T_1

1. 计算下列各式的数值，保留 10 位有效数字。

(1) 2^{200}

(2) $\log_5 135$

(3) e^{7-9i}

(4) $\ln(1+e^{-2})$

(5) $\sin 15^\circ + \cos 15^\circ$

(6) $\sqrt{|\ln \sin 35^\circ|}$

(7) $\cos\left(2\arccos\frac{1}{3} - \arccos\frac{1}{6}\right)$

(8) $\tan\left(\arctan\frac{\sqrt{2}}{2} + i\sin\frac{\sqrt{2}}{3}\right)$

(9) $10\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \div 6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(10) $12\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) \div 8\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

In[∘]:=
```
N[2^200, 10]
N[Log[5, 135], 10]
N[Exp[7 - 9 I], 10]
N[Log[1 + E^-2], 10]
N[Sin[15 °] + Cos[15 °], 10]
N[Sqrt[Abs[Log[Sin[35 °]]]], 10]
N[Cos[2 ArcCos[1 / 3] - ArcCos[1 / 6]], 10]
N[Tan[ArcTan[1 / Sqrt[2]] + I Sin[Sqrt[2] / 3]], 10]
N[10 (Cos[2 π / 3] + I Sin[2 π / 3]) / (6 (Cos[π / 3] + I Sin[π / 3])), 10]
N[12 (Cos[3 π / 2] + I Sin[3 π / 2]) / (8 (Cos[π / 6] + I Sin[π / 6])), 10]
```

Out[∘]= $1.606938044 \times 10^{60}$

Out[∘]= 3.047818583

Out[∘]= -999.1756568 - 451.9427961 i

Out[∘]= 0.1269280110

Out[∘]= 1.224744871

Out[∘]= 0.7455629221

Out[∘]= 0.4901185382

Out[∘]= 0.5311706528 + 0.5850327914 i

Out[∘]= 0.833333333 + 1.443375673 i

Out[∘]= -0.750000000 - 1.299038106 i

T2

2. 计算 861、1638、2415 的最大公约数。

In[ ]:=
```
GCD[861, 1638, 2415]
```

Out[ ]=
```
21
```

T3

3. 计算 48、105、120 的最小公倍数。

In[ ]:=
```
LCM[48, 105, 120]
```

Out[ ]=
```
1680
```

T4

4. 计算组合数 C_{10}^{3}、C_{12}^{5}、C_{15}^{7}。

In[ ]:=
```
Binomial[10, 3]
Binomial[12, 5]
Binomial[15, 7]
```

Out[ ]=
```
120
```

Out[ ]=
```
792
```

Out[ ]=
```
6435
```

T5

5. 计算 3!!/7!!、6!!/15!!、7!!/20!!。

In[∘]:=
```
3!!/7!!
6!!/15!!
7!!/20!!
```

Out[∘]= $\frac{1}{35}$

Out[∘]= $\frac{16}{675\,675}$

Out[∘]= $\frac{1}{35\,389\,440}$

T6

6. 对 $x=0.12$ 和 $x=67/100$ 分别计算 $e^{-x^2}\sin x$，计算过程中保留 50 位有效数字。

In[∘]:=
```
x1 = 0.12`50;
Exp[-x1^2] Sin[x1]
x2 = 67/100;
N[Exp[-x2^2] Sin[x2], 50]
Clear[x1, x2]
```

Out[∘]= 0.11800070390301374016560322148988766812084513075102

Out[∘]= 0.39639394070149074878098175890088678779227337181620

T7

7. 建立如下列表，并求所有元素的和与积。

(1) $\{1,3,5,7,\cdots,99\}$　　(2) $\{1,4,9,25,\cdots,100\}$

(3) $\{1/2,1/4,1/6,1/8,\cdots,1/100\}$　　(4) {小于 100 的素数}

In[ ]:=
```
a1 = Table[2 n - 1, {n, 1, 50}];
Print["The sum of a1 is"]
Total[a1]
Print["The product of a1 is"]
Apply[Times, a1]
Print[]

a2 = Table[n^2, {n, 1, 10}];
Print["The sum of a2 is"]
Total[a2]
Print["The product of a2 is"]
Apply[Times, a2]
Print[]

a3 = Table[1 / (2 n), {n, 1, 50}];
Print["The sum of a3 is"]
Total[a3]
Print["The product of a3 is"]
Apply[Times, a3]
Print[]

num = PrimePi[100];
a4 = Table[Prime[n], {n, 1, num}];
Print["The sum of a4 is"]
Total[a4]
Print["The product of a4 is"]
Apply[Times, a4]
Clear[a1, a2, a3, a4, num]
```

The sum of a1 is

Out[ ]= 2500

The product of a1 is

Out[ ]= 2 725 392 139 750 729 502 980 713 245 400 918 633 290 796 330 545 803 413 734 328 823 443 106 201 171 875

The sum of a2 is

Out[ ]= 385

The product of a2 is

Out[ ]= 13 168 189 440 000

The sum of a3 is

Out[ ]= $\frac{13\,943\,237\,577\,224\,054\,960\,759}{6\,198\,089\,008\,491\,993\,412\,800}$

