

# 光学

## 第一章

$$n_1 \sin i_1 = n_2 \sin i_2 \quad n = \frac{c}{v}$$

$$i_c = \arcsin \frac{n_2}{n_1} \quad \text{临界角}$$

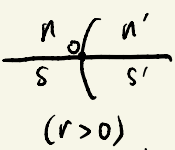


$$n = \frac{\sin \frac{\alpha + \delta_{\min}}{2}}{\sin \frac{\alpha}{2}} \quad \delta_{\min} = (n-1)\alpha \quad \text{最小偏向角}$$

$[L] = nS$  费马原理: 空间两点间实际光线路径是光程为平稳值的路径。  
 即  $dL = 0$   
 ↑ 光程

符号规则: 以球面顶点O为基准, S在O左为实物,  $S > 0$ , S'在O右为实像,  $S' > 0$   
 折射球面球心C在O右,  $r > 0$ , 在主光轴向上,  $y > 0$

### 单球面傍轴



$$\frac{n'}{S'} + \frac{n}{S} = \frac{n' - n}{r} \quad \text{光焦度 } P = \frac{n' - n}{r} = \frac{n' - n}{r}$$

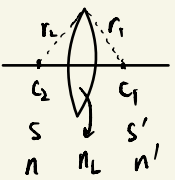
$$\text{物方焦距 } f = \frac{n}{n' - n} r = \frac{n}{P} \quad \text{像方焦距 } f' = \frac{n'}{n' - n} r = \frac{n'}{P}$$

$$\text{高斯公式: } \frac{f'}{S'} + \frac{f}{S} = 1 \quad \text{牛顿公式: } x'x = ff' \quad x = S - f, \quad x' = S' - f'$$

$$\text{横向放大率 } V = -\frac{nS'}{n'S} \quad V < 0 \text{ 倒立, } V > 0 \text{ 正立}$$

$$\text{角放大率 } \gamma = -\frac{S}{S'}$$

### 薄透镜



$$\frac{n'}{S'} + \frac{n}{S} = \frac{n_1 - n}{r_1} + \frac{n' - n_1}{r_2} \quad P_1 = \frac{n_1 - n}{r_1} \quad P_2 = \frac{n' - n_1}{r_2} \quad P = P_1 + P_2$$

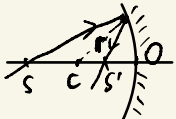
$$\text{物方焦距 } f = \frac{n}{\frac{n_1 - n}{r_1} + \frac{n' - n_1}{r_2}} \quad \text{像方焦距 } f' = \frac{n'}{\frac{n_1 - n}{r_1} + \frac{n' - n_1}{r_2}}$$

$$P = \frac{n}{f} = \frac{n'}{f'} \quad \frac{n'}{S'} + \frac{n}{S} = P \quad \text{高斯公式 } \frac{f'}{S'} + \frac{f}{S} = 1$$

$$\text{空气中高斯公式 } \frac{1}{S'} + \frac{1}{S} = \frac{1}{f}$$

球面反射

$\frac{1}{s'} + \frac{1}{s} = -\frac{2}{r}$  . 物方. 像方. 球面反射  $f = f' = -\frac{r}{2}$



$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f}$

显微镜

放大率  $M = M_{物} \times M_{目}$  .  $M_{物} = -\frac{L}{f_{物}}$  .  $L$ : 镜筒长度, 即为物镜后焦点到目镜前焦点的距离

$M_{目} = \frac{D}{f_{目}}$  .  $D$ : 人眼的明视距离 .  $M = -\frac{LD}{f_{物}f_{目}}$

望远镜

视角放大率  $M = -\frac{f_{物}}{f_{目}} = -\frac{f'_{物}}{f_{目}}$

开普勒型望远镜  $f_E = f'_E > 0$  . 凸透镜  
伽利略型望远镜  $f_E = f'_E < 0$  . 凹透镜

第二章

$\psi = A \cos(\omega t - kx + \varphi_0)$  .  $k = \frac{2\pi}{\lambda}$  空间角频率

偏振度  $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$  . (反衬度)

偏振片:

线偏振光:  $I = I_0 \cos^2 \alpha$  (马吕斯定律)

