Introduction to the Controlled thermal Fusion

Homework 2.6

2.6 To test the sensitivity of the nuclear force model to the shape of the binding energy curve, repeat the derivation of Eq. (2.27) using the following central force:

$$\mathbf{F}_j^{(\mathrm{N})} = -\frac{1}{4\pi\varepsilon_0 r_\mathrm{c}^2}\operatorname{cosech}^2\left(\frac{r^2}{r_\mathrm{c}^2}\right)\mathbf{e}_R.$$

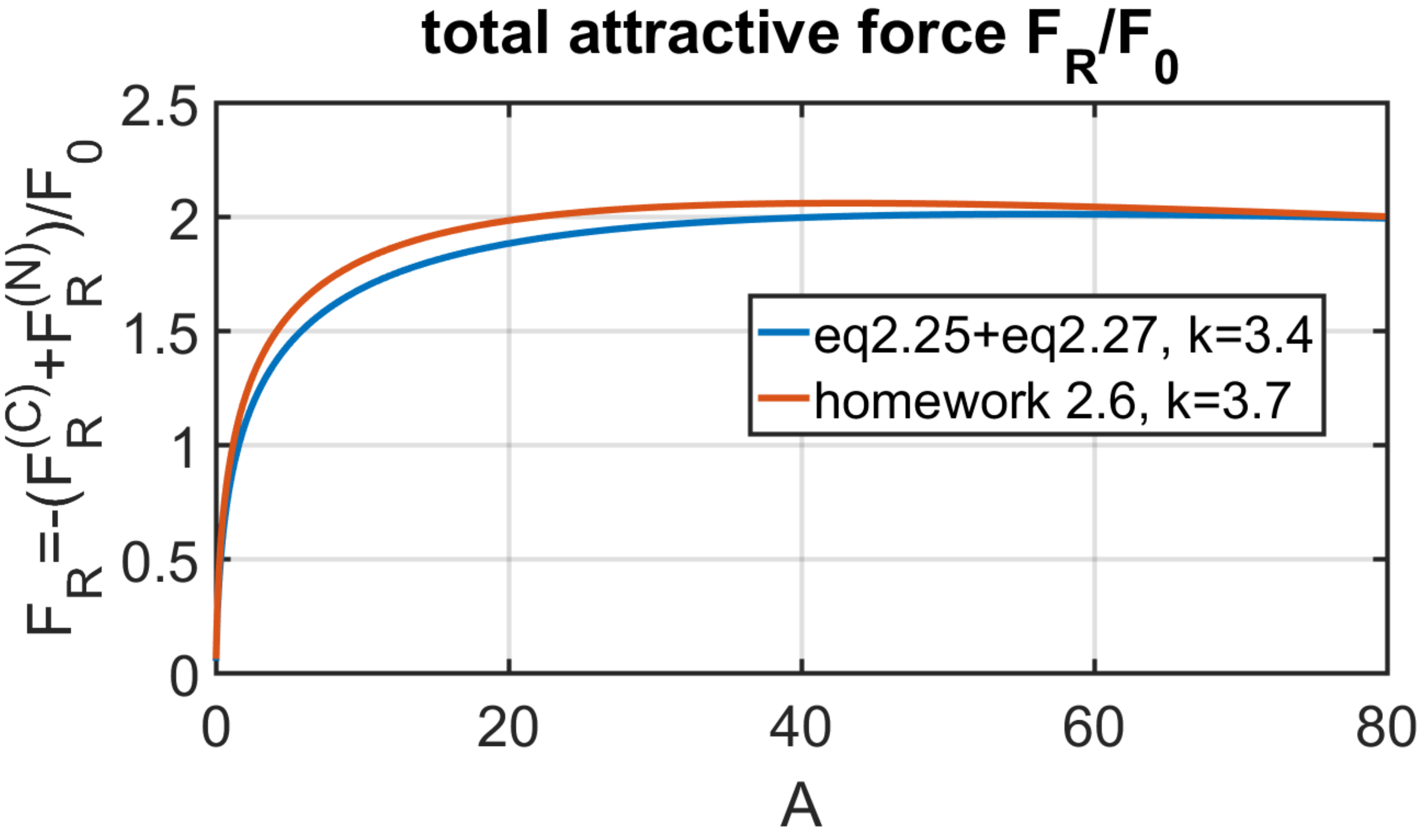
As in the text write $r_\mathrm{c} = kr_0$ and adjust k so that the maximum of the binding energy curve occurs for iron. This will probably involve a numerical calculation. Compare the result with the one obtained in the text.

$$F_R^{(\mathrm{N})} = -F_0\frac{k^2 A}{(A^{1/3}+1)^2(2A^{1/3}+1)}. \tag{2.27}$$

$$\mathbf{F}_R^{(\mathrm{C})} \equiv F_R^{(\mathrm{C})}\mathbf{e}_R = \frac{F_0}{2}\frac{A}{(A^{1/3}+1)^2}\mathbf{e}_R, \tag{2.25}$$

Homework 2.6

$$F_R^{(N)} = \frac{1}{4\pi\epsilon_0 r_c^2}\frac{3}{4\pi r_0^3}\int_V cosech^2\left(\frac{r^2}{r_c^2}\right) cos\alpha d\vec{r}\,\vec{e_R} \sim \frac{1}{k^2}\int_{r_0}^{A^{1/3}r_0} cosech^2\left(\frac{r^2}{r_c^2}\right) cos\alpha r^2 d\vec{r}$$



Homework 3.3

3.3 The purpose of this problem is to investigate the effect of plasma profiles on the alpha power density. The idea behind the calculation is to replace the 0-D model where all

quantities are equal to their average value by a volume-averaged 1-D model where the density and temperature have known profiles that vary in space. Specifically, in a plasma with a circular cross section the volume-averaged alpha power density is defined as

$$\overline{S}_\alpha = \frac{2}{a^2}\int_0^a \left(\frac{E_\alpha}{4} n^2 \langle \sigma v \rangle\right) r \, \mathrm{d}r.$$

Assume now that the density and temperature profiles are given by

$$n = (1+\nu_n)\overline{n}(1 - r^2/a^2)^{\nu_n},$$
$$T = (1+\nu_T)\overline{T}(1 - r^2/a^2)^{\nu_T},$$

where $\overline{n}$ and $\overline{T}$ are the volume-averaged density and temperature respectively. To determine the effect of temperature profile on alpha power density numerically evaluate $\overline{S}_\alpha$ for $\nu_n = 0$ and $0 \le \nu_T \le 4$ using $\langle \sigma v \rangle$ from Problem 3.1. For each ν_T find the optimum value of $\overline{T}$ that maximizes $\overline{S}_\alpha$ at fixed average pressure: $\overline{p} = 2\left[(1+\nu_n)(1+\nu_T)/(1+\nu_n+\nu_T)\right]\overline{n}\,\overline{T} = \text{const}$. Plot the optimum $\overline{T}$(keV) and corresponding $\overline{S}_\alpha/\overline{p}^2 \equiv C_\mathrm{p}$ as a function of ν_T showing the 0-D limit $\nu_n = \nu_T = 0$ for reference. To determine the variation of $\overline{S}_\alpha$ with density repeat the above calculation for $\nu_T = 2$ and $0 \le \nu_n \le 4$. Are peaked profiles good, bad, or unimportant in maximizing $\overline{S}_\alpha$?

