图论 2021.1.13

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1

R(3,3)=6

2

$\delta(G) \geq 2$, 则 G 有长至少为 $\delta + 1$ 的圈

3

G 简单平面 2 部图，至多 2n-4 条边，举例该界是紧的

4

网络流圈 C 定正向，证明 f' 是流且流值不变

5

色多项式
(1) 证明递推公式 G-e, G*e
(2)n 阶树的色多项式

6

bondy1972, 子集族 A_i, 存在 x 使得 $A_i \cup$x 互不相同

7

G,n,m 有至少 $\frac{m}{2}$ 的边割集

8

3 正则 Hamilton 图有 3 边染色

9

n≥2s,P_n 表示 n 个点的路，I_1,...,I_s 是 P_n 的 s 个 s 元独立集，证明：存在 P_n 的 s 元独立集 I 使得 |I∩I_i|≥1,i=1,2....,s.

Pf: Induction on s.

$s=1$ ✓

Assume it holds for $s \le t-1$. $t \ge 2$. and $n \ge 2s-1$.

When $s=t$

Let $P_n = \underset{1}{\circ} - \underset{2}{\circ} - \underset{3}{\circ} \cdots \underset{n-1}{\circ} - \underset{n}{\circ}$

Let i be the smallest index s.t $\exists I_j$, $1 \le j \le s$ s.t $i \in I_j$

~~Then put i into I.~~

Patition $\mathcal{I} = \{I_1, \cdots, I_s\}$ into $\mathcal{I}' \cup \mathcal{I}'' \cup \tilde{\mathcal{I}}$ ($\tilde{\mathcal{I}}$ may be empty)

where $\mathcal{I}' = \{I_j : i \in I_j\}$ $\mathcal{I}'' = \{I_j : i+1 \in I_j\}$ $\tilde{\mathcal{I}} = \{I_j : i \notin I_j \text{ and } i+1 \notin I_j\}$

Let $P' = P \setminus \{[i+1]\}$

Since for $\forall I_j \in \mathcal{I}'$, $I_j' = I_j \setminus \{i\} \subseteq P'$. We have $|P'| = n' \ge 2(s-1)-1$

For $\forall I_j \in \mathcal{I}''$, $I_j' = I_j \setminus \{i+1\} \subseteq P'$ (By the minimality of i)

For $\forall I_j \in \tilde{\mathcal{I}}$ $I_j \subseteq P'$

We arbitrarily choose a v_j in each $I_j \in \tilde{\mathcal{I}}$ and let $I_j' = I_j \setminus \{v_j\}$.

Thus $|I_j'| = s-1$ for $1 \le j \le s$

Without loss of generation, assume $I_s \in \mathcal{I}'$.

Then by induction, $\exists$ an $(s-1)$- independent set $I' \subseteq P'$ s.t

$|I' \cap I_j'| \ge 1$ for $1 \le j \le s-1$. i.e $|I' \cap I_j| \ge 1$

Let $I = I' \cup \{i\}$

Then $I' \cap I_j \subseteq I \cap I_j$ for $1 \le j \le s-1$ and $i \in I \cap I_s$