自然光:  $I = \frac{1}{2} I_0$

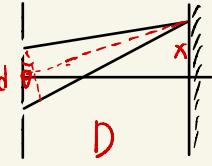
布儒斯特定律: 反射光与折射光垂直时, 反射光为线偏振光.  $i_0 = \arctan \frac{n_2}{n_1}$

第三章

相干条件: (1) 频率相同 (2) 振动方向不垂直 (3) 有稳定的初相差

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta\varphi$  . 当  $I_1 = I_2 = I_0$  时  $I = 2I_0 (1 + \cos \Delta\varphi) = 4I_0 \cos^2 \frac{\Delta\varphi}{2}$

$\Delta\varphi = \frac{2\pi}{\lambda} \Delta L$



$\Delta L = d \sin \theta$  .  $\sin \theta = \frac{x}{D}$  .  $\Rightarrow \Delta L = d \frac{x}{D}$  .  $\Delta\varphi = \frac{2\pi}{\lambda} \frac{dx}{D}$

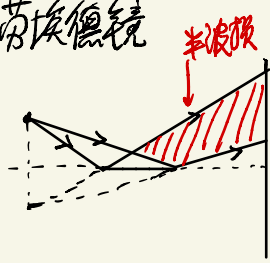
$I = 4I_0 \cos^2 \frac{\pi dx}{\lambda D}$

干涉极大:  $\frac{\pi dx}{\lambda D} = m\pi \Rightarrow x = m \frac{D}{\lambda}$  . 干涉极小:  $\frac{\pi dx}{\lambda D} = (m + \frac{1}{2})\pi \Rightarrow x = (m + \frac{1}{2}) \frac{D}{\lambda}$

相邻两明纹间距  $\Delta x = \frac{D}{f} \lambda$

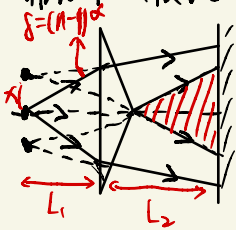
对不同波长光, 各级条纹最高级次:  $(m+1) \frac{D}{f} \lambda = m \frac{D}{f} (\lambda + \Delta \lambda) \Rightarrow m = \frac{\lambda}{\Delta \lambda}$

劳埃德镜



$$\Delta x = \frac{D}{f} \lambda \quad x=0 \text{ 处为干涉极小}$$

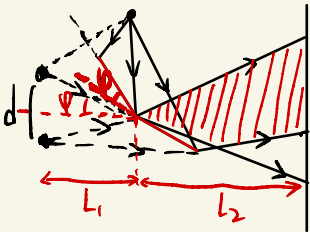
菲涅耳双棱镜



$$x = L_1(n-1)\alpha \quad D = L_1 + L_2$$

$$\Delta x = \frac{D}{d} \lambda = \frac{\lambda(L_1 + L_2)}{2L_1(n-1)\alpha}$$

菲涅耳双面镜

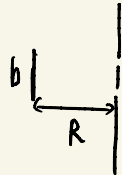


$$d = L_1 \cdot 2\varphi \quad D = L_1 + L_2$$

$$\Delta x = \frac{D}{d} \lambda = \frac{\lambda(L_1 + L_2)}{2L_1\varphi}$$

空间相干性

对比度  $Y = \left| \frac{\sin u}{u} \right|$  .  $u = \frac{\pi b d}{\lambda R} = \frac{\pi b \beta}{\lambda}$  .  $b$ : 光源宽度 .  $\beta = \frac{d}{R}$



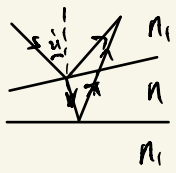
极限宽度为  $u = \pi$  时 ( $Y = 0$ ) .  $b_c = \frac{\lambda R}{d}$

时间相干性:

谱线宽度为  $\Delta \lambda$  的光源, 持续时间  $T = \frac{\lambda^2}{c \Delta \lambda}$  . 波列长度  $L_c = cT = \frac{\lambda^2}{\Delta \lambda}$