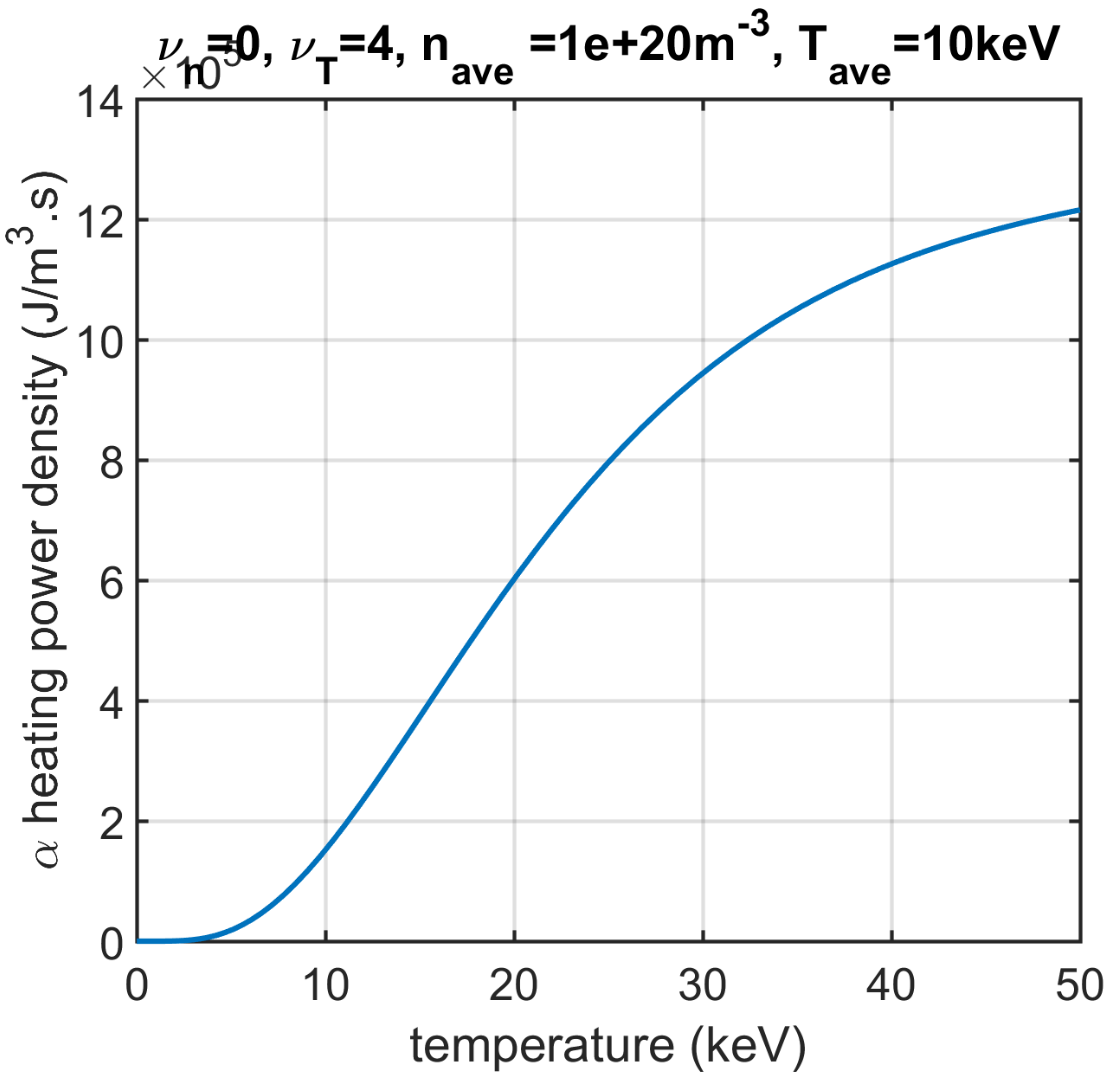
Homework 3.3

fusion reaction rate

$$\langle \sigma v \rangle = 10^{-6} \exp\left(\frac{a_{-1}}{T_i^{\alpha}} + a_0 + a_1 T_i + a_2 T_i^2 + a_3 T_i^3 + a_4 T_i^4\right) \text{m}^3/\text{s},$$

where $T_i = T_i\,(\text{keV})$ and **[Nuclear Fusion, 17, 873]**

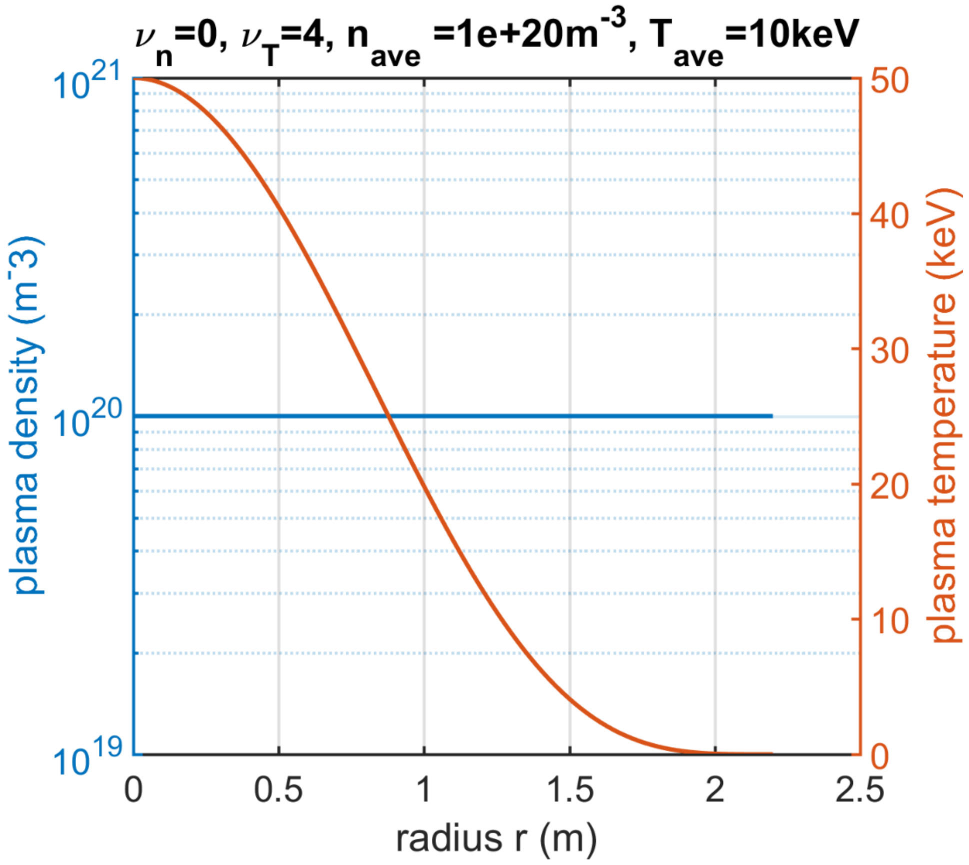
α	a_{-1}	a_0	a_1	a_2	a_3	a_4
0.2935	−21.38	−25.20	-7.101×10^{-2}	1.938×10^{-4}	4.925×10^{-6}	-3.984×10^{-8}

