}{x}$$

均匀分布

$$qU_3'+\underset{\Delta}{Q_1}U_1'+\underset{\Delta}{Q_2}U_2'=\underset{0}{q'}U_3+\underset{Q}{Q_1'}\underset{0}{U_1}+\underset{-Q}{Q_2'}\underset{0}{U_2}$$



$$V(h)=\frac{\sigma}{4\pi\varepsilon}\int_0^R\frac{2\pi r\,dr}{\sqrt{r^2+h^2}}$$

$$=\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+h^2}-|h|\right)$$

$$V_P=\frac{\lambda}{4\pi\varepsilon_0}\int_{l_1}^{l_2}\frac{dx'}{\sqrt{y^2+(x')^2}}$$

$$=\frac{\lambda}{4\pi\varepsilon_0}\ln\left(x'+\sqrt{(x')^2+y^2}\right)\Big|_{x'=l_1}^{l_2}$$

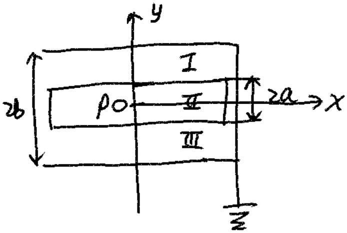
$$=\frac{\lambda}{4\pi\varepsilon_0}\ln\left[\frac{l_2+\sqrt{l_2^2+y^2}}{l_1+\sqrt{l_1^2+y^2}}\right]$$

$$\begin{cases} l_1=-\frac{l}{2}-x \\ l_2=\frac{l}{2}-x \end{cases}$$

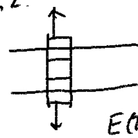
$$=\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\frac{l}{2}-x+\sqrt{\left(\frac{l}{2}-x\right)^2+y^2}}{-\frac{l}{2}-x+\sqrt{\left(\frac{l}{2}+x\right)^2+y^2}}\right)=\phi_0$$

有发散点

等势面：椭圆



法2.



$2E\Delta S = 2a\Delta S\rho/\varepsilon_0$

$E = 2a\rho/\varepsilon_0 \quad |y|\in(a,b]$

$E(y) = \frac{2y\rho}{\varepsilon_0} \quad y\in[-a,a]$

$V(x)$　　法1

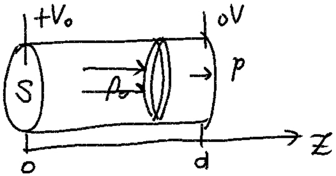
$V(x)=V(-x)$

$V(b)=0$　　V连续　E连续无面电荷

$$\frac{d^2V}{dy^2}=\begin{cases} 0 & (a,b] \rightarrow V=k_1y+C_1 \\ -\frac{\rho}{\varepsilon_0} & [-a,a] \rightarrow V=-\frac{\rho}{2\varepsilon_0}y^2+k_2y+C_2 \end{cases}$$

$k_2=0$

$$\begin{cases} k_1b+C_1=0 \\ k_1a+C_1=-\frac{\rho}{2\varepsilon_0}a^2+C_2 \Rightarrow k_1\ C_1\ C_2. \\ k_1=-\frac{\rho}{\varepsilon_0}a+C_2 \end{cases}$$



$V(z) \quad E(z)$

$$\frac{d^2V}{dz^2} = -\frac{\rho_0}{\varepsilon_0} \qquad V(z) = -\frac{\rho_0}{2\varepsilon_0}z^2 + az + b$$

$$\begin{cases} V(0) = V_0 \\ V(d) = 0 \end{cases} \Rightarrow \begin{cases} b = V_0 \\ -\frac{\rho_0}{2\varepsilon_0}d^2 + ad + b = 0 \end{cases}$$

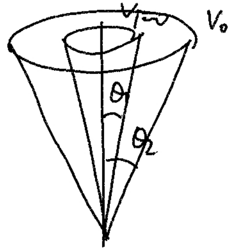
$$\begin{cases} b = V_0 \\ a = \frac{1}{d}\left(\frac{\rho_0}{2\varepsilon_0}d^2 - b\right) \end{cases}$$

$$E(z) = -\frac{dV}{dz} = \frac{\rho_0}{\varepsilon_0}z - \frac{\rho_0 d}{2\varepsilon_0} + \frac{b}{d}$$

$$p = \frac{1}{S}\int_0^d S\,dz\,\rho_0 E(z)$$

$$= \rho_0 \int_0^d E(z)\,dz$$

$$= \rho_0 V_0$$



$V = V(\theta)$

$$\nabla^2 V = \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}_{0}$$

$$+ \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)$$

$$+ \underbrace{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}}_{0} = 0$$

$$\frac{1}{r^2\sin\theta}\underbrace{\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)}_{0} = 0$$

$$\sin\theta\frac{\partial V}{\partial\theta} = A$$

$$V = A\ln\left|\tan\frac{\theta}{2}\right| + B = V_0\frac{\ln\left(\frac{\tan(\theta/2)}{\tan(\theta_1/2)}\right)}{\ln\left(\frac{\tan(\theta_2/2)}{\tan(\theta_1/2)}\right)}$$

$$\begin{cases} V(\theta_1) = 0 = A\ln\left(\tan\frac{\theta_1}{2}\right) + B \\ V(\theta_2) = V_0 = A\ln\left(\tan\frac{\theta_2}{2}\right) + B \end{cases}$$

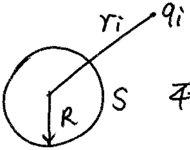
$$\begin{cases} A = V_0\left(\ln\left(\frac{\tan\frac{\theta_2}{2}}{\tan\frac{\theta_1}{2}}\right)\right)^{-1} \\ B = -V_0\frac{\ln\left(\tan\frac{\theta_1}{2}\right)}{\ln\left(\frac{\tan\theta_2/2}{\tan\theta_1/2}\right)} \end{cases}$$

电势函数(无电荷时)为调和函数

无极值点 ⇒ 带电构形不可能稳定静电平衡

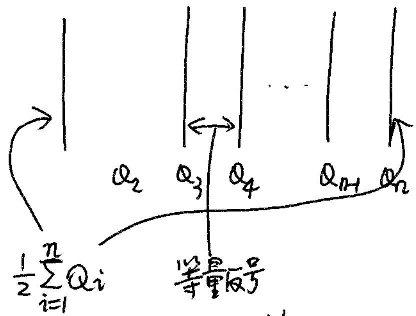
平均值定理

↘ 推广

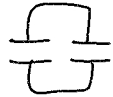


$$\frac{1}{4\pi R^2}\int_S u\,ds = \overbrace{\underbrace{\frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}}_{\text{outside}}}^{u(0)} + \underbrace{\frac{\sum_i q_i}{4\pi\varepsilon_0}\frac{1}{R}}_{\text{inside}}$$

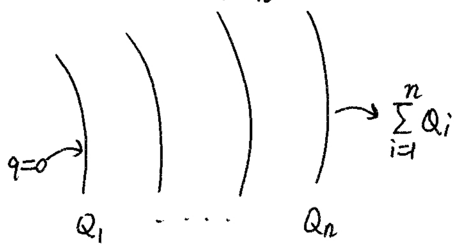
无限大平板

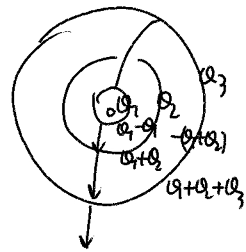


导线连接内外球面



球壳





能量最小

$$W=\frac{1}{2}Q_3\frac{Q_1+Q_2+Q_3}{4\pi\varepsilon_0 R_3}+\frac{1}{2}\frac{Q_2}{4\pi\varepsilon_0}\left[\frac{Q_1+Q_2+Q_3}{R_3}+(Q_1+Q_2)\left(\frac{1}{R_2}-\frac{1}{R_3}\right)\right]$$

$$+\frac{1}{2}Q_1\frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1+Q_2+Q_3}{R_3}+(Q_1+Q_2)\left(\frac{1}{R_2}-\frac{1}{R_3}\right)+Q_1\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]$$

$$=\frac{1}{8\pi\varepsilon_0}\frac{(Q_1+Q_2+Q_3)^2}{R_3}+\underbrace{\frac{(Q_1+Q_2)^2}{8\pi\varepsilon_0}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)+\frac{Q_1^2}{8\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}\to min$$

$$\frac{1}{8\pi\varepsilon_0}\left[\left(\frac{1}{R_1}-\frac{1}{R_3}\right)Q_1^2+2\left(\frac{1}{R_2}-\frac{1}{R_3}\right)Q_2Q_1+\left(\frac{1}{R_2}-\frac{1}{R_3}\right)Q_1^2\right]$$

$+$

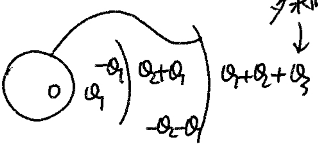
$$Q_1 = -\frac{\frac{1}{R_2}-\frac{1}{R_3}}{\frac{1}{R_1}-\frac{1}{R_3}} Q_2$$

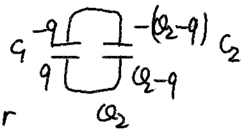
$$= \frac{R_3-R_2}{R_3-R_1}\frac{R_1}{R_2}(-Q_2)$$

与(Q_1+Q_3)总量无关

多余的都在外面

↓ 对内部只有电势的整体升降





$$C = \frac{Q}{\Delta U} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{R}\right)} = \frac{4\pi\varepsilon_0 Rr}{R-r}$$

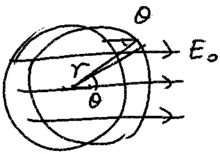
$$W = \frac{1}{2C_1}q^2 + \frac{1}{2C_2}(Q_2-q)^2$$

$$= \frac{1}{8\pi\varepsilon_0}\left[\frac{R_2-R_1}{R_1R_2}q^2 + \frac{R_3-R_2}{R_2R_3}(Q_2-q)^2\right]$$

同理 …

$$\begin{cases} \frac{q_1}{C_1} = U_1 = U_2 = \frac{q_2}{C_2} \\ q_1+q_2 = Q_2 \end{cases}$$

中性导体球在均匀电场 E_0 中
表面感应电荷分布



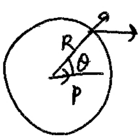
$$F_1 = qE_0 = F_2 = q\cdot\frac{\rho}{3\varepsilon_0}r$$

$$\Rightarrow \vec{r} = \frac{3\varepsilon_0\vec{E}_0}{\rho} \longrightarrow \sigma = r\cos\theta\,\rho = 3\varepsilon_0 E_0\cos\theta$$

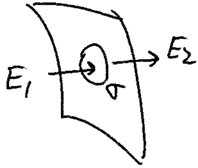
$$\vec{p} = q\vec{r} = 4\pi\varepsilon_0 R^3\vec{E}_0$$

或

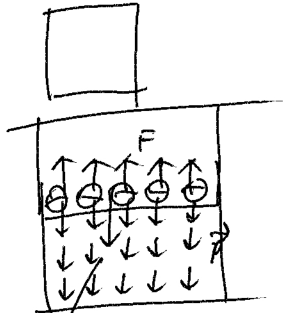
$$\sigma = \varepsilon_0 E_n = \varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{1}{R^3}\underbrace{(3(\vec{p}\cdot\hat{r})\hat{r}-\vec{p})_n}_{2p\cos\theta} + E_0\cos\theta\right) = 3\varepsilon_0 E_0\cos\theta$$



带电平面的受力



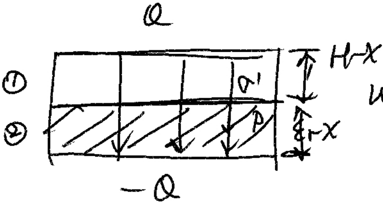
$F=\frac{1}{2}\sigma(E_1+E_2)$



适用于带电导体表面等自由电荷面

极化电荷不可.