$\Delta L \leq L_c$  时才可干涉

### 分振幅干涉



$$\Delta L = 2h \sqrt{n^2 - n_1^2 \sin^2 i}$$

### 等厚干涉

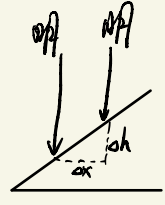


$$\Delta L = 2h + \frac{\lambda}{2} \quad \text{干涉极大: } 2h + \frac{\lambda}{2} = m\lambda$$

$$\text{干涉极小: } 2h + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda$$

$$\text{相邻明条纹对应厚度差: } \Delta h = \frac{\lambda}{2n}$$

$$\text{相邻明条纹间距: } \Delta x = \frac{\lambda}{2n \sin \theta} \approx \frac{\lambda}{2n\theta}$$



### 牛顿环



$$\Delta L = 2h + \frac{\lambda}{2} \quad \text{干涉极大: } 2h + \frac{\lambda}{2} = m\lambda$$

$$\text{干涉极小: } 2h + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda$$

$$r_m^2 = R^2 - (R - h_m)^2 = 2Rh_m - h_m^2$$

牛顿环中心点为暗点

$$\text{当 } R \gg h_m \text{ 时, } r_m^2 \approx 2Rh_m, \quad h_m = \frac{r_m^2}{2R}$$

$$\text{干涉极大: } \frac{r_m^2}{2R} = \frac{1}{2}(m - \frac{1}{2})\lambda \quad \text{于是 } r_m = \sqrt{\frac{(2m-1)R\lambda}{2}}$$

$$\text{干涉极小: } \frac{r_m^2}{2R} = \frac{1}{2}m\lambda \quad \text{于是 } r_m = \sqrt{mR\lambda}$$

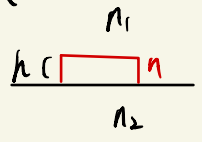
$$\text{在第 } m \text{ 个暗环处, 相邻暗条纹间距为 } \Delta r_m = \frac{dr_m}{dm} = \frac{R\lambda}{2r_m}$$

$$\text{曲面半径 } R = \frac{r_{m+1}^2 - r_m^2}{N\lambda}$$

### 等倾干涉

$$\Delta r_N = r_{N+1} - r_N = \frac{fn\lambda}{2n^2 h i_1}$$

### 增透膜



$n_1 < n < n_2$  时, 无半波损

$$\Delta L = 2nh \quad \text{反射光未相消时, } \Delta L = (m + \frac{1}{2})\lambda \quad h = (m + \frac{1}{2}) \frac{\lambda}{2n}$$



# 迈克尔逊干涉仪

每移动半个波长, 则出现/消失一个条纹.

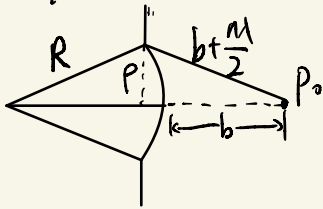
若移动  $N$  个, 则移动距离  $D = N \frac{\lambda}{2}$

能级数  $1/a$  个条纹变化时, 则长精度为  $\lambda/2a$

# 第四章

## 菲涅耳衍射

半波带法  $A = A_1 - A_2 + A_3 - \dots = \frac{1}{2} (A_1 + (-1)^{n+1} A_n)$



$$P = \sqrt{\frac{Rb}{R+b}} n \lambda \quad n = \left(\frac{1}{R} + \frac{1}{b}\right) \frac{P^2}{\lambda}$$

圆屏衍射  $A = (-1)^k \frac{A_{k+1}}{2}$

菲涅耳波带  $P_n = \sqrt{\frac{Rb}{R+b}} n \lambda \quad P_n = \sqrt{n} P_1 \quad n = \left(\frac{1}{R} + \frac{1}{b}\right) \frac{P_n^2}{\lambda}$

$$\text{令 } f = \frac{P_n^2}{n \lambda} = \frac{P_1^2}{\lambda} \quad \frac{1}{R} + \frac{1}{b} = \frac{1}{f}$$

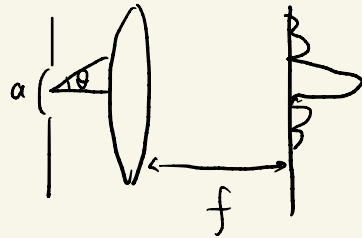
## 单缝的夫琅禾费衍射

$$I = I_0 \left(\frac{\sin \alpha}{\alpha}\right)^2 \quad \alpha = \frac{\pi a \sin \theta}{\lambda}$$