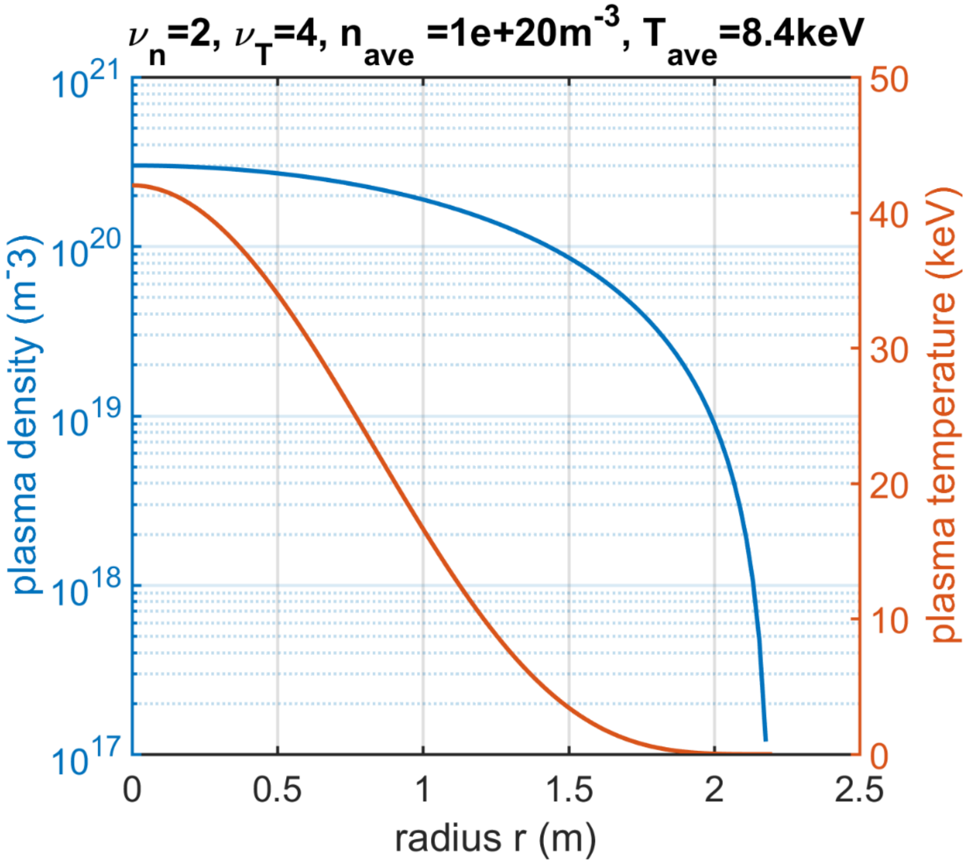


Homework 3.3. the effect of temperature profile

$$n = (1 + \nu_n)\overline{n}(1 - r^2/a^2)^{\nu_n},$$
$$T = (1 + \nu_T)\overline{T}(1 - r^2/a^2)^{\nu_T},$$

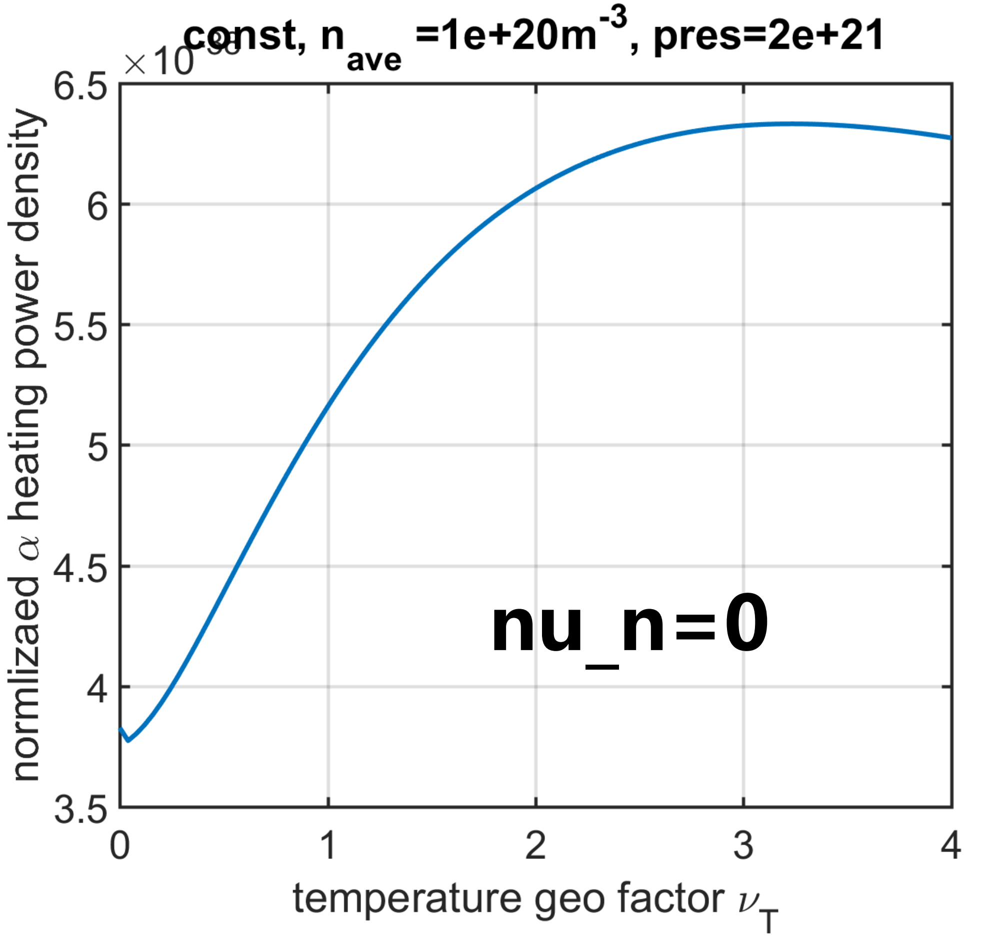
nu_n=0, constant density profile

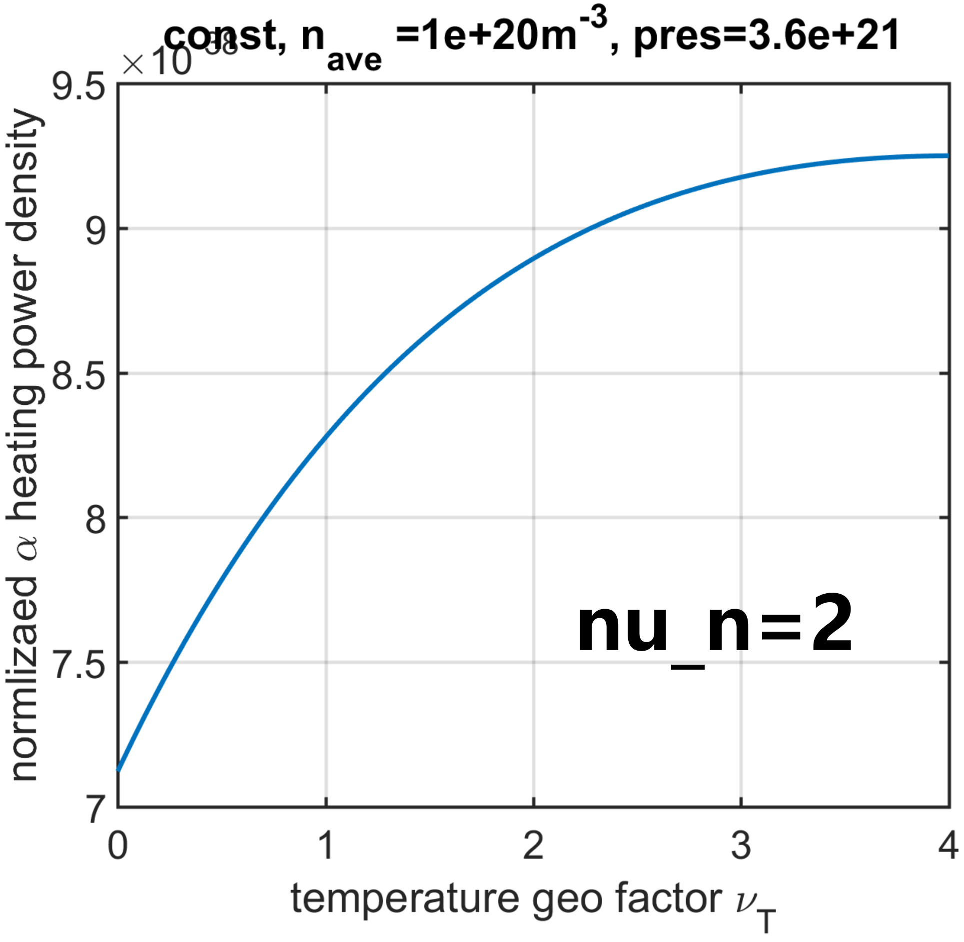




Homework 3.3. the effect of temperature profile

Peaked Ti profile leads to high alpha power, however, extremum exists.