对导体：$p=w_e=\frac{1}{2}\sigma E_n$ (向外)



$W_e=\boxed{\frac{\varepsilon_0}{2}\int E^2 dV}+\boxed{\frac{1}{2}\int P\cdot E\, dV}$

$D=\sigma$

$E_1=\frac{1}{\varepsilon_0}D=\frac{1}{\varepsilon_0}\sigma$

$E_2=\frac{1}{\varepsilon_0\varepsilon_r}D=\frac{1}{\varepsilon_0\varepsilon_r}\sigma$

$\frac{\varepsilon_0}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 S(H-x)+\left(\frac{\sigma}{\varepsilon_0\varepsilon_r}\right)^2 Sx$

$=\frac{\sigma^2 S}{2\varepsilon_0}\left((H-x)+\frac{1}{\varepsilon_r^2}x\right)=W$

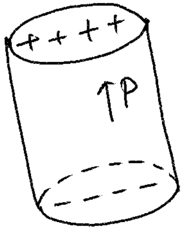
$\frac{dW}{dx}=\frac{\sigma^2 S}{2\varepsilon_0}\frac{\varepsilon_r^2-1}{\varepsilon_r^2}$

$\sigma'=P=\varepsilon_0(\varepsilon_r-1)E_2=\frac{\varepsilon_r-1}{\varepsilon_r}\sigma$

$F=\frac{\varepsilon_r-1}{\varepsilon_r}\sigma\cdot\frac{1}{2}\frac{\sigma}{\varepsilon_0}\left(\frac{1}{\varepsilon_r}+1\right)S$

$\frac{\varepsilon_r^2-1}{\varepsilon_r^2}\frac{\sigma^2 S}{2\varepsilon_0}>\frac{\varepsilon_r-1}{\varepsilon_r}\frac{\sigma^2 S}{2\varepsilon_0}$

真



$D \equiv \varepsilon_0 E + P.$

P表示为 $\varepsilon_0(\varepsilon_r - 1)E$

$$\int_V \frac{\rho(x')\,\delta(x-x')}{|x-x'|^2}\,dx'$$

‖

$$\nabla^2\phi = -\frac{1}{\varepsilon_0}\delta$$

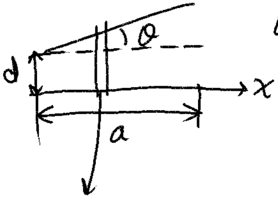
$$\phi = k\frac{1}{r}$$

$\phi =$

$$\nabla^2\phi = -\frac{1}{\varepsilon_0}\rho$$

非平行板电容器

$\theta \ll 1$



$$dC = \frac{\varepsilon_0 b\, dx}{d + x\theta} \quad \text{并联}$$

$$C = \int dC = \frac{\varepsilon_0 b}{\theta} \ln\left(1 + \frac{a\theta}{d}\right)$$

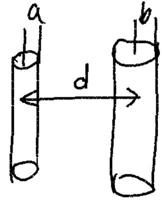
$$\sim \frac{\varepsilon_0 b}{\theta}\left(\frac{a\theta}{d} - \frac{1}{2}\frac{a^2\theta^2}{d^2}\right)$$

球形电容器

$$C = 4\pi\varepsilon_0 \frac{1}{\frac{1}{R_1} - \frac{1}{R_2}}$$

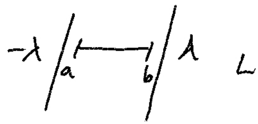
圆柱形

$$C = 2\pi\varepsilon_0 \frac{L}{\ln\frac{b}{a}}$$



$d \gg a, b$

单位长度电容值



$$U = \frac{\lambda L}{2\pi\varepsilon_0}\left(\ln\frac{d-a}{b} + \ln\frac{d-b}{a}\right)$$

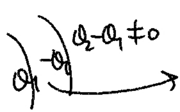
$$= \frac{\lambda L}{2\pi\varepsilon_0} \ln\frac{(d-a)(d-b)}{ab}$$

$$C = \frac{2\pi\varepsilon_0}{\ln\frac{(d-a)(d-b)}{ab}}$$

$a = b = r$ 且 $d \gg r$

$$\rightarrow \frac{\pi\varepsilon_0}{\ln\frac{d}{r}}$$

带电量不等的两导体



计算电容：设极板电势 $\varphi|_{\partial\Sigma}$

$\downarrow$

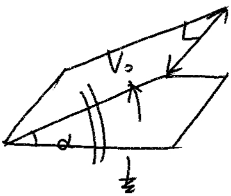
解Laplace方程 $\nabla^2\varphi=0$

$\downarrow$

$$\varepsilon_0\oint_{\partial\Sigma}\frac{\partial\varphi}{\partial n}dS=Q$$

$\downarrow$

$$C=\frac{Q}{U_1-U_2}$$



$V(\varphi)$

柱坐标

$$\frac{1}{r}\underset{0}{\underbrace{\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)}}+\frac{1}{r^2}\underbrace{\left(\frac{\partial^2 u}{\partial\varphi^2}\right)}+\underset{0}{\underbrace{\frac{\partial^2 u}{\partial z^2}}}=0$$

$$\frac{\partial^2 u}{\partial\varphi^2}=0\Rightarrow u=k\varphi+b$$

$$\begin{cases}u(0)=b=0\\ u(\alpha)=k\alpha+b=V_0\end{cases}\Rightarrow u=\frac{V_0}{\alpha}\varphi$$

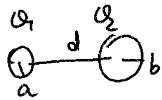
$$\frac{1}{r}\frac{\partial u}{\partial\varphi}=\frac{V_0}{\alpha}\frac{1}{r}=E_n=\frac{\sigma}{\varepsilon_0}$$

$$Q=\frac{V_0\varepsilon_0 L}{\alpha}\ln\frac{b}{a}\quad C_L=\frac{\varepsilon_0}{\alpha}\ln\frac{b}{a}$$

Maxwell电容矩阵

$$\begin{cases}V_i=p_{ij}Q_j\\ Q_i=C_{ij}V_j\end{cases}$$

$$C_{ij}=\frac{1}{p_{ii}+p_{jj}-2p_{ji}}$$



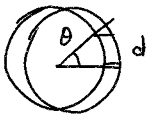
$d\gg a,b$

$$\begin{pmatrix}V_1\\V_2\end{pmatrix}=\frac{1}{4\pi\varepsilon_0}\begin{pmatrix}\frac{1}{a}&\frac{1}{d}\\\frac{1}{d}&\frac{1}{b}\end{pmatrix}\begin{pmatrix}Q_1\\Q_2\end{pmatrix}$$

$$\Rightarrow C_{12}=\frac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$$

均匀介质极化球内的退极化场

$\sigma = P\cos\theta$



$\sigma = \rho d\cos\theta$

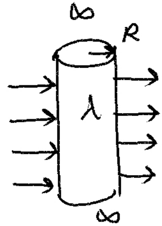
$E' = -\dfrac{\rho d}{3\varepsilon_0}$

$\Rightarrow E' = -\dfrac{P}{3\varepsilon_0}$

$\sigma = 3\varepsilon_0|E'|\cos\theta$

$\downarrow$

$\dfrac{\varepsilon_r-1}{\varepsilon_r+2}E_0$



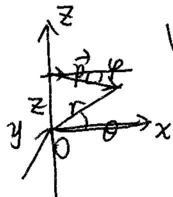
均匀介质无限长圆柱

法1

猜解

内部匀强 外部 E_0+线偶极

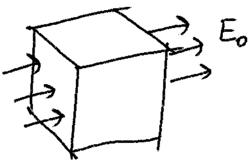
线偶极(外部)

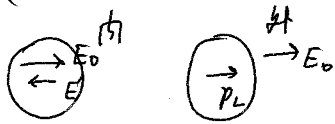


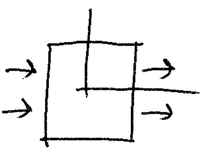
$$V(r,\theta) = \frac{p_L}{4\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{r\cos\theta\, dz}{(r^2+z^2)^{\frac{3}{2}}}$$

$$= \frac{p_L}{4\pi\varepsilon_0}\frac{2r\cos\theta}{r^2} = \frac{p_L\cos\theta}{2\pi\varepsilon_0 r}$$

$$\left(\int\frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{x^2+a^2}}+C\right)$$







(假设)

$$u(r,\theta) = \begin{cases} -E_1 r\cos\theta + C_1 & \text{内} \\ -E_0 r\cos\theta + \dfrac{p_L\cos\theta}{2\pi\varepsilon_0 r} + C_2 & \text{外} \end{cases}$$

$$\begin{cases} u_{in}(R,\theta) = u_{out}(R,\theta) \\ \varepsilon_0\varepsilon_r \dfrac{\partial u_{in}}{\partial r}\Big|_R = \varepsilon_0\dfrac{\partial u_{out}}{\partial r}\Big|_R \end{cases}$$

$$\begin{cases} -E_1 R\cos\theta \equiv -E_0 R\cos\theta + \dfrac{p_L\cos\theta}{2\pi\varepsilon_0 R} \\ \varepsilon_r E_1\cos\theta \equiv E_0\cos\theta + \dfrac{p_L\cos\theta}{2\pi\varepsilon_0 R^2} \end{cases} \qquad C_1 = C_2$$

$$\Leftrightarrow \begin{cases} E_1 R = E_0 R - \dfrac{p_L}{2\pi\varepsilon_0 R} \\ \varepsilon_r E_1 = E_0 + \dfrac{p_L}{2\pi\varepsilon_0 R^2} \end{cases}$$

$$\Leftrightarrow \begin{cases} p_L = \dfrac{\varepsilon_r-1}{\varepsilon_r+1}\, 2\varepsilon_0 E_0 \pi R^2 \\ E_1 = \dfrac{2}{\varepsilon_r+1} E_0 \end{cases}$$

可满足边值关系

由唯一性定理

假设正确✓

$$\Rightarrow \begin{cases} P = \varepsilon_0(\varepsilon_r-1)E_1 = 2\varepsilon_0 E_0 \dfrac{\varepsilon_r-1}{\varepsilon_r+1} \overset{!}{=} p_L/\pi R^2 \\ E' = E_1 - E_0 \overset{!}{=} -\dfrac{P}{2\varepsilon_0} \end{cases}$$

假设解.

E_1 p_L 未知

E'与P. P与p_L关系未知

↓

为使满足边值关系

解得 E_1, p_L (唯一)

↓

根据 $E' := E_1 - E_0$

$P := \varepsilon_0(\varepsilon_r-1)E_1$

解得 E与P 的关系

P与p_L

法2.

认为正负电荷柱整体偏移了d (合理的模型)

$$E' = -\frac{\rho d}{2\varepsilon_0} = -\frac{P}{2\varepsilon_0} \quad 匀强$$

$$P = \varepsilon_0(\varepsilon_r-1)\left(E_0 - \frac{P}{2\varepsilon_0}\right) \Rightarrow P$$

$$p_L = P\pi R^2$$

↓

E'与P的关系

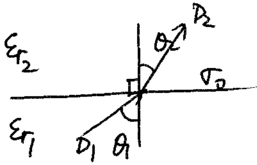
$p_L = P\pi R^2$

↓

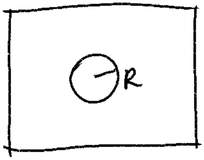
得解 (唯一性)

$$\frac{\varepsilon_r-1}{\varepsilon_r+2}=\eta\frac{\varepsilon_{r1}-1}{\varepsilon_{r1}+2}+(1-\eta)\frac{\varepsilon_{r2}-1}{\varepsilon_{r2}+2}$$ why?

介质界面对 E.D 的折射



无限大均匀介质中 q_f 均匀带电球 (R)



$q_f \to \frac{q_f}{\varepsilon_r}$

$E_f \to \frac{E_f}{\varepsilon_r}$

$$\begin{cases}\frac{D_2\sin\theta_2}{\varepsilon_{r2}}=\frac{D_1\sin\theta_1}{\varepsilon_{r1}}\\ D_2\cos\theta_2=D_1\cos\theta_1\end{cases}$$

$$D_2\cos\theta_2-D_1\cos\theta_1=\sigma_0$$

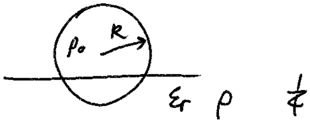
$$\frac{\tan\theta_1}{\tan\theta_2}=\frac{\varepsilon_{r1}}{\varepsilon_{r2}}\left(1+\frac{\sigma_0}{\varepsilon_0\varepsilon_{r1}E_1\cos\theta_1}\right)$$

$\left.\begin{array}{l}\text{全空间充满均匀介质}\\ \text{介质界面为等势面}\end{array}\right\}$ $\nabla\times D=0$

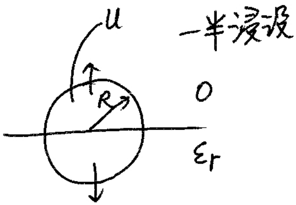
空间中存在多种介质时

界面 $\begin{cases}\text{平行于等势面}\\ \text{垂直于等势面}\end{cases}$

无限大介质去掉一个球



$\rho_0 = \frac{1}{4}\rho$



E相同

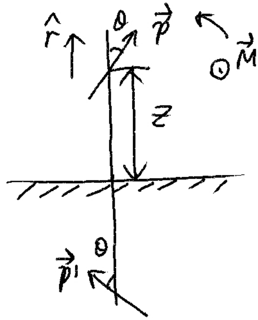
$$E = k\frac{1}{r^2} = \frac{u_0 R}{r^2} \qquad u_0 = k\frac{1}{R}$$

$$\begin{cases} P_上 = w_e = \frac{\varepsilon_0}{2}E^2 \\ P_下 = \frac{1}{2}D\cdot E = \frac{\varepsilon_0}{2}\varepsilon_r E^2 \end{cases}$$

$$F = \frac{\varepsilon_0}{2}\frac{u_0^2 R^2}{R^4}(\varepsilon_r - 1)\pi R^2 = \frac{1}{4}\cdot\rho\cdot\frac{4}{3}\pi R^3 g$$