主极大:  $\theta = 0$  时,  $\alpha = 0$

极小:  $\alpha = \pm k\pi$ , 即  $\sin \theta = \pm k \frac{\lambda}{a}$

次极大:  $\sin \theta = \pm 1.43 \frac{\lambda}{a} \dots$



## 各级主极大亮斑宽度:

两侧极小位置:  $\sin \theta = \pm \frac{\lambda}{a}$ , 傍轴  $\sin \theta \approx \theta$ , 半角宽度  $\Delta \theta = \frac{\lambda}{a}$ , 角宽度  $2\Delta \theta = \frac{2\lambda}{a}$ .

$$\text{线宽度 } \Delta x_0 = f \cdot 2\Delta \theta = 2f \frac{\lambda}{a}$$

次极大亮斑宽度:  $\Delta x = \frac{1}{2} \Delta x_0$

### 圆孔夫琅禾费衍射

$$I = I_0 \left( \frac{2J_1(u)}{u} \right)^2, \quad u = \frac{2\pi}{\lambda} a \sin\theta, \quad a \text{ 为圆孔半径}$$

第一级主极大为圆斑，光能约占84%，称为艾里斑。

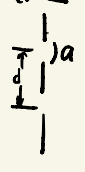
第一暗斑位置:  $u = 1.22\pi, \quad \Delta\theta = 0.61 \frac{\lambda}{a} = 1.22 \frac{\lambda}{D}$

艾里斑半径  $r = f \Delta\theta = 1.22 \frac{\lambda f}{D}$

光学仪器的分辨率本领: 瑞利判据

最小分辨角:  $\Delta\theta = 1.22 \frac{\lambda}{D}$

### 双缝夫琅禾费衍射



$$I = 4I_0 \left( \frac{\sin\alpha}{\alpha} \right)^2 \cos^2\beta, \quad \beta = \frac{\pi d \sin\theta}{\lambda}, \quad \alpha = \frac{\pi a \sin\theta}{\lambda}$$

$\uparrow$  单缝衍射
 $\uparrow$  双缝干涉

双缝衍射: 单缝衍射对双缝干涉调制

双缝衍射极大 (即双缝干涉极大):  $d \sin\theta = k\lambda$

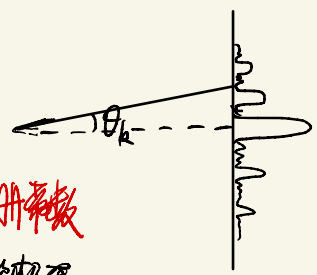
双缝衍射极小 (即双缝干涉极小):  $d \sin\theta = (k + \frac{1}{2})\lambda$

缺级 (即单缝衍射极小 && 双缝干涉极大):  $\begin{cases} d \sin\theta = k\lambda \\ a \sin\theta = m\lambda \end{cases}$

### 光栅的夫琅禾费衍射: 单缝衍射对缝间干涉调制

$$I = I_0 \left( \frac{\sin\alpha}{\alpha} \right)^2 \left( \frac{\sin N\beta}{\sin\beta} \right)^2, \quad \alpha = \frac{\pi a \sin\theta}{\lambda}, \quad \beta = \frac{\pi d \sin\theta}{\lambda}$$

$\uparrow$  单缝衍射
 $\uparrow$  缝间干涉因子
 $\rightarrow$  光栅方程
 $d$ : 光栅常数



主极大:  $d \sin\theta = k\lambda$ . 每两个主极大之间有  $N-1$  个零点,  $N-2$  个次极大

零点:  $\sin N\beta = 0$  且  $\sin\beta \neq 0$ . 即  $\beta = k\pi + \frac{m}{N}\pi$ .  $N$  为槽数

主极大的半角宽度  $\Delta\theta_k$  ( $k$ 级):  $\sin\theta_k = k\frac{\lambda}{d}$  (极大) 到  $\sin(\theta_k + \Delta\theta_k) = (k + \frac{1}{N})\frac{\lambda}{d}$  (边上第一零点,  $m=1$ )

于是  $\sin(\theta_k + \Delta\theta_k) - \sin\theta_k \approx (\sin\theta_k)' \Delta\theta_k = \cos\theta_k \cdot \Delta\theta_k = \frac{\lambda}{Nd}$ . 衍射半角宽  $\Delta\theta_k = \frac{\lambda}{Nd \cos\theta_k}$