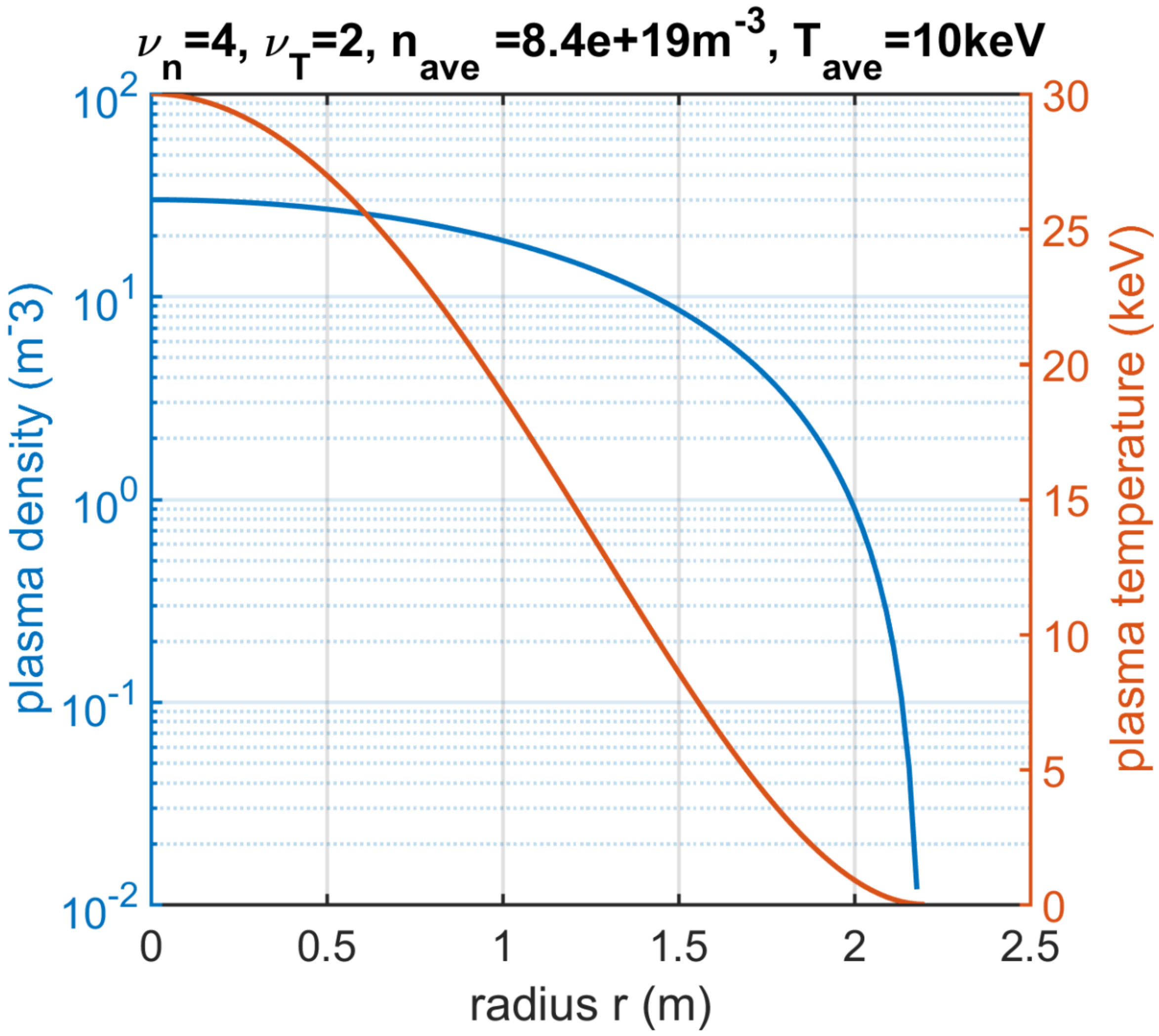




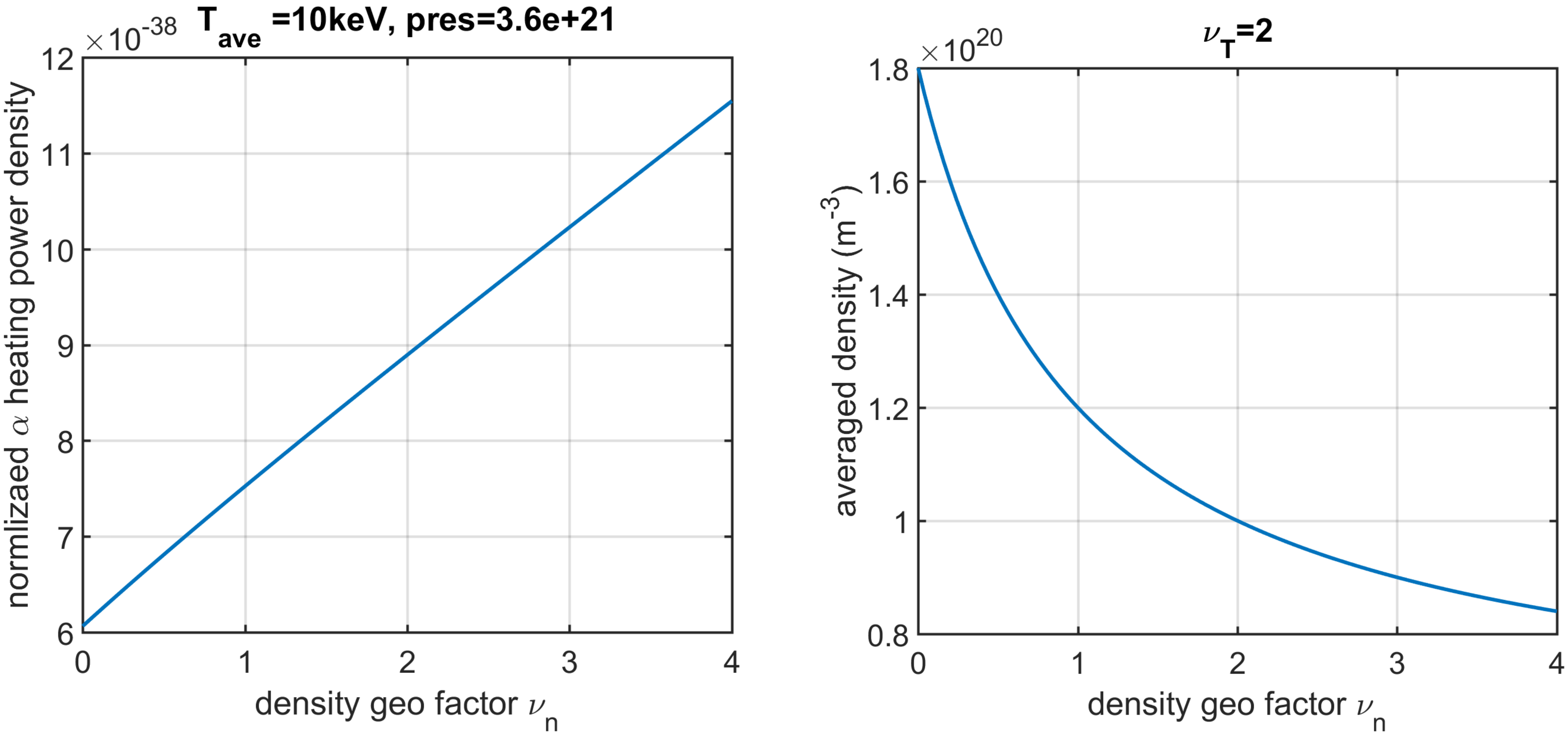
Homework 3.3. the effect of density profile