向下

$\Rightarrow u_0.$

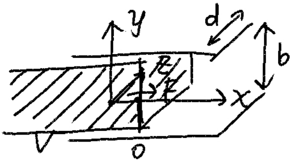


$$W = -p\cdot E\cdot\frac{1}{2}$$
$$= -\frac{1}{8\pi\varepsilon_0(2z)^3}\left(3(\vec{p}\cdot\hat{r})(\vec{p}'\cdot\hat{r}) - \vec{p}\cdot\vec{p}'\right)$$
$$= -\frac{p^2(\cos^2\theta+1)}{64\pi\varepsilon_0 z^3}$$

$$F = -\frac{dW}{dz} = (-)\frac{3p^2(\cos^2\theta+1)}{64\pi\varepsilon_0 z^4}(\hat{r})$$

引力

$$M = -\frac{dW}{d\theta} = (-)\frac{p^2}{64\pi\varepsilon_0 z^3}\sin 2\theta$$



$$f_x = \frac{\varepsilon_r - 1}{\varepsilon_r} \partial_x w$$

$$F_x = \int_V f_x \, dV = \frac{\varepsilon_r - 1}{\varepsilon_r} \int_0^b dy \int_0^d dz \underbrace{\int_{-\infty}^0 \partial_x w \, dx}_{w(0) - w(-\infty)}$$

$$= \frac{\varepsilon_r - 1}{\varepsilon_r} b d \frac{\varepsilon_0 \varepsilon_r}{2} \left(\frac{V}{d}\right)^2$$

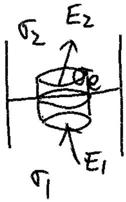
$w(0)$ ↓ 介质内

电流

$\vec{j}=\rho\vec{v}$

$\vec{j}=\sigma\vec{E}$ (部分导体)

在不同导体介面上



$$\vec{n}\cdot(\underbrace{E_2}_{=\frac{j_2}{\sigma_2}}-\underbrace{E_1}_{=\frac{j_1}{\sigma_1}})=\frac{\sigma_e}{\varepsilon_0}$$

稳恒时 $j_2=j_1=\frac{I}{S}$

$$\Rightarrow \sigma=\frac{I\varepsilon_0}{S}\left(\frac{1}{\sigma_2}-\frac{1}{\sigma_1}\right)$$

稳恒电流时

欧姆定律 $\Rightarrow$ 焦耳热最小

利用 $R=\int\frac{\rho dl}{S}$ 时

导体截面S应为等势面

$$R=\frac{U}{I}=\frac{\int_L \vec{E}\cdot d\vec{l}}{\int\vec{j}\cdot d\vec{S}}$$

$\vec{E} \rightarrow \vec{j}\rho$

取L为一电流线/电场线

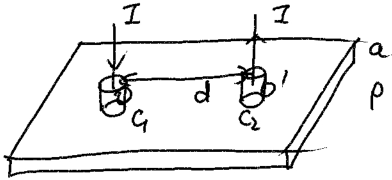
S为等势面 $\Rightarrow \vec{j}\parallel d\vec{S}$

$$\frac{\int\rho j\,dl}{\int j\,dS}=\int\frac{\rho j}{\int_{S_x} j\,dS}dl$$

同一截面上各处j相同时

$$\int\frac{\rho j}{j\int_{S_x}dS}dl=\int\frac{\rho\,dl}{S}$$

看成电阻串联



① $E=\frac{Q}{4\pi\varepsilon r^2}$ $\quad U_0=\frac{Q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right)$

$$Q=4\pi\varepsilon U_0\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$E=\frac{U_0}{r^2}\frac{b-a}{ab}$$

$$j=\sigma E$$

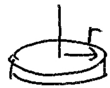
$$I=\sigma\frac{U_0}{\cancel{a^2}}\frac{b-a}{ab}4\pi\cancel{a^2}=4\pi\sigma U_0\frac{b-a}{ab}$$

②

$$R=\frac{U_0}{I}=\frac{ab}{4\pi\sigma(b-a)}=\rho\varepsilon\frac{1}{C}$$

$$RC=\rho\varepsilon$$

③ $$R=\int\frac{\rho\,dl}{S}=\rho\int_a^b\frac{dr}{4\pi r^2}=\frac{\rho}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$



$$2\pi r a\, j_1=I$$

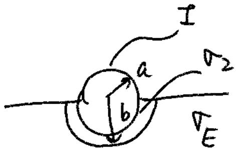
$$j_1=\sigma E_1$$

$$E_1=\frac{I}{2\pi\sigma a r_1}\hat{r}_1 \qquad E_2=\frac{I}{2\pi\sigma a}\frac{-\hat{r}_2}{r_2}$$

$$j=j_1+j_2 \Rightarrow E=E_1+E_2$$

$$U_{12}=\frac{I}{2\pi\sigma a}\int_{c_1}^{d-c_2}\left(\frac{1}{r_1}+\frac{1}{d-r_1}\right)dr$$

$$=\underbrace{\frac{I}{2\pi\sigma a}\ln\left(\frac{d-c_2}{c_1}\cdot\frac{d-c_1}{c_2}\right)}_{R}$$



$\vec{j} = \frac{I\hat{r}}{2\pi r^2}$ $\vec{E} = \frac{1}{\sigma}\vec{j}$

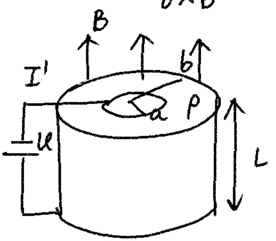
$$U = \int_{+\infty}^{a} E\,dr = \underbrace{\frac{I}{2\pi}\left[\frac{1}{\sigma_2}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{\sigma_E}\frac{1}{b}\right]}_{R}$$

含源电路

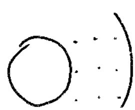
$$\vec{j} = \sigma\left(\vec{E}+\vec{K}\right)$$

$$\downarrow$$

$$\vec{v}\times\vec{B}$$



L>>b>a



(r,θ)

$$\vec{E}(r,\theta) = \frac{U_0}{\ln\frac{b}{a}}\frac{\hat{r}}{r}$$

$$\vec{v} = v_r\hat{r} + v_\theta\hat{\theta}$$

$$\vec{K} = \vec{v}\times\vec{B} = v_\theta B\hat{r} - v_r B\hat{\theta}$$

$$\vec{j} = ne\vec{v}$$

$$\vec{j} = \sigma\left(\vec{E}+\vec{K}\right)$$

$$\begin{cases} nev_r = \sigma\left(\frac{U_0}{\ln\frac{b}{a}}\frac{1}{r} + v_\theta B\right) \\ nev_\theta = \sigma\left(-v_r B\right) \end{cases}$$

$$\left(ne + \frac{\sigma^2 B^2}{ne}\right)v_r = \frac{\sigma}{r}\frac{U_0}{\ln\frac{b}{a}}$$

$$j_r = nev_r = \frac{\sigma}{r}\frac{U_0}{\ln\frac{b}{a}}\frac{1}{1+\frac{\sigma^2 B^2}{n^2e^2}}$$

$$I' = j_r(a)\,2\pi a l$$

$$= \frac{2\pi\sigma L U_0}{\ln\left(\frac{b}{a}\right)\left(1+\frac{\sigma^2 B^2}{n^2 e^2}\right)}$$

$$j_\theta = -\frac{\sigma B}{ne}j_r \propto \frac{1}{r}$$

环向电流 $I'' = \int_a^b j_\theta(r)\,L\,dr$

能量损耗

$$p = \left(j_r^2 + \underline{j_\theta^2}\right)\rho$$

$$P \neq (I')^2 r$$

二维电子气体均匀磁场中的运动

散射力 $\vec{f}=-m\vec{v}/\tau$

$$\begin{cases} m\ddot{x}=-eE_x-ev_yB-m\dfrac{v_x}{\tau} \\ m\ddot{y}=-eE_y+ev_xB-m\dfrac{v_y}{\tau} \end{cases}$$

$$\begin{cases} j_x=-env_x \\ j_y=-env_y \end{cases}$$

稳态:
$$\begin{cases} 0=-eE_x-eBv_y-\dfrac{m}{\tau}v_x \\ 0=-eE_y+eBv_x-\dfrac{m}{\tau}v_y \end{cases}$$

① 已知 σ

用 $\vec{j}=\sigma(\vec{E}+\vec{K})$ 列方程 ↘ 已是稳态

$\parallel$

$ne\vec{v}$

如 $ne\vec{v}=\sigma(\vec{E}+\vec{v}\times\vec{B})\Rightarrow\vec{v}\Rightarrow\vec{j}$

② 未知 σ

列运动方程

稳态,令加速度为0 $\Rightarrow\vec{v}=\vec{j}$

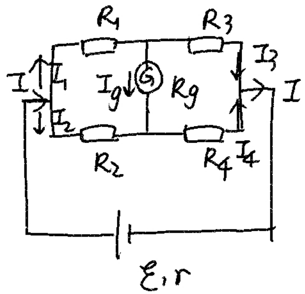
由 $\vec{j}$ 与 $\vec{E}$ 关系得 σ

$$\begin{pmatrix} \frac{m}{\tau} & eB \\ eB & -\frac{m}{\tau} \end{pmatrix}\begin{pmatrix} v_x \\ v_y \end{pmatrix}=\begin{pmatrix} -eE_x \\ eE_y \end{pmatrix}$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}=\frac{1}{\left(\frac{m}{\tau}\right)^2+(eB)^2}\begin{pmatrix} \frac{m}{\tau} & eB \\ eB & -\frac{m}{\tau} \end{pmatrix}\begin{pmatrix} -eE_x \\ eE_y \end{pmatrix}$$

$$\begin{cases} \sigma_0=\dfrac{ne^2\tau}{m} \\ \omega_c=\dfrac{eB}{m} \end{cases}$$

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}=\frac{-ne^2}{\left(\frac{m}{\tau}\right)^2+(eB)^2}\begin{pmatrix} -\frac{m}{\tau} & eB \\ -eB & -\frac{m}{\tau} \end{pmatrix}=\frac{\sigma_0}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & -R_3 & R_4 & R_g \\ 0 & 0 & R_2 & -R_1 & 0 & -R_g-R_1 \\ 0 & -r & -r-R_2 & 0 & -R_4 & 0 \end{pmatrix}$$

r $\quad$ R_2

$$I = I_1 + I_2$$

$$I_1 = I_g + I_3$$

$$I_4 = I_g + I_2$$

$$-I_1R_1 - I_gR_g + I_2R_2 = 0$$

$$+I_gR_g - I_3R_3 + I_4R_4 = 0$$

$$-I_2R_2 - I_4R_4 - Ir + \varepsilon = 0$$

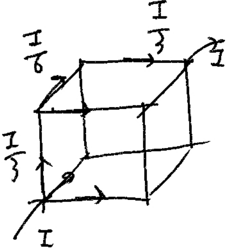
$$\begin{matrix} 0 & 0 & 0 & -R_1 & R_2 & -R_2-R_g-R_1 \\ 0 & 0 & -r-R_2 & -r & -R_4 & -r \\ 0 & 0 & 0 & -r & -R_4-r-R_2 & R_4 \end{matrix}$$

$$I_g = \frac{\Delta g}{\Delta} \rightarrow \begin{vmatrix} -R_3 & R_4 & 0 \\ -R_1 & R_2 & 0 \\ -r & -r-R_2-R_4 & \varepsilon \end{vmatrix}$$

$$\begin{matrix} 1 & -1 & -1 & 0 & 0 & 0 & I & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 & I_1 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 & I_2 & 0 \\ 0 & -R_1 & R_2 & 0 & 0 & -R_g & I_3 & 0 \\ 0 & 0 & 0 & -R_3 & R_4 & R_g & I_4 & 0 \\ -r & 0 & -R_2 & 0 & -R_4 & 0 & I_g & \varepsilon \end{matrix}$$

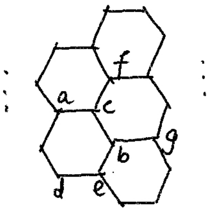
R_1 $\quad$ r

$$I_g = 0 \;\Updownarrow\; R_2R_3 = R_1R_4$$



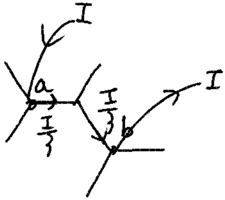
$$U=\left(\frac{I}{3}+\frac{I}{6}+\frac{I}{3}\right)R=\frac{5}{6}IR$$

$$R'=\frac{5}{6}R$$



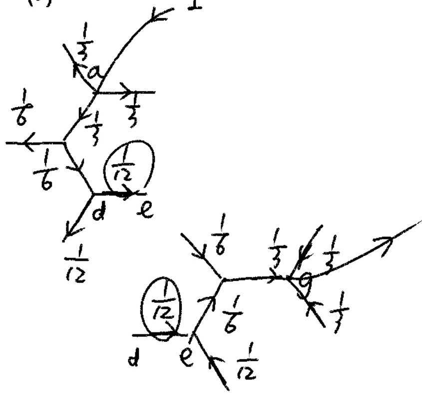
(1)