The product of a3 is

Out[•]=
```
1/
 34 243 224 702 511 976 248 246 432 895 208 185 975 118 675 053 719 198 827 915 654 463 488 000
  000 000 000
```

The sum of a4 is

Out[•]=
```
1060
```

The product of a4 is

Out[•]=
```
2 305 567 963 945 518 424 753 102 147 331 756 070
```

T8

8. 随机删除第 7 题中每个表中的 3 个元素。

In[•]:=
```
a1 = Table[2 n - 1, {n, 1, 50}];
Print[a1]
Print["Delete three elements randomly."]
For[i = 1, i ≤ 3, i++, a1 = Drop[a1, {RandomInteger[{1, 50 + 1 - i}]}];
 Print[a1]]
Print[]

a2 = Table[n^2, {n, 1, 10}];
Print[a2]
Print["Delete three elements randomly."]
For[i = 1, i ≤ 3, i++, a2 = Drop[a2, {RandomInteger[{1, 10 + 1 - i}]}];
 Print[a2]]
Print[]

a3 = Table[1 / (2 n), {n, 1, 50}];
Print[a3]
Print["Delete three elements randomly."]
For[i = 1, i ≤ 3, i++, a3 = Drop[a3, {RandomInteger[{1, 50 + 1 - i}]}];
 Print[a3]]
Print[]

num = PrimePi[100];
a4 = Table[Prime[n], {n, 1, num}];
Print[a4]
Print["Delete three elements randomly."]
For[i = 1, i ≤ 3, i++, a4 = Drop[a4, {RandomInteger[{1, num + 1 - i}]}];
 Print[a4]]
Print[]

Clear["Global`*"]
```

```
{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53,
 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99}
```

Delete three elements randomly.

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 87, 89, 91, 93, 95, 97, 99}

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 87, 89, 91, 93, 95, 97, 99}

{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 87, 89, 91, 93, 95, 97, 99}

{1, 4, 9, 16, 25, 36, 49, 64, 81, 100}

Delete three elements randomly.

{1, 4, 9, 25, 36, 49, 64, 81, 100}

{1, 4, 9, 25, 49, 64, 81, 100}

{4, 9, 25, 49, 64, 81, 100}

$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}, \frac{1}{24}, \frac{1}{26}, \frac{1}{28}, \frac{1}{30}, \frac{1}{32}, \frac{1}{34}, \frac{1}{36}, \frac{1}{38}, \frac{1}{40}, \frac{1}{42}, \frac{1}{44}, \frac{1}{46}, \frac{1}{48}, \frac{1}{50}, \frac{1}{52}, \frac{1}{54}, \frac{1}{56}, \frac{1}{58}, \frac{1}{60}, \frac{1}{62}, \frac{1}{64}, \frac{1}{66}, \frac{1}{68}, \frac{1}{70}, \frac{1}{72}, \frac{1}{74}, \frac{1}{76}, \frac{1}{78}, \frac{1}{80}, \frac{1}{82}, \frac{1}{84}, \frac{1}{86}, \frac{1}{88}, \frac{1}{90}, \frac{1}{92}, \frac{1}{94}, \frac{1}{96}, \frac{1}{98}, \frac{1}{100}\right\}$$

Delete three elements randomly.

$$\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}, \frac{1}{24}, \frac{1}{26}, \frac{1}{28}, \frac{1}{32}, \frac{1}{34}, \frac{1}{36}, \frac{1}{38}, \frac{1}{40}, \frac{1}{42}, \frac{1}{44}, \frac{1}{46}, \frac{1}{48}, \frac{1}{50}, \frac{1}{52}, \frac{1}{54}, \frac{1}{56}, \frac{1}{58}, \frac{1}{60}, \frac{1}{62}, \frac{1}{64}, \frac{1}{66}, \frac{1}{68}, \frac{1}{70}, \frac{1}{72}, \frac{1}{74}, \frac{1}{76}, \frac{1}{78}, \frac{1}{80}, \frac{1}{82}, \frac{1}{84}, \frac{1}{86}, \frac{1}{88}, \frac{1}{90}, \frac{1}{92}, \frac{1}{94}, \frac{1}{96}, \frac{1}{98}, \frac{1}{100}\right\}$$

$$\left\{\frac{1}{2}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}, \frac{1}{24}, \frac{1}{26}, \frac{1}{28}, \frac{1}{32}, \frac{1}{34}, \frac{1}{36}, \frac{1}{38}, \frac{1}{40}, \frac{1}{42}, \frac{1}{44}, \frac{1}{46}, \frac{1}{48}, \frac{1}{50}, \frac{1}{52}, \frac{1}{54}, \frac{1}{56}, \frac{1}{58}, \frac{1}{60}, \frac{1}{62}, \frac{1}{64}, \frac{1}{66}, \frac{1}{68}, \frac{1}{70}, \frac{1}{72}, \frac{1}{74}, \frac{1}{76}, \frac{1}{78}, \frac{1}{80}, \frac{1}{82}, \frac{1}{84}, \frac{1}{86}, \frac{1}{88}, \frac{1}{90}, \frac{1}{92}, \frac{1}{94}, \frac{1}{96}, \frac{1}{98}, \frac{1}{100}\right\}$$