$$n = (1+\nu_n)\overline{n}(1-r^2/a^2)^{\nu_n},$$
$$T = (1+\nu_T)\overline{T}(1-r^2/a^2)^{\nu_T},$$

nu_T=2, peaked Ti profile



Peaked density profile leads to monotonic alpha power increase



Homework 4.1

4.1 This problem involves the derivation of a generalized version of the Lawson criterion. Consider a subignited reactor in which $S_\alpha < S_\kappa$. In the plasma power balance include alpha heating, external heating, and thermal conduction losses. Also, include the power produced by breeding tritium from Li^6. However, assume that of the total alpha power only a fraction f deposits its energy in the plasma while $1 - f$ is immediately lost to the first wall and converted to heat. Assume a thermal conversion efficiency η_t and an input electricity to plasma heating conversion efficiency η_h (i.e., $\eta_h = \eta_e, \ \eta_a = 1$).

(a) Derive an expression for $p\tau_E = G(Q_E, f)$ for steady state operation.

(b) Assume $T = 15$ keV, $\eta_t = 0.35, \ \eta_h = 0.5$. Plot curves of $p\tau_E$ vs. Q_E for $f = 0, 0.5, 1$. Compare the required $p\tau_E$ values at $Q_E = 20$ with the fully ignited value ($Q_E = \infty, \ f = 1$) and the Lawson breakeven criterion ($Q_E = 1, \ f = 0$).

Homework 4.1

$$P_{out}^{E} = [S_n + S_{Li} + S_B + S_K + (1-f)S_\alpha]\eta_t \quad \times \eta_e = 0.4 \quad + (1-\eta_\alpha)\eta_e P_{in}^{(F)}$$

$\eta_\alpha = 1 / 0$

$$P_{in}^{E} = S_h.$$

steady state. $f S_\alpha + S_h = S_K = \frac{3}{2}\frac{P}{\tau_E}$.

$$Q_E = \frac{P_{out}^{E} - P_{in}^{E}}{S_h} = \frac{P_{out}^{E} - S_h}{S_h} = P_{out}^{E}/S_h - 1.$$

$$\Rightarrow S_h = P_{out}^{E}/(Q_E + 1).$$

$$f S_\alpha + P_{out}^{E}/(Q_E+1) = S_K$$

$$\Rightarrow f S_\alpha + \frac{[S_n + S_{Li} + S_B + S_K + (1-f)S_\alpha]\eta_t \times \eta_e}{Q_E + 1} = S_K$$

Homework 4.1

$E_n = 14.1\ MeV \quad S_{Li} = 4.8\ MeV \quad S_\alpha = 3.5\ MeV. \quad \frac{S_n + S_{Li}}{S_\alpha} = \beta S_\alpha, \quad \beta = 5.4$

$$\frac{\eta_t \eta_e}{Q_E+1} S_B + f \cancel{E_\alpha \frac{1}{16}\frac{p^2}{T^2}\langle\sigma v\rangle} + \frac{\eta_e \eta_t [(1-f) + \beta]}{Q_E+1} \frac{E_\alpha \frac{1}{16}\frac{p^2}{T^2}\langle\sigma v\rangle}{S_\alpha} = \left(1 - \frac{\eta_t \eta_e}{Q_E+1}\right) S_K$$

$$\frac{\eta_t \eta_e}{Q_E+1} S_B + \left[f + \frac{\eta_e \eta_t (1-f+\beta)}{Q_E+1}\right] S_\alpha = \left(1 - \frac{\eta_t \eta_e}{Q_E+1}\right) S_K$$

$$\Rightarrow \frac{\eta_t \eta_e}{Q_E+1} K_B \frac{p}{T^{3/2}} + \left[f + \frac{\eta_e \eta_t (1-f+\beta)}{Q_E+1}\right] E_\alpha \frac{1}{16}\frac{p^2}{T^2}\langle\sigma v\rangle = \left(1 - \frac{\eta_t \eta_e}{Q_E+1}\right) \frac{3}{2} \frac{p}{\tau_E}$$

$$\left[\frac{\eta_t \eta_e}{Q_E+1} \frac{K_B}{T^{3/2}} + \left[f + \frac{\eta_e \eta_t (1-f+\beta)}{Q_E+1}\right] \frac{E_\alpha \langle\sigma v\rangle}{16\ T^2}\right] p \tau_E = \left(1 - \frac{\eta_t \eta_e}{Q_E+1}\right) \frac{3}{2}$$

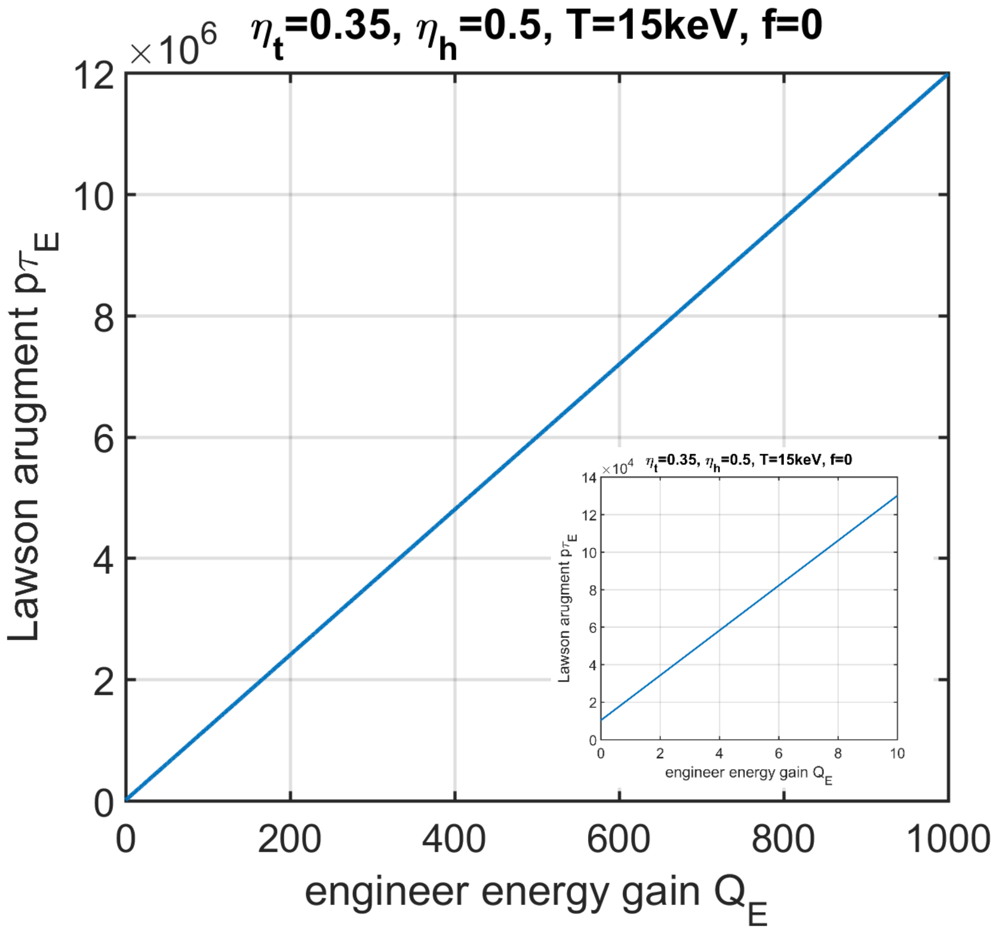
$\Rightarrow p\tau_E = G(Q_E, f). \quad \eta_e = 0.4, \ E_\alpha = 3.5\ MeV, \ K_B = 0.052$