中央主极大处半角宽:  $\omega_3 \theta \approx 1$ . 干涉  $\Delta \theta_0 = \frac{\lambda}{Nd}$

各级:  $\left\{ \begin{array}{l} \text{干涉明纹: } d \sin \theta = \pm k \lambda \\ \text{衍射暗纹: } a \sin \theta = \pm k' \lambda \end{array} \right.$

## 光栅仪

光栅方程:  $d \sin \theta = \pm k \lambda$

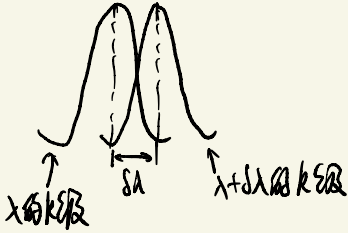
光栅的色散本领: 角色散本领:  $D_\theta \equiv \frac{\delta \theta}{\delta \lambda} = \frac{k}{d \cos \theta_k}$

线色散本领:  $D_L \equiv \frac{\delta L}{\delta \lambda} = f D_\theta = \frac{kf}{d \cos \theta_k}$   $f$ : 像焦距

光栅的分辨率:  $R \equiv \frac{\lambda}{\delta \lambda}$ . 能分辨的最小波长:  $\delta \lambda = \frac{\lambda}{kN}$

$$R = kN$$

$N$  为槽线条数

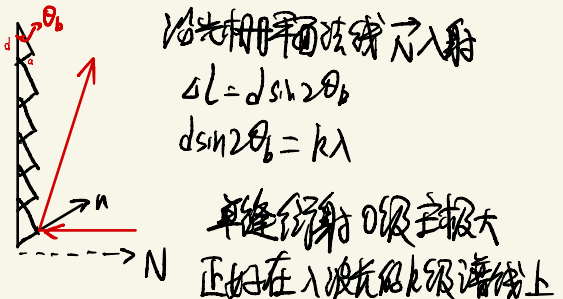
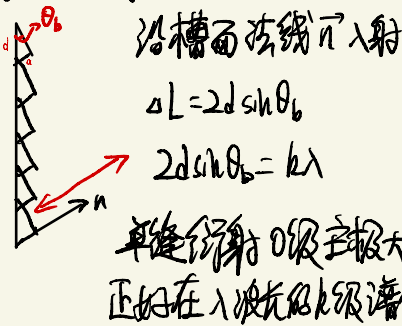


## 计算最高级数:

单波长最高级数: 利用  $\sin \theta_k = k \frac{\lambda}{d} \in [-1, 1]$  计算  $k_{max}$

多波长最高级数: 利用  $(k+1)\lambda_k = k\lambda_d$

## 闪耀光栅



# 第五章

主平面: 光线与光轴构成的平面

o光垂直主平面, e光在主平面内

正晶体:  $n_o < n_e$   
光轴



负晶体:  $n_o > n_e$



## 晶体相移器

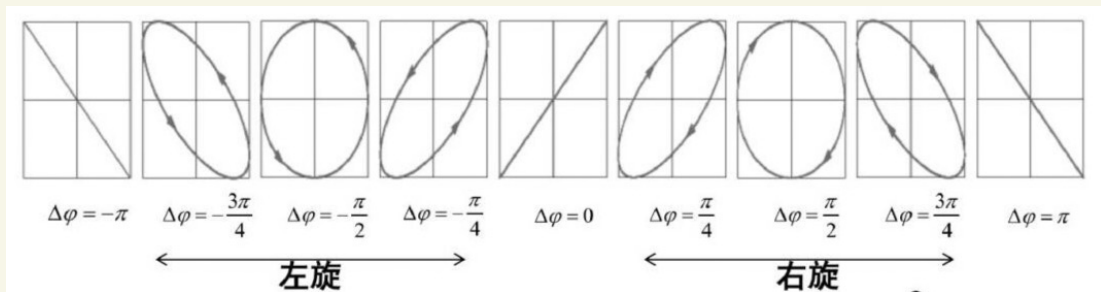
$$\Delta\varphi = \frac{2\pi}{\lambda}(n_e - n_o)d, \quad \Delta L = (n_e - n_o)d$$