R_{ab}

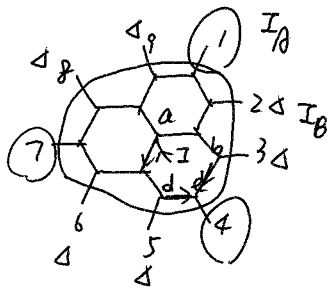


$$U=\frac{I}{3}2r \quad R_{ab}=\frac{2}{3}r$$

(2)



$$I_{de}=\frac{1}{6}I$$



对a $\begin{cases} 3I_A+6I_B=I \\ I'_{de}=I'_{be}=\frac{1}{2}I_A \end{cases}$

对g: $I''_{de}=I_B$

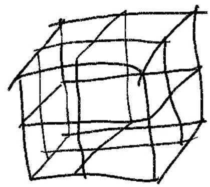
$$I_{de}=I'_{de}+I''_{de}=\frac{1}{2}I_A+I_B=\frac{1}{6}I$$

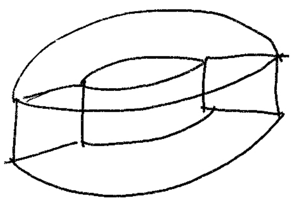
对称电阻网络

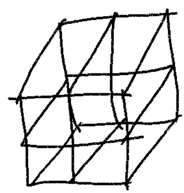
电流叠加 输入与输出电流叠加

对称分析 等势点/线的拆、合

无穷电阻网络

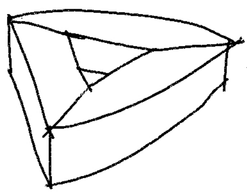






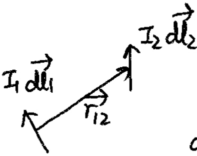
$V-E+F$

$8-16+8=0$



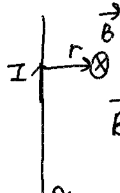
$$d\vec{F}_{12}=\frac{\mu_0}{4\pi}\frac{I_1 I_2\, d\vec{l}_2\times\left(d\vec{l}_1\times\hat{r}_{12}\right)}{r_{12}^2}$$

$$\vec{F}_{12}=-\vec{F}_{21}$$
$$=\frac{\mu_0}{4\pi}\oint_{l_1}\oint_{l_2}\frac{I_1 I_2\, d\vec{l}_2\times\left(d\vec{l}_1\times\hat{r}_{12}\right)}{r_{12}^2}$$

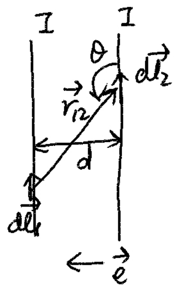


$$d\vec{B}_{12}=\frac{\mu_0}{4\pi}\frac{I_1\, d\vec{l}_1\times\hat{r}_{12}}{r_{12}^2}$$

$$d\vec{F}_{12}=I_2\, d\vec{l}_2\times d\vec{B}_{12}$$



$$\vec{B}=\frac{\mu_0 I}{2\pi r}$$



$$d\vec{F}_2=\int_1 d\vec{F}_{12}$$

$$B=\frac{\mu_0 I}{4\pi d}\left(\cos\theta_1-\cos\theta_2\right)$$

$$d\left(\frac{d}{\tan\theta}\right)$$

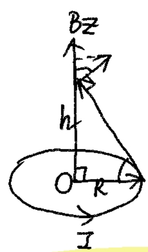
$$dF_{12}=\frac{\mu_0}{4\pi}I^2 dl_2\,\frac{dl_1\sin\theta}{(d/\sin\theta)^2}\vec{e}$$

$$=\frac{\mu_0}{4\pi}I^2\frac{\sin^3\theta}{d^2}\,d\,\frac{1}{\tan^2\theta}\,\frac{1}{\cos^2\theta}\,d\theta\, dl_2$$

$$=\frac{\mu_0 I^2}{4\pi d}\sin\theta\, d\theta\;dl_2$$

$(0,\pi)$

$$dF_2=\frac{\mu_0 I^2}{2\pi d}dl_2$$



$$B_z=\frac{\mu_0 I}{2}\frac{R^2}{(h^2+R^2)^{3/2}}$$

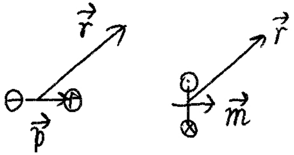
$h\gg R$

$$B_z\approx\frac{\mu_0 I}{2}\frac{R^2}{h^3}$$

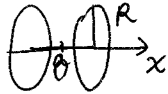
$$=\frac{\mu_0}{2\pi}\frac{m}{h^3}\qquad I\pi R^2$$

$d\ll R$

$$F\approx\frac{\mu_0 I^2}{\cancel{2\pi} d}\cancel{2\pi}R=\frac{\mu_0 I^2 R}{d}$$



Helmholtz 线圈



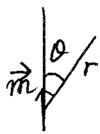
$a=R$时

$$B'(x)\big|_{x=0} = B''(x)\big|_{x=0} = 0$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\left(3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right)$$

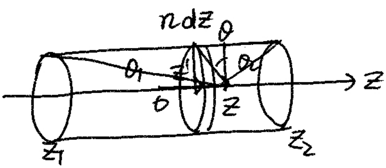
$$\vec{B} = \frac{\mu_0}{4\pi r^3}\left(3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right)$$

$$\frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\vec{e}_r + \sin\theta\,\vec{e}_\theta\right)$$



$$\vec{m} = I\vec{S}$$

$$= I\left(\frac{1}{2}\oint_L \vec{r}\times d\vec{r}\right)$$



$$\int_{z_1}^{z_2} \frac{\mu_0 I}{2}\frac{R^2}{\left(R^2+(z-z')^2\right)^{3/2}}\, n\,dz' = B(z)$$

$$z'-z = R\tan\theta$$

$$\frac{\mu_0 I n}{2}\left(\sin\theta_2 - \sin\theta_1\right) = B_p$$

① $\theta_1 \to -\frac{\pi}{2}$ $\theta_2 \to \frac{\pi}{2}$ $B_p \to \mu_0 n I$

② $\theta_1 = 0$ $\theta_2 \to \frac{\pi}{2}$ $B_p \to \frac{1}{2}\mu_0 n I$

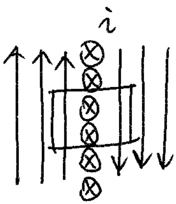
$$\vec{B} = \frac{\mu_0}{4\pi}\oint_L \frac{I d\vec{l}\times\hat{r}}{r^2} \Rightarrow \nabla\cdot B = 0$$

电流产生的磁场无源

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{R^3} \frac{\vec{j}\times\hat{r}\, dV}{r^2}$$

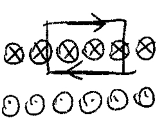
$$\Downarrow$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{j}$$



$$\mu_0 i\Delta l = 2B\Delta l$$

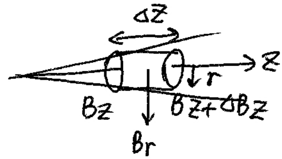
$$B = \frac{1}{2}\mu_0 i$$



理想螺线管

$$B = \mu_0 n I$$

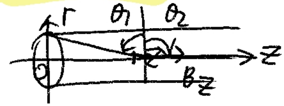
轴对称磁场 B_z B_r



$$B_r 2\pi r\Delta z + \Delta B_z \pi r^2 = 0$$

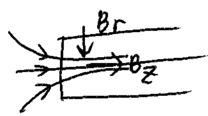
r很小

$$B_r = -\frac{r}{2}\frac{dB_z}{dz}$$

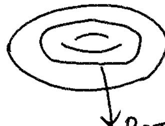


$$B_z = \frac{\mu_0 n I}{2}\left(1 + \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$B_r(z) = -\frac{r}{2}\frac{\mu_0 n I}{2}\frac{R^2}{(z^2+R^2)^{3/2}}$$



螺线环



$$B 2\pi r = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r}$$

$$\vec{B}=\frac{\mu_0 I}{4\pi}\oint_{L'}\frac{d\vec{l}'\times\vec{e}_r}{R^2}$$

$$=\frac{\mu_0 I}{4\pi}\oint_{L'}\nabla\times\left(\frac{d\vec{l}'}{R}\right)$$

$$=\nabla\times\vec{A}$$

$$\vec{A}=\frac{\mu_0 I}{4\pi}\oint_{L'}\frac{d\vec{l}'}{R}$$

$$\nabla\cdot A=0$$

$$\underbrace{\nabla\times(\nabla\times A)}_{\nabla\times B=\mu_0\vec{j}}=\nabla\underbrace{(\nabla\cdot A)}_{0}-\nabla^2 A$$

$$\nabla^2 A=-\mu_0\vec{j}$$

$$\left(\nabla^2 A_i=-\mu_0 j_i\right)$$

e.g. 均匀 $\vec{B}=B_0\vec{e}_z$

$$\nabla\times A=B$$

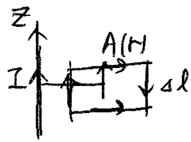
$$\begin{cases}\dfrac{\partial A_z}{\partial y}-\dfrac{\partial A_y}{\partial z}=0\\[2mm]\dfrac{\partial A_x}{\partial z}-\dfrac{\partial A_z}{\partial x}=0\\[2mm]\dfrac{\partial A_y}{\partial x}-\dfrac{\partial A_x}{\partial y}=B_0\end{cases}$$

有多解

$$\vec{A}=\begin{cases}-B_0 y\,\vec{e}_x\\ B_0 x\,\vec{e}_y\\ \frac{1}{2}\vec{B}_0\times\vec{r}\end{cases}$$

e.g. 无限长直导线

取 $\vec{A}$ 平行于导线



$$\left(A(r_0)-A(r)\right)\Delta l=\frac{\mu_0 I\Delta l}{2\pi}\int_{r_0}^{r}\frac{dr}{r}$$

$$\vec{A}(r)=\underbrace{\vec{A}(r_0)}_{\text{取为}\vec{0}}-\frac{\mu_0 I}{2\pi}\ln\frac{r}{r_0}\vec{e}_z$$

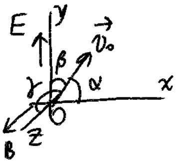
参考点

$$U(\vec{r})=\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

e.g 磁矩

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi r^3}(\vec{m}\times\vec{r})$$

带电粒子在磁场中的运动



$$\begin{cases} \vec{E}=(0,E,0) \\ \vec{B}=(0,0,B) \end{cases}$$

$$\vec{F}=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\begin{pmatrix} F_x \\ F_y \\ F_z \end{pmatrix}=q\begin{pmatrix} v_yB \\ E-v_xB \\ 0 \end{pmatrix}$$

$$\begin{cases} \dot{v}_x=\frac{qB}{m}v_y \\ \dot{v}_y=\frac{qE}{m}-\frac{qB}{m}v_x \end{cases} \qquad \omega=\frac{qB}{m}$$

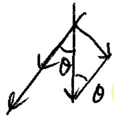
$$\ddot{v}_y=-\omega^2 v_y$$

$$\begin{cases} v_y=v_{\parallel}\cos(\omega t+\varphi) \\ v_x=\frac{E}{B}+v_{\parallel}\sin(\omega t+\varphi) \\ v_z=v_{\parallel}=v_0\cos\gamma \end{cases}$$