$\beta = 5.4.$

Homework 4.1

$p\tau_E$ increases linearly along with the engineering energy gain factor Q_E

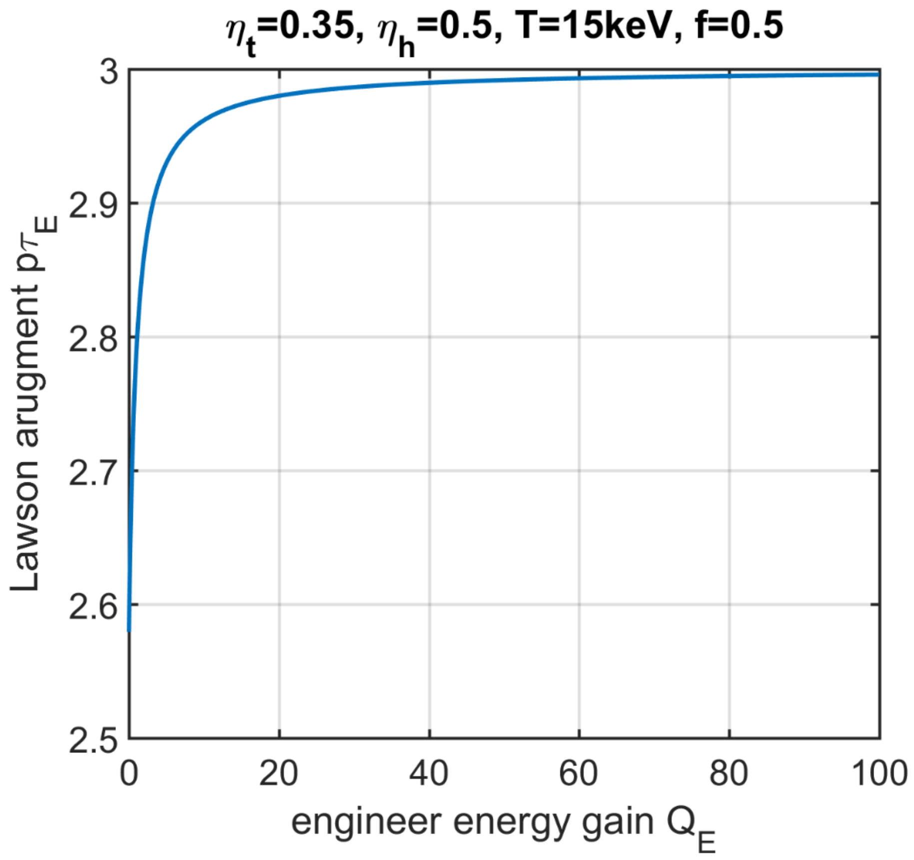
No alpha heating power, the self-sustained plasma completely depends on the external heating

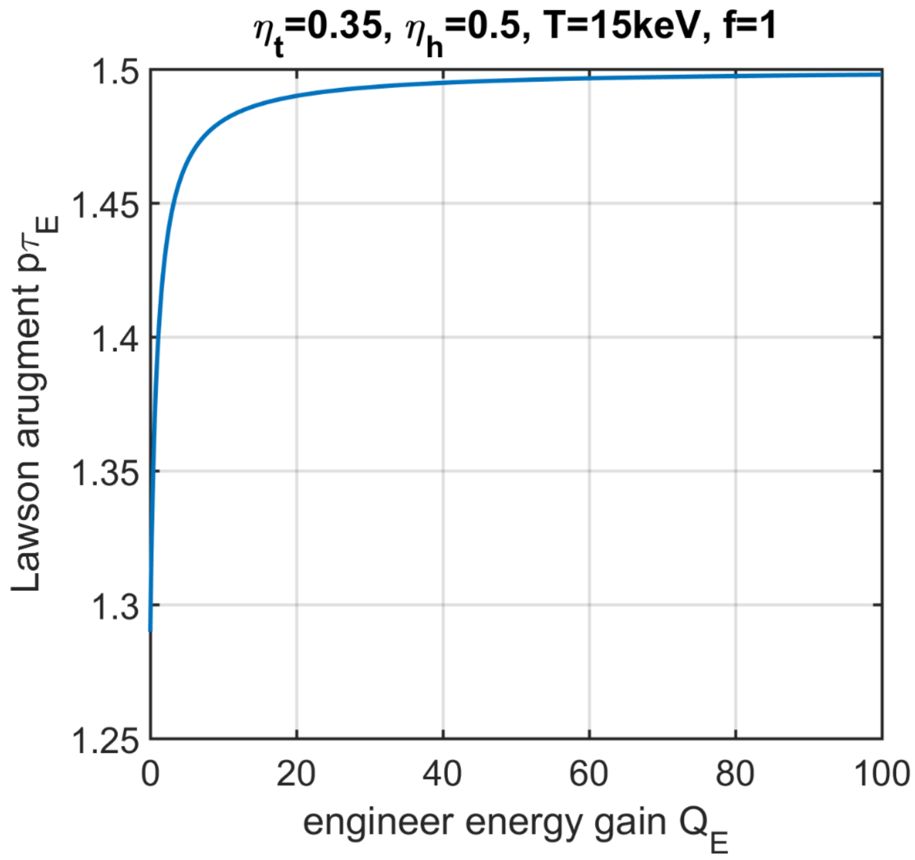


Homework 4.1

$p\tau_E$ increases rapidly in the regime $Q_E < 20$

Increased alpha heating power facilitates the achieve of the self-sustained burning plasma $Q_E \sim \infty$, because the only perturbation comes from thermal conduction losses and the bremsstrahlung is not considered, which is proportion to n^2 sqrt(T)





Homework 8.6

8.6 A plasma has a constant uniform magnetic field $\mathbf{B} = B_0\mathbf{e}_z$. Superimposed is an electrostatic electric field of the form $\mathbf{E} = E_0 \cos(\omega t - kz)\mathbf{e}_z$, where ω and k are known constants. Assume a positively charged particle is initially located at $z(0) = 0$ with a parallel velocity $v_z(0) = v_\parallel$. Show that for a sufficiently large value of E_0 the particle is trapped in the wave. Calculate the critical E_0.