全波片:  $\Delta\varphi = 2m\pi, \quad \Delta L = m\lambda$

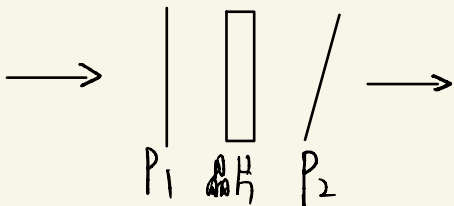
半波片:  $\Delta\varphi = (2m+1)\pi, \quad \Delta L = (2m+1)\frac{\lambda}{2}$

1/4波片:  $\Delta\varphi = (2m+1)\frac{\pi}{2}, \quad \Delta L = (2m+1)\frac{\lambda}{4}$

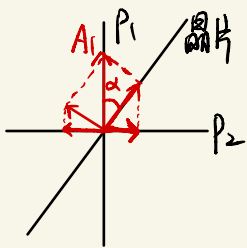
李萨如图形:



## 平行偏振光的干涉



$P_1 \perp P_2$  时.  $I_{2\perp} = A_1^2 \sin^2 2\alpha \sin^2 \frac{\Delta}{2}$  .  $\Delta = \frac{2\pi}{\lambda} (n_e - n_o) d$



$\Delta = (2m+1)\pi$  .  $d = \frac{2m+1}{|n_e - n_o|} \cdot \frac{\lambda_0}{2}$  干涉极大

$\Delta = 2m\pi$  .  $d = \frac{m\lambda_0}{|n_e - n_o|}$  干涉极小

$P_1 \parallel P_2$  时.  $I_{2\parallel} = A_1^2 - A_1^2 \sin^2 2\alpha \sin^2 \frac{\Delta}{2}$

$I_{2\perp} + I_{2\parallel} = A_1^2$

### 旋光效应

$\theta = \alpha d$  .  $\alpha$ : 旋光率.  $\theta$ : 光矢量转过角度.  $d$ : 样品池的长度

$\theta = [\alpha] dc$  .  $[\alpha]$ : 比旋光率.  $c$ : 浓度

### 第六章

线性吸收定律:

朗伯定律:  $-dI = \alpha I dx$  . 积分得  $I = I_0 e^{-\alpha z}$  .  $\alpha$ : 吸收系数.  $z$ : 通过长度

比尔定律:  $\alpha = kc$  .  $k$ : 常数.  $c$ : 浓度

朗伯-比尔定律: 吸光度  $A = \epsilon lc$  .  $A = \lg \frac{I_0}{I} = \lg \frac{1}{T}$

### 色散

柯西色散公式:  $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$

### 散射

散射粒子半径  $a$  与入射光波长  $\lambda$  之比  $a/\lambda < 0.1$  . 瑞利散射.  $I \propto \lambda^{-4}$

$a/\lambda = 0.1 \sim 10$  . 米氏散射

$I = I_0 e^{-(\alpha_s + \alpha_a) l}$  .  $\alpha_s$  为散射系数.  $\alpha_a$  为吸收系数

# 第七章

单色辐射本领

基尔霍夫辐射定律:  $\frac{r_1(\lambda, T)}{\alpha_1(\lambda, T)} = \frac{r_2(\lambda, T)}{\alpha_2(\lambda, T)} = \dots = f(\lambda, T)$

↑  
单色吸收本领

斯特藩-玻耳兹曼定律:  $R = \sigma T^4$  .  $\sigma = 2.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

维恩位移定律:  $T \lambda_m = b$  .  $b = 0.288 \text{ cm} \cdot \text{K}$

↑  
单色辐射本领  $r(\lambda, T)$  的极大值对应波长

瑞利-金斯定律:  $r_0(\lambda, T) d\lambda = \frac{2\pi c}{\lambda^4} K T d\lambda$

普朗克公式:  $r_0(\nu, T) = \frac{2\pi h \nu^3 / c^3}{\exp(h\nu / (kT)) - 1}$

## 光电效应:

$eV_0 = mV_m^2 / 2$  .  $V_0$ : 反向截止电压 .  $V_m$ : 最大速度

$mV_m^2 / 2 = eK(\nu - \nu_0)$  .  $\nu_0$ : 截止频率

爱因斯坦公式:  $mV_m^2 / 2 = h\nu - A$  .  $A$ : 脱出功