漂移

在任意场中的漂移速度

$$\vec{v}_F=\frac{\vec{E}\times\vec{B}}{B^2}$$

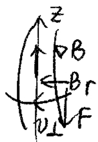


$$\vec{v}_F=\frac{\vec{F}\times\vec{B}}{qB^2}$$

$EB\sin\theta$

磁场不均匀引起的漂移

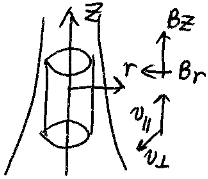
$$B_r=-\frac{r}{2}\frac{dB}{dz} \qquad F=qv_\perp \times B_r=-\mu\nabla B$$



$$v_F=\frac{\cancel{q}v_\perp\left(-\frac{1}{2}\frac{mv_\perp}{\cancel{q}B}\frac{dB}{dz}\right)\times B}{qB^2}$$

$$=\mu\frac{B\times\nabla B}{qB^2}$$

轴对称



时空缓变磁场中浸渐不变量

① $\mu = \dfrac{\frac{1}{2}mv_\perp^2}{B}$

② $\Phi = B\pi R^2 = 2\pi \dfrac{m}{q^2}\mu$

$$B_r = -\frac{r}{2}\frac{dB_z}{dz}$$

$$\mu = \frac{1}{2}\frac{mv_\perp^2}{B}$$

$$\frac{d\mu}{dt} = \frac{m}{2}\left(\frac{2v_\perp}{B}\frac{dv_\perp}{dt} - \frac{v_\perp^2}{B^2}\frac{dB}{dt}\right)$$

$$m\frac{dv_\perp}{dt} = -qv_\parallel B_r \qquad \frac{mv_\perp^2}{r} = qv_\perp B_z$$

$$\frac{dB}{dt} \approx \frac{dB_z}{dt} = \frac{dB_z}{dz}v_\parallel$$

$$\frac{m}{2}\left(\frac{2v_\perp}{B}\frac{-qB_r}{m}v_\parallel - \frac{v_\perp^2}{B^2}\frac{dB_z}{dz}v_\parallel\right) \approx 0$$

$$B_r = -\frac{r}{2}\frac{dB_z}{dz}$$

$$r = \frac{mv_\perp}{qB_z}$$

$B \approx B_z$

Hall 效应

磁场对电流的作用

$$\begin{cases} \text{体电流} \quad \vec{j} \quad \text{用该点的场} \\ \text{面电流} \quad \vec{i} \quad \text{用两侧平均场} \\ \text{线电流} \quad I d\vec{l} \times \text{发散} \end{cases}$$

对载流线圈

均匀B: $\vec{F}=\vec{0}$ $\qquad \vec{M}=\vec{\mu}\times\vec{B}$

非均匀B: $\vec{F}=\nabla(\vec{\mu}\cdot\vec{B})$ $\qquad \vec{M}=\vec{\mu}\times\vec{B}+\vec{r}\times\vec{F}$

(外场)

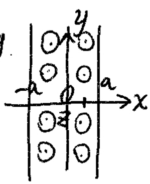
$$= B\times(\nabla\times\mu)+\mu\times(\nabla\times B)+(B\cdot\nabla)\mu+(\mu\cdot\nabla)B$$

若 μ不变 μ处 $\nabla\times B=0$

$\downarrow$

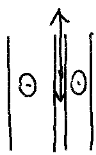
$(\mu\cdot\nabla)B$

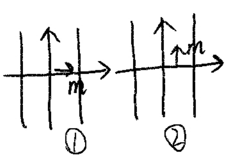
e.g.



$$\vec{j}(x)=\begin{cases} j_0\vec{e}_z & |x|\le a \\ 0 & |x|>a \end{cases}$$

$$\vec{B}(x)=\begin{cases} \frac{\mu_0}{2}\left(j_0(x+a)-j_0(a-x)\right)\vec{e}_y = \mu_0 j_0 x\vec{e}_y & |x|\le a \\ \mu_0 j_0 a\vec{e}_y & x>a \\ -\mu_0 j_0 a\vec{e}_y & x<-a \end{cases}$$





$\vec{F}_1=\nabla(\vec{m}\cdot\vec{B})=0$ 　不能用$(\mu\cdot\nabla)B$ 因为μ处 $\nabla\times B\neq0$

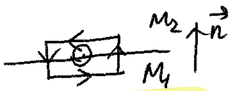
$\vec{F}_2=m\mu_0 j_0\vec{e}_x$

磁介质的磁化

$$\vec{j} = \nabla\times\vec{M}$$

$$\iint \vec{j}\cdot d\vec{S} = \oint \vec{M}\cdot d\vec{l}$$

界面上

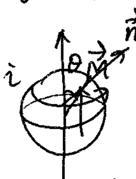


$$\vec{i} = \vec{n}\times(\vec{M}_2-\vec{M}_1)$$

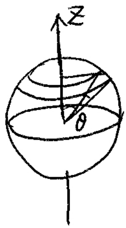
$$\begin{cases} B = \mu_0 H + \mu_0 M \\ D = \varepsilon_0 E + P \end{cases}$$

$$\begin{cases} H' = -\frac{1}{3}M \quad B' = \frac{2}{3}\mu_0 M \\ E' = -\frac{1}{3}\frac{P}{\varepsilon_0} \quad D' = \frac{2}{3}P \end{cases}$$

e.g. 均匀磁化球磁化电流在轴上产生的



$i = M\sin\theta$



$$R\,d\theta\, M\sin\theta\,\frac{\mu_0}{2}\frac{R^2\sin^2\theta}{(R^2\sin^2\theta+(z-R\cos\theta)^2)^{3/2}}$$

$$\frac{R^3 M\mu_0}{2}\int_0^{\pi}\frac{\sin^3\theta\,d\theta}{(R^2-2zR\cos\theta+z^2)^{3/2}}$$

$$= \begin{cases} \frac{\mu_0}{4\pi}\frac{2\mu}{|z|^3} & \text{球外} \\ \frac{2\mu_0 M}{3} & \text{球内} \end{cases} \qquad \mu = \frac{4}{3}\pi R^3 M$$

磁化规律

$M = \chi_m H$ 弱磁性.线性各向同性
无损耗

$$B = \mu_0 \mu_r H = \mu_0 H + \mu_0 \overbrace{\underbrace{(\mu_r - 1)}_{\chi_m} H}^{M}$$

$$\left(\begin{aligned} P &= \varepsilon_0 \chi_e E \\ D &= \varepsilon_0 \varepsilon_r E \\ &= \varepsilon_0 E + \overbrace{\varepsilon_0 \underbrace{(\varepsilon_r - 1)}_{\chi_e} E}^{P} \end{aligned} \right)$$

顺磁性 $\mu_r > 1$ $\chi_m > 0$ M与H同向

Curie's Law

$\chi_m \propto \frac{1}{T}$ (气体)

抗磁性 $\mu_r < 1$ $\chi_m < 0$ M与H反向

特殊磁性材料

超导材料

*: Cooper 对

$m^* = 2m$

$e^* = 2e$

$$\vec{j}_s = n_e^* e^* \vec{v}$$

$$m^* \frac{d\vec{v}}{dt} = e^* \vec{E}$$ 没有阻尼力

没有磁场？

$$\frac{d\vec{j}_s}{dt} = \frac{n_e^* e^{*2}}{m^*} \vec{E}$$ (London 第一方程)

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

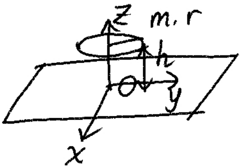
$$\frac{\partial}{\partial t}(\nabla \times j_s) = \frac{n_e^* e^{*2}}{m^*} \cancel{\nabla \times E}$$

$$-\frac{\partial B}{\partial t}$$

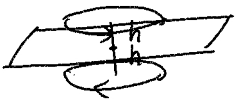
初始条件 $j_s = 0$ $B = 0$

$$\nabla \times j_s = -\frac{n_e^* e^{*2}}{m^*} B$$ (London 第二方程)

超导电像法



$r \gg h$



$$F = \frac{\mu_0}{\cancel{2\pi}} \frac{I}{2h} \cdot I \cdot \cancel{2\pi} R = mg$$

$$\Rightarrow I = \sqrt{\frac{2mgh}{\mu_0 R}}$$

$$F = \frac{\cancel{\mu_0 R}}{\cancel{2}h} \frac{\cancel{2}mgh_0}{\cancel{\mu_0 R}} \qquad h = h_0 + \delta$$

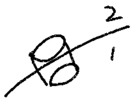
$$= mg\left(1 - \frac{\delta}{h_0}\right)$$

$$m\ddot{\delta} = F - mg = -\frac{mg}{h_0}\delta$$

$$\ddot{\delta} + \frac{g}{h_0}\delta = 0$$

$$T = 2\pi\sqrt{\frac{h_0}{g}}$$

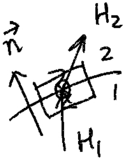
边值关系



$\nabla\cdot B=0 \Leftrightarrow B_{1n}=B_{2n}$

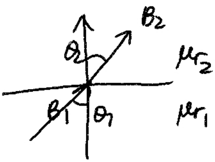
$\nabla\times H=j_0$ 传导电流

$\Updownarrow$



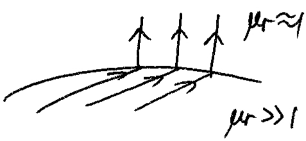
$\vec{n}\times(\vec{H_2}-\vec{H_1})=\vec{j_0}$

$j_0=0 \Rightarrow H_{1t}=H_{2t}$



$$\begin{cases} B_1\cos\theta_1=B_2\cos\theta_2 \\ \dfrac{B_1}{\mu_{r_1}}\sin\theta_1=\dfrac{B_2}{\mu_{r_2}}\sin\theta_2 \quad (j_0=0) \end{cases}$$

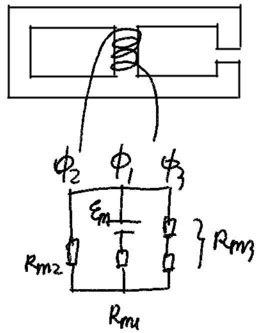
$$\frac{\tan\theta_1}{\mu_{r_1}}=\frac{\tan\theta_2}{\mu_{r_2}}$$



磁路

$$\Phi_m(R_m+r_m)=\mathcal{E}_m$$

$$R_m=\frac{l}{\mu S}$$



$$U_{m3}=\frac{(R_{m2}\parallel R_{m3})\mathcal{E}_m}{(R_{m2}\parallel R_{m3})+R_{m1}}=\phi_3 R_{m3}$$

介质中磁场的基本方程:

$$\begin{cases}\nabla\cdot B=0\\ \nabla\times H=j_0\end{cases}+\begin{matrix}\text{边值关系}\\ \text{本构方程}\end{matrix}$$

一、各向同性均匀介质充满空间

$$B=\mu_0\mu_r H=\mu_0(H+M)$$

$$\nabla\times B=\mu_0\underline{\mu_r j_0}$$

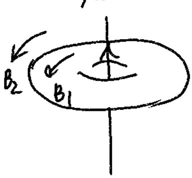
$$\vec{j}_0+\vec{j}'=\mu_r\vec{j}_0$$

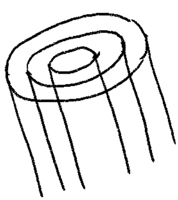
$$\begin{cases}\nabla\cdot H=0\\ \nabla\times H=j_0\end{cases}$$ H与真空中相同

二、介质界面与B平行

H不变

$$\vec{H}=\frac{\vec{B}_0}{\mu_0}\qquad \vec{B}=\mu_r\vec{B}_0$$



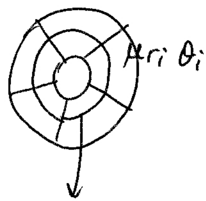


三、介质界面与B垂直

$\vec{B}$与$\vec{B}_0$差一常数

$$\oint\frac{\vec{B}}{\mu_0\mu_r}\cdot d\vec{l}=\sum I_0\Rightarrow B$$

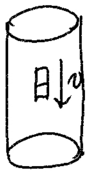
磁化电流仅出现在有传导电流出现的地方



$$\frac{Br}{\mu_0}\sum_{i=1}^{n}\frac{\theta_i}{\mu_{ri}}=I_0$$

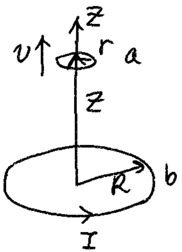
$$B=\frac{\mu_0 I_0}{r\sum_{i=1}^{n}\frac{\theta_i}{\mu_{ri}}}$$

电磁感应



$$\frac{\varepsilon^2}{R}=mgv$$

e.g.



$R \gg r$

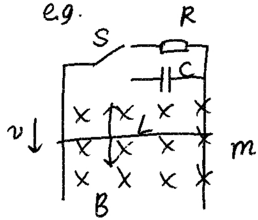
$z \gg R$

$$B=\frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\approx\frac{\mu_0 I}{2}\frac{R^2}{z^3}$$

$$\phi=\frac{\mu_0 I}{2}\frac{R^2}{z^3}\pi r^2$$

$$|\varepsilon|=\left|\frac{d\phi}{dt}\right|=\frac{\mu_0 I}{2}R^2\pi r^2\frac{-3}{z^4}v$$

e.g.