Homework 8.6

$$\frac{1}{2}mV_z^2 < q\int_0^{\pi/2k} E\cos(\omega t - kz)\,dz$$

$$V_z < \sqrt{\frac{2qE_0}{m}} + \frac{w}{k}$$

$$E_0 > \frac{m(V_z - \omega/k)^2}{2q}$$

Homework 8.13

8.13 A positive ion is placed in a sheared magnetic field given by

$$\mathbf{B} = B_0[\mathbf{e}_z + (x/L)\mathbf{e}_y].$$

(a) Write down the exact equations of motion describing the orbit of the particle.
(b) Find a relation between $v_z(t)$ and $x(t)$ assuming the following initial conditions: $v_y(0) = v_z(0) = x(0) = y(0) = z(0) = 0$ and $v_x(0) = v_0$.
(c) Using this relation derive a single, second order ODE for $x(t)$.
(d) Calculate the x location of the turning point of the orbit.

Homework 8.13

$a)$

$$\frac{d^2x}{dt^2} = \omega_c\left(\frac{dy}{dt} - \frac{dz}{dt}\frac{x}{L}\right)$$

$$\frac{d^2y}{dt^2} = -\omega_c\frac{dx}{dt}$$

$$\frac{d^2z}{dt^2} = \omega_c\frac{dx}{dt}\frac{x}{L}$$

$$\omega_c = \frac{qB_0}{m}$$

$b)$

$$V_{z(t)} = \frac{1}{2L}\omega_c x_{(t)}^2$$

$$V_{y(t)} = -\omega_c x_{(t)}$$

c)

$$\frac{d^2x}{dt^2} + \omega_c^2\left(x + \frac{x^3}{2L^2}\right) = 0$$

d)

$$V_x^2 + V_y^2 + V_z^2 = V_0^2$$

Turning point Vx=0

$$V_y^2 + V_z^2 = V_0^2$$

From b)

$$x^4 + 4L^2x^2 - \frac{4L^2V_0^2}{\omega_c^2} = 0$$

Homework 11.8

11.8 A straight 2-D non-circular plasma has an elliptic cross section with horizontal width $2a$ and vertical height $2\kappa a$, where κ is the elongation. The plasma is surrounded by a close fitting circular wall of radius $r = 2\kappa b$ with $b \approx a$. For simplicity assume the "toroidal" field $B_z = B_0 = \text{const}$. Now, note that the requirement $\nabla \cdot \mathbf{B} = 0$ implies that the magnetic field in the plasma can be written as $\mathbf{B} = \nabla A \times \mathbf{e}_z + B_0 \mathbf{e}_z$, where $A(r, \theta)$ is the vector potential.

(a) Using the MHD equilibrium equations and Maxwell's equations show that $p = p(A)$ and derive the partial differential equation satisfied by A.

(b) Solve the equation for A assuming that $\mu_0 p(A) = (C^2/2)(A_{\max}^2 - A^2)$, where C and $A_{\max}$ are constants. To obtain an analytic solution assume that $C\kappa a \ll 1$ and solve by expansion.

(c) Magnetic measurements on the wall surface indicate that $B_\theta(\kappa a, \theta) = (\mu_0 I / 2\pi\kappa a)(1 + \alpha \cos 2\theta)$, where I and α are measured constants. Solve the equation for A in the vacuum region between the wall and the plasma (where $p = 0$). Match the solutions across the plasma–vacuum interface and derive an expression for $\kappa = \kappa(\alpha)$.

Homework 11.8

(a). $\nabla p\cdot B = B\cdot(J\times B) = 0$

$\Rightarrow \nabla p\cdot B = \nabla p\cdot(\nabla A\times\hat{z} + B_0\hat{z})$

$= \nabla p\cdot(\nabla A\times\hat{z}) + \nabla p\cdot(B_0\hat{z})$ 　 $\nabla p\cdot\hat{z}=0$

$\Rightarrow \hat{z}\cdot(\nabla p\times\nabla A) = 0$

p与A面平行 . $\Rightarrow p = p(A)$

平衡 $\nabla p = J\times B = \frac{1}{\mu_0}(\nabla\times B)\times\vec{B}$

$\Rightarrow \mu_0\nabla p = \nabla\times(\nabla A\times\hat{z} + B_0\hat{z})\times\vec{B}$ 　 $\frac{\partial}{\partial z}=0$

$= [\nabla\times(\nabla A\times\hat{z})]\times\vec{B}$

$\because \nabla\times(\nabla A\times\hat{z}) = \nabla A(\nabla\cdot\hat{z}) - (\nabla A\cdot\nabla)\hat{z} + (\hat{z}\cdot\nabla)\nabla A - (\nabla\cdot\nabla A)\hat{z}$

$= -\nabla^2 A\,\hat{z}$

$\mu_0\nabla p = (-\nabla^2 A\hat{z})\times\vec{B} = (-\nabla^2 A\hat{z})\times(\nabla A\times\hat{z} + B_0\hat{z})$

$= (-\nabla^2 A\hat{z})\times(\nabla A\times\hat{z})$

$= -\nabla^2 A[\hat{z}\times(\nabla A\times\hat{z})]$

$= -\nabla^2 A\,[\nabla A(\hat{z}\cdot\hat{z}) - \hat{z}(\hat{z}\cdot\nabla A)]$

平衡: $\mu_0\nabla p = -\nabla^2 A\nabla A$, $A = A(r,\theta)$.

(b). 考虑柱位型 . (r,θ,z).

$\mu_0\nabla p/\nabla A = -\nabla^2 A$

$\Rightarrow \mu_0\frac{dp}{dA} = -\nabla^2 A$, $\mu_0 p(A) = \frac{C^2}{2}(A_m^2 - A^2)$

$\Rightarrow \nabla^2 A + C^2 A = 0$.

$\Rightarrow \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{1}{r^2}\frac{\partial^2 A}{\partial\theta^2} + C^2 A = 0$

对A分离变量 $A(r,\theta) = R(r)\,\Phi(\theta)$

$$\Rightarrow \begin{cases} \frac{1}{\Phi}\frac{d^2\Phi}{d\theta^2} = -m^2 \\ \frac{r^2}{R}\frac{d^2R}{dr^2} + \frac{r}{R}\frac{dR}{dr} + C^2r^2 - m^2 = 0 \end{cases}$$

Φ通解为 $e^{im\theta}$, R为贝塞尔方程，两个线性无关解为贝塞尔函数 $J_c(Cr)$, $Y_c(Cr)$, C为常数 .

(c). 真空区没有电流，求解磁场简化为拉普拉斯方程.

$$\nabla^2 A = 0.$$

分离变量其通解为:

$$A(r,\theta) = \sum_{l,m} [A_l J_l(kr) + B_l Y_l(kr)] e^{im\theta}$$

在磁轴处 $r=0$, 磁场有限值, $B_l=0$. 无穷远处, $r\to\infty$. $\vec{B}=0$.