(1) 接R

$$\varepsilon=BLv$$

$$F=BIL$$

$$mg-F=m\frac{dv}{dt}$$

$$\varepsilon=IR$$

$$\Rightarrow mg-\frac{B^2L^2v}{R}=m\dot{v}$$

$$v=\frac{mgR}{B^2L^2}\left(1-e^{-\frac{B^2L^2}{mR}t}\right)$$

(2)

$$q=Cu$$

$$u=\varepsilon=BLv$$

$$\dot{q}=I$$

$$F=BIL$$

$$mg-F=m\dot{v}$$

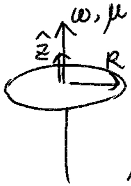
$$mg-B^2L^2C\dot{v}=m\dot{v}$$

$$\dot{v}=\frac{mg}{m+B^2L^2C}$$

$$\vec{E} = -\nabla u - \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\nabla \times \frac{\partial A}{\partial t}$$

e.g. 均匀带电圆盘匀加速旋转



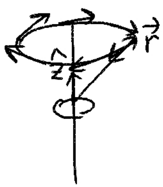
$q \quad \omega = \alpha t$

$$\mu = \int_0^R \underline{2\pi r\,dr\,\sigma}\;\underline{r\omega}\;\underline{\pi r^2}$$

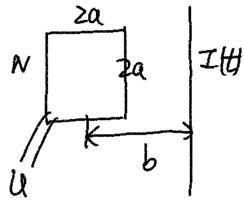
$$= 2\pi^2 \sigma \omega \frac{1}{5} R^5$$

远处: $A = \frac{\mu_0}{4\pi} \frac{\vec{\mu} \times \vec{r}}{r^3}$

$$E = -\frac{\partial A}{\partial t} = -\frac{\mu_0}{4\pi r^3} \frac{2}{5} \pi^2 R^5 \underset{\downarrow\ \frac{q}{\pi R^2}}{\sigma} \alpha\, \hat{z} \times \vec{r}$$



脉冲电流的测量



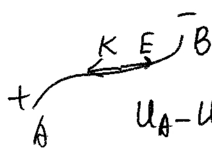
$$B = \frac{\mu_0 I(t)}{2\pi r}$$

$$u = N \frac{d\Phi}{dt}$$

$$\Phi = \int_{b-a}^{b+a} \frac{\mu_0 I(t)}{\cancel{2}\pi} \frac{\cancel{2}a\,dr}{r}$$

$$= \frac{\mu_0}{\pi} I(t)\, a \ln\left(\frac{b+a}{b-a}\right)$$

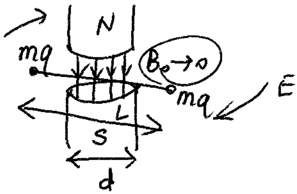
$$u = \frac{N \mu_0 a}{\pi} \ln\left(\frac{b+a}{b-a}\right) \frac{dI}{dt}$$

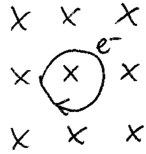


$$u_A - u_B = -\int_B^A E \cdot dl$$

$$= \int_B^A K \cdot dl$$

电磁场具有角动量:





$$\begin{cases} E\pi L = -\frac{dB}{dt}\cdot\pi\frac{d^2}{4} \\ 2qE\cdot\frac{1}{2}L = 2m\left(\frac{L}{2}\right)^2\frac{d\omega}{dt} \end{cases}$$

$$\dot{\omega} = -\frac{qd^2}{2mL^2}\dot{B}$$

$$\omega = \frac{qd^2}{2mL}B_0 \quad (\text{只与}\Delta B\text{有关})$$

$$2m\frac{L}{2}\omega = J = \frac{qd^2}{2}B_0$$

$$\begin{cases} \frac{mv^2}{R} = evB \\ \frac{\frac{1}{2}mv^2}{B} = \mu = Const \end{cases}$$

$$\Rightarrow \frac{1}{2m}\frac{(eBR)^2}{B} = \frac{e^2}{2m}BR^2 = \mu$$

$$B_1R_1^2 = B_2R_2^2$$

1 1 3 $\frac{1}{\sqrt{3}}$

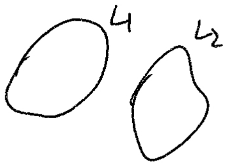
$S = E\times H$

$g = \frac{S}{c^2}$

$l = r\times g$

互感

$\Phi_{21}=M_{21}I_1$ 磁通定义



$$\begin{cases}\Phi_{21}=M_{21}I_1\\ \Phi_{12}=M_{12}I_2\end{cases}$$

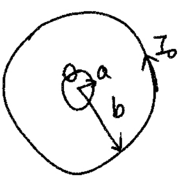
互感系数的对称性

$$M_{21}=M_{12}=\frac{\mu_0}{4\pi}\oint_{L_1}\oint_{L_2}\frac{d\vec{l}_1\cdot d\vec{l}_2}{r}$$

互感系数的计算

利用$M_{12}=M_{21}$ 简化问题

e.g.



$a<<b$

b在a中磁场

近似为常数

$$\Phi=\pi a^2\frac{\mu_0 I}{2b}$$

$$M=\frac{\mu_0\pi a^2}{2b}$$

e.g. 互绕螺线管

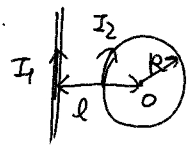
$S,\ l,\ N_1\ N_2$

$$\mu_0\frac{N_1}{l}I\cdot N_2 S=\Phi$$

$$M=\frac{\mu_0 S N_1 N_2}{l}$$

$$=\mu_0 n_1 n_2 V$$

e.g.



$$B=\frac{\mu_0 I}{2\pi r}$$

$$\Phi_{21}=MI_1$$

互感电动势

$$\begin{cases}\varepsilon_1=-M_{12}\frac{dI_2}{dt}\\ \varepsilon_2=-M_{21}\frac{dI_1}{dt}\end{cases}$$

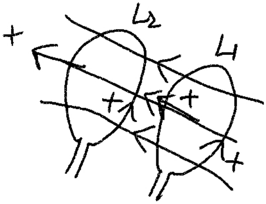
电动势定义

实际测量

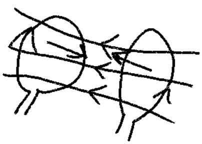
磁能定义

$$W=\iiint_V w\,dV=\pm MI_1I_2+\frac{1}{2}L_1I_1^2+\frac{1}{2}L_2I_2^2$$

M 何时为正
何时为负



M>0



M<0

取决于线圈中电流正方向的定义

自感

$\Phi = LI$ 磁通定义 $\varepsilon = -L\frac{dI}{dt}$ 电动势定义

$L = M_{11} = \frac{\mu_0}{4\pi}\oint_{L}\oint_{L}\frac{d\vec{l_1}\cdot d\vec{l_2}}{r}$ L总大于0

电磁惯量

可能发散 导线的曲线模型失效 必须考虑粗细

导线内有一定的电流分布，与不同层电流相对应的磁通不同

此时自感是个平均效应

磁能定义（方能）

$\frac{1}{2}LI^2 = \iiint_V w_m\, dV = \frac{1}{2}\iiint_V \vec{B}\cdot\vec{H}\, dV$

$L = \frac{W_m}{2I^2}$

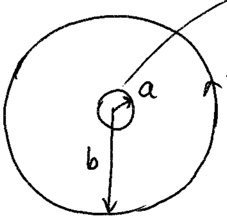
e.g.

理想单螺线管

$\mu\frac{N}{l}I\cdot N\cdot S = \Phi$

$L = \mu\frac{N^2S}{l} = \mu n^2 V$

e.g.

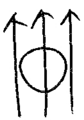


μ_r, a,
顺磁介质小球 ($\mu_r > 1$)

$a \ll b$

(1) 求小球 M

(2) 求 ΔL



$$B_0 = \frac{\mu_0 I}{2b}$$

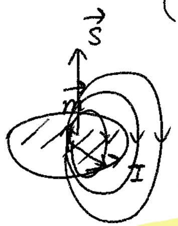
$$H_0 = \frac{I}{2b}$$

$$H' = -\frac{M}{3} \qquad B' = \frac{2}{3}\mu_0 M$$

$$\begin{cases} H = H_0 + H' \\ M = H(\mu_r - 1) \end{cases} \qquad \mu_0(H' + M) = B$$

$$\Rightarrow H = \frac{3}{\mu_r + 2} H_0$$

$$M = \frac{3(\mu_r - 1)}{\mu_r + 2} \frac{I}{2b}$$



$$\Delta\Phi = \Delta L I$$

$$\vec{m} = \frac{4}{3}\pi a^3 M = \frac{2\pi a^3 I}{b} \frac{\mu_r - 1}{\mu_r + 2}$$

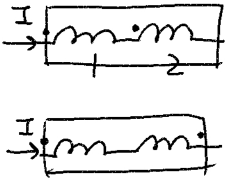
$$A \doteq \frac{\mu_0 \vec{m} \times \vec{r}}{4\pi r^3} \quad (a \ll b) \text{ 远场}$$

$$\iint_S \vec{B} \cdot d\vec{S} = \oint_{\partial S} \vec{A} \cdot d\vec{l}$$

$$= \mu_0 \frac{\pi a^3 I}{b^2} \frac{\mu_r - 1}{\mu_r + 2}$$

$$\Delta L = \mu_0 \frac{\pi a^3}{b^2} \frac{\mu_r - 1}{\mu_r + 2}$$

线圈的串并联

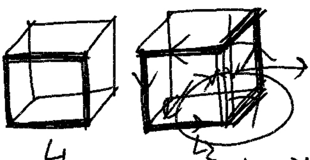


$$\Phi = \Phi_1 + \Phi_2$$

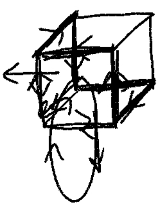
$$\Phi = \underbrace{(L_1 + L_2 + 2M)}_{L'} I$$

$$\Phi = \underbrace{(L_1 + L_2 - 2M)}_{L'} I$$

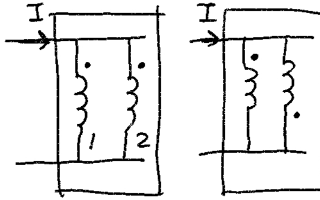
磁通定义



$$L_2 = 2L_1 - 2M$$



$$L_3 = 3L_1 - 6M$$
$$= 3L_1 - 3(2L_1 - L_2)$$
$$= 3(L_2 - L_1)$$



同名　　　　电动势定义

$$\varepsilon = -\left(L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}\right)$$
$$= -\left(L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}\right)$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

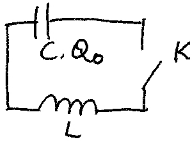
$$\Rightarrow \varepsilon = -L' \frac{dI}{dt} \qquad L' = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

异名

$$L' = \frac{L_1 L_2 - (-M)^2}{L_1 + L_2 - 2(-M)} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$\begin{cases} \text{无耦合} & M = 0 \\ \text{理想耦合} & M = \sqrt{L_1 L_2} \end{cases}$$

暂态过程



RL

$$\tau = \frac{L}{R}$$

$$I = -\frac{dq}{dt}$$

RC

$$\tau = RC$$

$$-L\frac{dI}{dt} + \frac{q}{C} = 0$$

RLC

$$\ddot{q} + \frac{1}{LC}q = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

磁能

载流线圈的磁能　　已考虑电源

$\sum_i \frac{1}{2} L_i I_i^2 + \frac{1}{2} \sum_{ij} M_{ij} I_i I_j$

载流线圈在外磁场中的势能
I_1　　　外磁场 ↓ 也是由电流产生 I_2

$W_{互} = M I_1 I_2 = I_1 \Phi_1$

均匀外场　$W = I_1 \vec{B} \cdot \vec{S} = \vec{m} \cdot \vec{B}$

磁能属于磁场

$w_m = \frac{1}{2} B \cdot H$

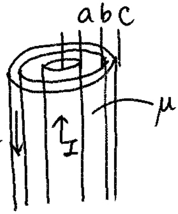
$W_e = \frac{1}{2} \sum_{k=1}^{N} Q_k u_k \leftrightarrow W_m = \frac{1}{2} \sum_{k=1}^{N} I_k \Phi_k$　　$u \leftrightarrow I$

$\frac{1}{2} \sum_k u_k \int_{V_k} \rho \, dV$　　$\frac{1}{2} \sum_k I_k \oint_{L_k} \vec{A} \cdot d\vec{l}$　　$Q \leftrightarrow \Phi$

$W_e = \frac{1}{2} \int_{V'} \rho u \, dV \leftrightarrow W_e = \frac{1}{2} \int_{V'} \vec{A} \cdot \vec{j} \, dV$

用自感、互感的磁能定义求 L、M

e.g.



$$L=\frac{2W_m}{I^2}$$

$0\le r\le a$

$$H=\frac{\pi r^2}{\pi a^2}\cdot I\frac{1}{2\pi r}=\frac{Ir}{2\pi a^2}$$

$$w=\frac{1}{2}\mu_0 H^2=\frac{\mu_0}{2}\frac{I^2}{4\pi^2 a^4}r^2$$

磁介质存在时的磁能

介质改变 L 与 M

电源在元过程中的做功

$$dA = d\underbrace{\left(\frac{1}{2}\mu_0 H^2\right)}_{\text{宏观磁能}} + \underbrace{\mu_0 \vec{H}\cdot d\vec{M}}_{\text{磁化功}}$$

对于线性无损耗介质

可定义磁化能 $\vec{H}\cdot d\vec{M} = \vec{M}\cdot d\vec{H}$

$$\mu_0 \vec{H}\cdot d\vec{M} = d\left(\frac{1}{2}\mu_0 \vec{H}\cdot\vec{M}\right)$$

$$w_m = \frac{1}{2}\mu_0 H^2 + \frac{1}{2}\mu_0 H\cdot M$$

$$= \frac{1}{2}\vec{B}\cdot\vec{H}$$

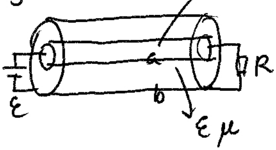
不仅内部有电流

表面还有电荷

$I = \frac{\varepsilon}{R}$

视作均匀分布

e.g.



$$D = \frac{\lambda}{2\pi r} \qquad H = \frac{I}{2\pi r}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r r} \qquad B = \frac{\mu_0\mu_r I}{2\pi r}$$

$$U = \frac{\lambda}{2\pi\varepsilon_0\varepsilon_r}\ln\frac{b}{a} = \varepsilon$$

$$\Downarrow$$

$$\lambda$$

载流线圈有源

“无线圈” 如基本粒子的固有磁矩

无源 无热效应 无感应电动势

区分

小磁矩在外磁场中的受力

$W=-\vec{m}\cdot\vec{B}$

$F=-(\nabla W)$

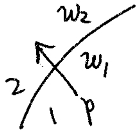
$F=\nabla(m\cdot B)=m\times\underbrace{(\nabla\times B)}_{0}+B\times(\nabla\times m)+(m\cdot\nabla)B+(B\cdot\nabla)m$

源在别处

m处无电流

固定$\vec{m}$ $F=(\vec{m}\cdot\nabla)\vec{B}$

$M=\vec{m}\times\vec{B}$

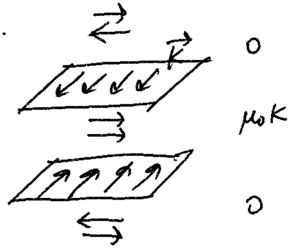


$P_{1\to2}=w_1-w_2$

电流平面受力

倾向使磁场能量增加

e.g.

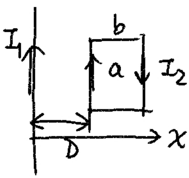


$\uparrow f \quad \downarrow \nabla w$

$f=-\nabla w$ 压力 $\frac{1}{2}\mu_0K^2=\frac{\Delta F}{\Delta S}$

$= -\frac{1}{2}\frac{(\mu_0K)^2}{\mu_0}\delta s$

e.g.



作用力与有源还是无源无关

$$W=\underbrace{\frac{1}{2}L_1I_1^2+\frac{1}{2}L_2I_2^2}_{不变}+\underbrace{\overset{I_2\Phi}{MI_1I_2}}_{>0}$$

$$F=\left(\frac{\partial}{\partial D}W\right)_{I_2}=\frac{\mu_0 a}{2\pi}I_1I_2\frac{1}{1+\frac{b}{D}}\left(-\frac{b}{D^2}\right)=-\frac{\mu_0 a}{2\pi}I_1I_2\frac{b}{D^2+Db}$$

引力

有源 保持I_2不变 (要考虑电源做功)

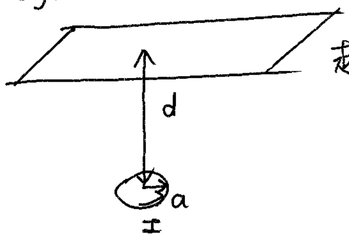
$$F=-\left(\frac{\partial}{\partial D}W\right)_{\Phi}$$ = 同. 引力

无源. 保持Φ_2不变

$I\longleftrightarrow U$

$\Phi\longleftrightarrow Q$

e.g.



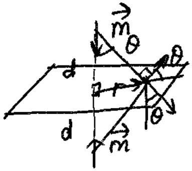
超导平面

$$F=\int_0^{+\infty} f(a)\,2\pi r\,dr$$

$$=\frac{9}{8}\mu_0 I^2 d^2 a^4\pi \underbrace{\int_0^{+\infty}\frac{r^2\,2r}{(r^2+d^2)^5}dr}_{=\frac{1}{12d^6}}$$

$$=\frac{3}{32}\frac{\mu_0 I^2 a^4\pi}{d^4}$$

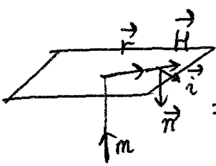
(1) 平面上感应电流



$$\vec{B}=\frac{\mu_0 m}{4\pi R^3}\left(2\cos\theta\,\vec{e}_r+\sin\theta\,\vec{e}_\theta\right)$$

$$B=\frac{\mu_0 m}{4\pi(r^2+d^2)^{3/2}}\,3\cdot\frac{rd}{r^2+d^2}\times 2$$

$$H=B/\mu_0=\frac{3I\pi a^2}{2\pi}\frac{rd}{(r^2+d^2)^{5/2}}\,\hat{r}$$

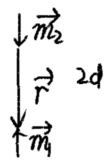


$$\vec{i}=\vec{n}\times(\vec{H}-0)$$

$$=\frac{3I}{2}\frac{a^2 rd}{(r^2+d^2)^{5/2}}\underbrace{\vec{n}\times\hat{r}}_{\hat{\theta}}$$

(2) 平面受力 $f=w_m=\frac{1}{2}\mu_0 H^2$ 斥力

$$\frac{1}{2}\mu_0\left(\frac{3}{2}I\frac{a^2 rd}{(r^2+d^2)^{5/2}}\right)^2$$



圆环受力

$$\vec{F}=\frac{3\mu_0}{4\pi r^4}\left[(\vec{m}_1\cdot\hat{r})\vec{m}_2+(\vec{m}_2\cdot\hat{r})\vec{m}_1+(\vec{m}_1\cdot\vec{m}_2)\hat{r}-5(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})\hat{r}\right]$$

$$\| $$

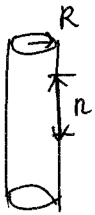
$$\frac{3\mu_0}{64\pi d^4}\left(-m^2-m^2-m^2+5m^2\right)\hat{r}$$

$$=\frac{3}{32}\frac{\mu_0 m^2}{\pi d^4}\hat{r}$$

斥力

$$\begin{cases} \nabla\cdot D=\rho \\ \nabla\times E=-\dfrac{\partial B}{\partial t} \\ \nabla\cdot B=0 \\ \nabla\times H=j_0+\underbrace{\dfrac{\partial D}{\partial t}}_{j_D} \end{cases}$$

e.g.



$I=I_0\sin\omega t$

求 j_D

只考虑一阶感应

$B=\mu_0 n I_0 \sin\omega t$

感应电场

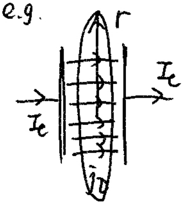
$$E\,2\pi r=-\pi r^2\frac{\partial B}{\partial t}$$

$$\vec{E}=\begin{cases} -\dfrac{r}{2}\mu_0 n I_0\omega\cos\omega t\,\vec{e}_\theta & (0\le r\le R) \\ -\dfrac{R^2}{2r}\mu_0 n I_0\omega\cos\omega t\,\vec{e}_\theta & (R<r) \end{cases}$$

$$j_D=\frac{\partial D}{\partial t}=\varepsilon_0\frac{\partial E}{\partial t}$$

$$=\begin{cases} \dfrac{1}{2}r\,\varepsilon_0\mu_0 n I_0\omega^2\sin\omega t\,\vec{e}_\theta & 0\le r\le R \\ \dfrac{R^2}{2r}\varepsilon_0\mu_0 n I_0\omega^2\sin\omega t & r>R \end{cases}$$

e.g.



$d\ll a$

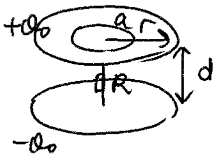
$$Ed=U=\frac{Q}{C} \qquad \frac{\partial E}{\partial t}=\frac{1}{dC}I_c$$

$$j_D=\frac{\varepsilon_0}{dC}I_c=\frac{I_c}{S}, \qquad C\to\frac{\varepsilon_0 S}{d}$$

$$H=\begin{cases} \dfrac{\pi a^2 j_D}{2\pi r}=\dfrac{I_c}{2\pi r} & r>a \\ \dfrac{\pi r^2 j_D}{2\pi r}=\dfrac{r}{2}\dfrac{I_c}{\pi a^2} & r<a \end{cases}$$

就相当于只由 I_c 激发 $d\ll a$ 间隙忽略

e.g.



$$B = \frac{r^2-a^2}{r^2}\frac{Q_0\mu_0}{2\pi aRC}e^{-\frac{t}{RC}}$$

$d \ll r$

R很大 τ很大

忽略高阶感应

(1) $U = \frac{Q}{C}$

$U = IR$

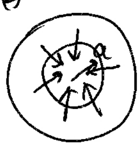
$I = -\dot{Q}$

$-\dot{Q}R = \frac{1}{C}Q$

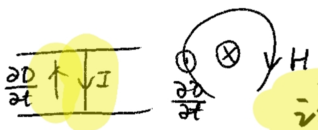
$\dot{Q} + \frac{1}{RC}Q = 0$ $\quad E = \frac{U}{d} = \frac{Q_0}{Cd}e^{-\frac{t}{RC}}$

$Q = Q_0 e^{-\frac{t}{RC}}$ $\quad \dot{E} = \frac{Q_0}{\varepsilon_0\pi r^2}\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$

(2)



$I_a = \frac{r^2-a^2}{r^2}I$



(3) $\nabla\times H = j_0 + \frac{\partial D}{\partial t}$

注意j_0

$H\cdot 2\pi a = I + \cancel{\pi}a^2\cancel{\varepsilon_0}\frac{Q_0}{\cancel{\varepsilon_0}\cancel{\pi}r^2}\left(-\frac{1}{RC}\right)e^{-\frac{t}{RC}}$

注意方向

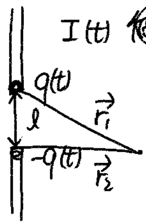
$= \frac{r^2-a^2}{r^2}\frac{Q_0}{RC}e^{-\frac{t}{RC}}$

似稳条件

毕萨定律仍成立

位移电流对磁场贡献近似为0

e.g.



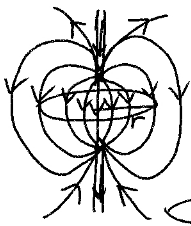
$I(t)$ 低频交流电

求 j_D

忽略高阶感应

$$\vec{E}(t) \approx \frac{q(t)}{4\pi\varepsilon_0}\left(\frac{\vec{r}_1}{r_1^3}-\frac{\vec{r}_2}{r_2^3}\right)$$

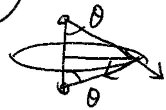
$$\vec{j}_D = \varepsilon_0 \frac{\partial \vec{E}(t)}{\partial t} = \frac{1}{4\pi}\left(\frac{\vec{r}_1}{r_1^3}-\frac{\vec{r}_2}{r_2^3}\right)\frac{dq(t)}{dt}$$



j_D

$$H\,2\pi r = \underbrace{\frac{dq(t)}{dt}}_{=I}\,\frac{2\pi(1-\cos\theta)}{4\pi}\times 2$$

$$H = \frac{I}{2\pi r}(1-\cos\theta)$$



由 B-S 定律计算 $H = \frac{I}{2\pi r}(1-\cos\theta)$

故相当于只考虑传导电流即可

电磁波

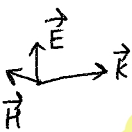
真空中电磁波

$$\begin{cases} \dfrac{\partial^2 E}{\partial t^2} - c^2\nabla^2 E = 0 \\ \dfrac{\partial^2 B}{\partial t^2} - c^2\nabla^2 B = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

平面波 $u = u_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}$

$$\begin{cases} \nabla\cdot E = 0 \quad \nabla\cdot H = 0 \\ \nabla\times E = -\mu_0\dfrac{\partial H}{\partial t} \\ \nabla\times H = \varepsilon_0\dfrac{\partial E}{\partial t} \end{cases} \Longrightarrow \begin{cases} \vec{k}\cdot\vec{E} = \vec{k}\cdot\vec{H} = 0 \\ \vec{k}\times\vec{E} = \mu_0\mu_r\omega\vec{H} \\ \vec{k}\times\vec{H} = -\varepsilon_0\varepsilon_r\omega\vec{E} \end{cases}$$

介质中 $\mu_0, \varepsilon_0 \to \mu_0\mu_r, \varepsilon_0\varepsilon_r$



$$\sqrt{\varepsilon_0\varepsilon_r}E_0 = \sqrt{\mu_0\mu_r}H_0$$

$$v = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} = \frac{c}{\underbrace{\sqrt{\varepsilon_r\mu_r}}_{n}}$$

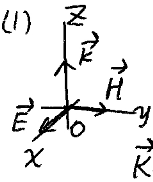
$$\frac{E_0}{B_0} = \frac{c}{n}$$

$$\sqrt{\frac{\omega}{\sigma\mu_0}} = \frac{E_0}{B_0}$$ 良导体

e.g.