芯部磁通为0. 取. $A = A_v J_1(kr) e^{im\theta}$

$r=ka,\ \theta=0.$ $B_\theta = \frac{\partial A}{\partial r}$. 确定常数 A_v 值:

$$\Rightarrow (1+\alpha)\frac{\mu_0 I}{2\pi ka} = A_v \frac{\partial J_1}{\partial r}\Big|_{r=ka}. \quad \left[\frac{d}{dr}(rJ_1) = rJ_0\right].$$

~~等离子体-真空区匹配. $r=a,\ \theta=0$, $[B_{\theta,plasma}(r=a) = B_{\theta,vacuum}(r=a)]$~~

~~$B_{\theta,v}(r=a) = \frac{\partial A_v}{\partial r} = A_v\frac{\partial J_1}{\partial r}\Big|_{(r=a)}$, $A_v = (1+\alpha)\frac{\mu_0 I}{2\pi ka}\left(\frac{1}{J_0 - \frac{J_1}{ka}}\right)\Big|_{r=ka}$~~

$$\Rightarrow A_v = (1+\alpha)\frac{\mu_0 I}{2\pi}\cdot\frac{1}{kaJ_0(ka) - J_1(ka)}$$

等离子体-真空区匹配, $r=a,\ \theta=0$, $[B_{\theta,p} = B_{\theta,v}]_{r=a}$.

$$\Rightarrow \left[\frac{\partial A_{,p}}{\partial r} = \frac{\partial A_{,v}}{\partial r}\right]_{r=a}.$$

$$\Rightarrow \alpha = 1 - \frac{2\pi[kaJ_0(ka) - J_1(ka)]A_p}{\mu_0 I}, \quad A_p = const.$$

Homework 11.9

11.9 The purpose of this problem is to derive the Grad–Shafranov equation, a famous partial differential equation describing the MHD equilibrium of configurations possessing toroidal symmetry: $Q(R, Z, \phi) \rightarrow Q(R, Z)$.

(a) Using $\nabla \cdot \mathbf{B} = 0$ prove that the magnetic field can be written in terms of a flux function $\psi(R, Z)$ as follows: $\mathbf{B} = \nabla\psi \times \mathbf{e}_\phi / R + B_\phi(R, Z)\mathbf{e}_\phi$.

(b) From Ampères law derive an expression for $\mu_0 J_\phi$ in terms of ψ.

(c) From the momentum equation prove that $p(R, Z) = p(\psi)$, where $p(\psi)$ is an arbitrary function.

(d) From the momentum equation prove that $B_\phi(R, Z) \rightarrow F(\psi)/R$, where $F(\psi)$ is an arbitrary function.

(e) From the momentum equation derive the Grad–Shafranov equation:

$$R^2 \nabla \cdot (\nabla\psi / R^2) = -\mu_0 R^2 (dp/d\psi) - F(dF/d\psi).$$

Homework 11.9

(a). $\nabla\cdot B=0 \Rightarrow \frac{1}{R}\frac{\partial}{\partial R}(RB_R)+\frac{\partial B_Z}{\partial Z}=0$

磁通: $\psi=\int B_Z\cdot 2\pi R\,dR \Rightarrow B_Z=\frac{1}{R}\frac{\partial\psi}{\partial R}$.

带入 $\nabla\cdot B=0$: $\frac{1}{R}\frac{\partial}{\partial R}(RB_R)+\frac{\partial}{\partial Z}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)=0$

$\Rightarrow B_R=-\frac{1}{R}\frac{\partial\psi}{\partial Z}$

$\vec{B}=B_R\hat{R}+B_Z\hat{Z}+B_\phi\hat{\phi}$

$=\frac{1}{R}\left(-\frac{\partial\psi}{\partial Z}\hat{R}+\frac{\partial\psi}{\partial R}\hat{Z}\right)+B_\phi\hat{\phi}$

$=\nabla\psi\times\hat{\phi}/R+B_\phi\hat{\phi}$ $\quad (R,\phi,Z)$.

(b). $\mu_0 J=\nabla\times B=\nabla\times\left[\frac{\nabla\psi}{R}\times\hat{\phi}+B_\phi\hat{\phi}\right]$

$$=\begin{vmatrix}\frac{\hat{R}}{R} & \hat{\phi} & \frac{\hat{Z}}{R}\\ \frac{\partial}{\partial R} & \frac{1}{R}\frac{\partial}{\partial\phi} & \frac{\partial}{\partial Z}\\ -\frac{1}{R}\frac{\partial\psi}{\partial Z} & RB_\phi & \frac{1}{R}\frac{\partial\psi}{\partial R}\end{vmatrix} \qquad \frac{\partial}{\partial\phi}=0$$

$=\frac{1}{R}\left[\frac{\partial(RB_\phi)}{\partial R}\hat{Z}-\frac{\partial(RB_\phi)}{\partial Z}\hat{R}\right]-\left[\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{1}{R}\frac{\partial^2\psi}{\partial Z^2}\right]\hat{\phi}$

$=-\frac{1}{R}\Delta^*\psi\hat{\phi}+\frac{1}{R}\nabla(RB_\phi)\times\hat{\phi}$

$\Delta^*\psi=R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Z^2}$

(c). $B\cdot\nabla p=B\cdot(J\times B)=0$

$\Rightarrow B\cdot\nabla p=\left[\nabla\psi\times\hat{\phi}/R+B_\phi\hat{\phi}\right]\cdot\nabla p=0$

$\Rightarrow \hat{\phi}\cdot\nabla\psi\times\nabla p=0 \qquad \hat{\phi}\cdot\nabla p=0$

$\Rightarrow p=P(\psi)$.

Homework 11.9

(c).
$$\begin{cases} B_R = -\frac{1}{R}\frac{\partial \psi}{\partial Z} \\ B_\phi = \frac{F}{R} \\ B_Z = \frac{1}{R}\frac{\partial \psi}{\partial R} \end{cases}$$

$$\mu_0 J = -\frac{1}{R}\Delta^* \psi \hat{\phi} + \frac{1}{R}\nabla F \times \hat{\phi} \Rightarrow \begin{cases} J_R = -\frac{1}{\mu_0 R}\frac{\partial F}{\partial Z} \\ J_\phi = -\frac{1}{\mu_0 R}\Delta^* \psi \\ J_Z = \frac{1}{\mu_0 R}\frac{\partial F}{\partial R} \end{cases}$$

$$\nabla p = \vec{J} \times \vec{B}$$

$$\frac{\partial p}{\partial R} = J_\phi B_Z - J_Z B_\phi$$

$$= -\frac{1}{\mu_0 R}\Delta^* \psi \frac{1}{R}\frac{\partial \psi}{\partial R} - \frac{1}{\mu_0 R}\frac{\partial F}{\partial R}\frac{F}{R}.$$

~~$$= -\frac{1}{\mu_0 R}\left[R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2}\right]\frac{1}{R}\frac{\partial \psi}{\partial R} - \frac{1}{\mu_0 R}\frac{F}{R}\frac{\partial F}{\partial R}$$~~

$$\Rightarrow \mu_0 R^2 \frac{\partial p}{\partial R} = -\Delta^* \psi \frac{\partial \psi}{\partial R} - F\frac{\partial F}{\partial R}.$$

$$\mu_0 R^2 \frac{dp}{d\psi} = -\Delta^* \psi - F\frac{dF}{d\psi}.$$

$$\Rightarrow \Delta^* \psi = -\mu_0 R^2 \frac{dp}{d\psi} - F\frac{dF}{d\psi}$$

$$\Delta^* \psi = R^2 \nabla \cdot \left(\frac{\nabla \psi}{R^2}\right) = R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2}.$$

(d). $J \cdot \nabla p = J \cdot (J \times B) = 0$

$\Rightarrow [\nabla(RB_\phi) \times \hat{\phi}] \cdot \nabla p = 0.$ 由于 $p = p(\psi)$.

$\Rightarrow [\nabla(RB_\phi) \times \hat{\phi}] \cdot \nabla \psi = 0.$

$\Rightarrow \hat{\phi} \cdot [\nabla \psi \times \nabla(RB_\phi)] = 0.$

$\Rightarrow \mu_0 I = RB_\phi = \mu_0 F(\psi)$