$$\vec{E} = E_0\cos(\omega\sqrt{\mu_0\varepsilon_0}z - \omega t)\vec{e}_x$$

(1)



$$\vec{k} = \omega\sqrt{\mu_0\varepsilon_0}\,\vec{e}_z$$

$$\vec{k}\times\vec{E} = \mu_0\omega\vec{H}$$

$$\vec{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}E_0\cos(\omega\sqrt{\mu_0\varepsilon_0}z - \omega t)\vec{e}_y$$

(2)

$$w = \frac{1}{2}(\varepsilon_0E^2 + \mu_0H^2)$$

$$= \varepsilon_0E_0^2\cos^2(\omega\sqrt{\mu_0\varepsilon_0}z - \omega t)$$

$$\langle w\rangle = \frac{1}{2}\varepsilon_0E_0^2$$

(3)

$$S = E\times H$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\cos^2(\omega\sqrt{\mu_0\varepsilon_0}z - \omega t)\,\vec{e}_z$$

$$\langle S\rangle = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_0^2\vec{e}_z$$

电磁波的能量与动量

$\vec{S}=\vec{E}\times\vec{H}$ $\quad w=\frac{1}{2}\left(\varepsilon E^2+\frac{1}{\mu}B^2\right)$

$$-\frac{\partial w}{\partial t}=\nabla\cdot\vec{S}+\underbrace{\vec{j}\cdot\vec{E}}_{\text{对电流做功}}$$

$\vec{S}=w\vec{v}$

真空中 $\vec{S}=\frac{\vec{E}\times\vec{B}}{\mu_0}$

平面电磁波

$$\begin{cases}\vec{E}=E_0\cos(kx-\omega t)\,\vec{j}\\ \vec{B}=B_0\cos(kx-\omega t)\,\vec{k}\end{cases}$$

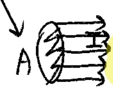
$$\vec{S}=\frac{E_0B_0}{\mu_0}\cos^2(kx-\omega t)\,\vec{i}$$

定义波的强度 $I=\langle S\rangle=\frac{E_0B_0}{2\mu_0}$

$\langle\cos^2(kx-\omega t)\rangle=\frac{1}{2}$

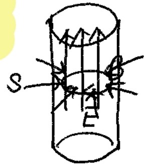
时间平均值

$$\begin{cases}\text{动量密度}\quad \vec{g}=\frac{1}{c^2}\vec{S}\\ \text{角——}\quad \vec{l}=\vec{r}\times\vec{g}\end{cases}$$



$p=IA$

e.g. 螺线管



$I=kt$

$B=\mu_0 nI \quad r<R$

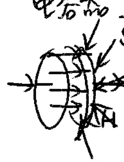
$E=\frac{r}{2}\mu_0 nk \quad r<R$

$$S=\frac{E\times B}{\mu_0}=-\frac{r}{2}\mu_0 n^2k^2t\,\vec{e}_r \quad r<R$$

W_e 不变

$W_m \nearrow$

e.g. 电容器



$I=\text{Const.}$

$Q=It$

$D=\frac{Q}{S}=\frac{I}{S}t \quad (r<R)$

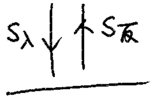
$H=\frac{r}{2}\frac{I}{S} \quad (r<R)$

$$S=-\frac{1}{\varepsilon_0}D\times H=-\frac{1}{\varepsilon_0}\frac{I^2}{S^2}\frac{r}{2}t\,\vec{e}_r \quad r<R$$

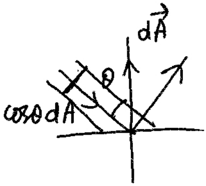
π^2R^4

W_m 不变

$W_e \nearrow$

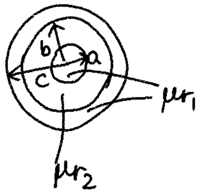


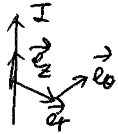
$$\frac{1}{c^2}\left(\vec{S}_入 - \vec{S}_反\right) = \Delta p$$



$$d\vec{F} = -\underbrace{\frac{(1+R)\,\bar{S}\cos\theta}{c^2}\cos\theta}_{\cos^2\theta}\, d\vec{A}$$

例





$r<a$

$$H\,2\pi r=\frac{r^2}{a^2}I$$

$$\vec{H}=\frac{r}{2\pi a^2}I\vec{e}_\theta$$

$$\vec{B}=\mu_0\mu_{r_1}\frac{r}{2\pi a^2}I\vec{e}_\theta$$

$$\vec{M}=(\mu_{r_1}-1)\vec{H}=(\mu_{r_1}-1)\frac{rI}{2\pi a^2}\vec{e}_\theta$$

$a<r<b$

$$\vec{H}=\frac{I}{2\pi r}\vec{e}_\theta$$

$$\vec{B}=\mu_0\mu_{r_2}\frac{I}{2\pi r}\vec{e}_\theta$$

$$\vec{M}=(\mu_{r_2}-1)\frac{I}{2\pi r}\vec{e}_\theta$$

$b<r<c$

$$\vec{H}=\frac{1}{2\pi r}\left(1-\frac{r^2-b^2}{c^2-b^2}\right)I\vec{e}_\theta$$

$B\sim\quad M\sim$

$r>c$

$$\vec{H}=0\quad B=M=0$$

$$w_m=\frac{1}{2}B\cdot H=\frac{1}{2}\mu_0\mu_r H^2$$

$$W_1=\int_0^a\frac{1}{2}\mu_0\mu_{r_1}\left(\frac{I}{2\pi a^2}\right)^2 r^2\,l\,2\pi r\,dr$$

$$=\frac{1}{2}\mu_0\mu_{r_1}\frac{I^2}{2\pi \cancel{a^4}}l\,\frac{1}{4}\cancel{a^4}$$

$$=\frac{\mu_0\mu_{r_1}}{16\pi}I^2 l$$

$$W_2=\int_a^b\frac{1}{2}\mu_0\mu_{r_2}\left(\frac{I}{2\pi r}\right)^2 l\,2\pi r\,dr$$

$$=\frac{1}{2}\mu_0\mu_{r_2}\frac{I^2}{2\pi}l\ln\left(\frac{b}{a}\right)$$

$$W_3=\int_b^c\frac{1}{2}\mu_0\mu_{r_1}\left(\frac{I}{2\pi r}\right)^2\left(\frac{c^2-r^2}{c^2-b^2}\right)^2 l\,2\pi r\,dr$$

$$=\frac{1}{2}\mu_0\mu_{r_1}\frac{I^2 l}{2\pi}\frac{1}{(c^2-b^2)^2}\int_b^c\left(\frac{c^4}{r}-2c^2r+r^3\right)dr$$

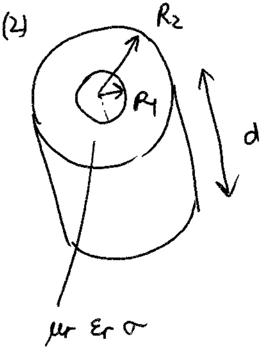
$$\left(c^4\ln\left(\frac{c}{b}\right)-c^2(c^2-b^2)+\frac{1}{4}(c^4-b^4)\right)$$

$$=\frac{\mu_0\mu_{r_1}I^2 l}{4\pi}\left(\frac{c^4}{(c^2-b^2)^2}\ln\left(\frac{c}{b}\right)-\frac{c^2}{c^2-b^2}+\frac{1}{4}\frac{c^2+b^2}{c^2-b^2}\right)$$

$$W_4=0$$

$$L=\frac{2W}{I^2 l}=\frac{\mu_0\mu_{r_1}}{8\pi}+\mu_0\mu_{r_2}\frac{1}{2\pi}\ln\left(\frac{b}{a}\right)+\frac{\mu_0\mu_{r_1}}{2\pi}\left(\frac{e^4}{(c^2-b^2)^2}\ln\left(\frac{c}{b}\right)-\frac{c^2}{c^2-b^2}+\frac{1}{4}\frac{c^2+b^2}{c^2-b^2}\right)$$

(2)



$$H=\frac{I}{2\pi r}$$

$$W=\int_{R_1}^{R_2}\frac{1}{2}\mu_0\mu_r\frac{I^2}{(2\pi r)^2}\,d\,2\pi r\,dr$$

$$=\frac{\mu_0\mu_r}{4\pi}I^2 d\ln\left(\frac{R_2}{R_1}\right)$$

$$L=\frac{\mu_0\mu_r d}{2\pi}\ln\left(\frac{R_2}{R_1}\right)$$

$$E=\frac{Q/d}{2\pi\varepsilon_0\varepsilon_r r}$$

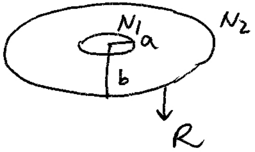
$$R=\int_{R_1}^{R_2}\frac{dr}{d\cdot 2\pi r\cdot\sigma}$$

$$=\frac{1}{2\pi\sigma d}\ln\left(\frac{R_2}{R_1}\right)$$

$$U=\int_{R}^{R_2}\frac{Q/d}{2\pi\varepsilon_0\varepsilon_r r}\,dr=\frac{Q/d}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow C=\frac{Q}{U}=\frac{2\pi\varepsilon_0\varepsilon_r d}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$\begin{cases} RC=\dfrac{\varepsilon_0\varepsilon_r}{\sigma} \\ L/R=\mu_0\mu_r\sigma \\ LC=\mu_0\mu_r\varepsilon_0\varepsilon_r \end{cases}$$



$a \ll b$

(1)

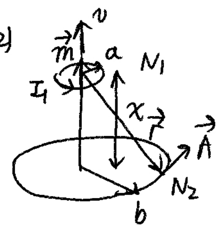
$M = M_{12}$

$\phi_{12} = M_{12} I_2$

$\phi_{12} = N_1 \pi a^2 \dfrac{\mu_0 N_2 I_2}{2b}$

$\Rightarrow M = \dfrac{\pi a^2}{2b}\mu_0 N_1 N_2$

(2)



$\Phi_2 = \oint_{L_2} \vec{A}\cdot d\vec{l} = \dfrac{\mu_0 I_1 \pi a^2}{2}\dfrac{b}{x^2+b^2} N_1 N_2$

$\varepsilon = -\dfrac{d\Phi_2}{dt} = -\dfrac{\mu_0 I_1 \pi a^2 b}{\cancel{2}}\left(-\dfrac{\cancel{2}x}{(x^2+b^2)^2}\right)\cdot v N_1 N_2$

$= \dfrac{\mu_0 I_1 \pi a^2 b x}{(x^2+b^2)^2} v N_1 N_2$

(3)

$i = \varepsilon / R$

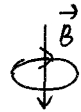
$\int_0^{+\infty} i\,dt = \dfrac{1}{R}\int_0^{+\infty} \varepsilon\,dt$

$x = vt$

$\underbrace{\int_0^{+\infty} \mu_0 I_1 \pi a^2 b \dfrac{x}{(x^2+b^2)^2}dx\, N_1 N_2}_{}$

$= \mu_0 I_1 \pi a^2 b \dfrac{1}{2}\dfrac{1}{b^2} N_1 N_2$

$Q = \dfrac{\mu_0 I_1 \pi a^2 N_1 N_2}{2Rb}$



(4) $\vec{B} = \dfrac{\mu_0 N_2 I_2}{2}\dfrac{b^2}{(x^2+b^2)^{3/2}}\left(-\vec{e}_z\right)$

$W = -m\cdot B$

$F = -\nabla(m\cdot B) = (m\cdot\nabla)B$

$= -N_1 I_1 \pi a^2 \dfrac{d}{dx}\vec{B}$

$= N_1 I_1 \pi a^2 \dfrac{\mu_0 N_2 I_2}{2}\dfrac{b^2\, 3x}{(x^2+b^2)^{5/2}}$

$= mg \quad (x=h)$

$I_2 = \dfrac{2mg\,(h^2+b^2)^{5/2}}{N_1 N_2 \mu_0 I_1 \pi a^2 b^2\, 3h}$

$2h > b$

(5)

$$F(h+\delta)=\frac{1}{2}N_1N_2I_1I_2\,3\pi a^2b^2\mu_0\,\frac{h+\delta}{((h+\delta)^2+b^2)^{5/2}}$$

$$\frac{h}{(h^2+b^2)^{5/2}}\left(1+\frac{\delta}{h}\right)\left(1-\frac{5}{2}\frac{2h\delta}{h^2+b^2}\right)$$

$$=mg\left(1-\frac{4h^2-b^2}{h(h^2+b^2)}\delta\right) \qquad 1+\left(\frac{1}{h}-\frac{5h}{h^2+b^2}\right)\delta$$

$$\Rightarrow T=2\pi\sqrt{\frac{h(h^2+b^2)}{(4h^2-b^2)g}}$$ $2h>b$时 可稳定平衡