$$\left\{\frac{1}{2}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}, \frac{1}{18}, \frac{1}{20}, \frac{1}{22}, \frac{1}{24}, \frac{1}{26}, \frac{1}{28}, \frac{1}{32}, \frac{1}{34}, \frac{1}{36}, \frac{1}{40}, \frac{1}{42}, \frac{1}{44}, \frac{1}{46}, \frac{1}{48}, \frac{1}{50}, \frac{1}{52}, \frac{1}{54}, \frac{1}{56}, \frac{1}{58}, \frac{1}{60}, \frac{1}{62}, \frac{1}{64}, \frac{1}{66}, \frac{1}{68}, \frac{1}{70}, \frac{1}{72}, \frac{1}{74}, \frac{1}{76}, \frac{1}{78}, \frac{1}{80}, \frac{1}{82}, \frac{1}{84}, \frac{1}{86}, \frac{1}{88}, \frac{1}{90}, \frac{1}{92}, \frac{1}{94}, \frac{1}{96}, \frac{1}{98}, \frac{1}{100}\right\}$$

{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}

Delete three elements randomly.

{3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}

{3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}

{3, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 73, 79, 83, 89, 97}

T9

9. 设 a ={Pi/4,Pi/2,Pi},写出下列运行结果:

(1) Apply[Plus,a]　　(2) Apply[Times,a]

(3) a^2　　(4) Sin[a]

In[∘]:=
```
a = {π / 4, π / 2, π};
Apply[Plus, a]
Apply[Times, a]
a^2
Sin[a]

Clear[a]
```

Out[∘]= $\frac{7\pi}{4}$

Out[∘]= $\frac{\pi^3}{8}$

Out[∘]= $\left\{\frac{\pi^2}{16}, \frac{\pi^2}{4}, \pi^2\right\}$

Out[∘]= $\left\{\frac{1}{\sqrt{2}}, 1, 0\right\}$

T10

10. 建立表格

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

In[∘]:=
```
Grid[Table[10 i + j, {i, 1, 4}, {j, 1, 4}]]
```

Out[∘]=

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

T11

11. 建立表格

11			
21	22		
31	32	33	
41	42	43	44

In[ ]:=
```
Grid[Table[10 i + j, {i, 1, 4}, {j, 1, i}]]
```

Out[ ]=
```
11
21  22
31  32  33
41  42  43  44
```

T12

In[ ]:=
```
For[i = 1, i ≤ 4, i++, a[i] = RandomReal[{5.2, 9.7}, 4]]
table = Table[a[i], {i, 1, 4}];
Grid[table]
Print["The maximum is " <> ToString[Max[table]]]
Print["The minimum is " <> ToString[Min[table]]]

Clear[i, table]
```

Out[ ]=
```
7.16768   7.0084   9.36209  6.14217
9.52658  8.88718  8.05697  7.94933
6.70161  7.87653  6.25568  8.47267
8.01317   7.3939   7.77296  7.89804
```

The maximum is 9.52658

The minimum is 6.14217

T13

In[∘]:=
```
Print["The original array is"]
a = RandomSample[100 ;; 200, 60]
Print["The ascend order is"]
Sort[a]
Print["The decrease order is"]
%% // Reverse
```

The original array is

Out[∘]=
```
{168, 151, 135, 172, 197, 189, 158, 138, 116, 108, 107, 179, 182, 198, 125,
 171, 103, 122, 142, 111, 141, 144, 193, 191, 166, 178, 137, 153, 176, 100,
 160, 134, 136, 200, 173, 115, 170, 113, 147, 162, 148, 120, 140, 118, 130,
 155, 105, 119, 146, 194, 165, 159, 101, 186, 183, 106, 139, 145, 180, 109}
```

The ascend order is

Out[∘]=
```
{100, 101, 103, 105, 106, 107, 108, 109, 111, 113, 115, 116, 118, 119, 120,
 122, 125, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146,
 147, 148, 151, 153, 155, 158, 159, 160, 162, 165, 166, 168, 170, 171, 172,
 173, 176, 178, 179, 180, 182, 183, 186, 189, 191, 193, 194, 197, 198, 200}
```

The decrease order is

Out[∘]=
```
{200, 198, 197, 194, 193, 191, 189, 186, 183, 182, 180, 179, 178, 176, 173,
 172, 171, 170, 168, 166, 165, 162, 160, 159, 158, 155, 153, 151, 148, 147,
 146, 145, 144, 142, 141, 140, 139, 138, 137, 136, 135, 134, 130, 125, 122,
 120, 119, 118, 116, 115, 113, 111, 109, 108, 107, 106, 105, 103, 101, 100}
```

T14

14. 写出与下列数学条件等价的 Mathematica 逻辑表达式。

(1) $m>s$ 且 $m<t$，即 $m\in(s,t)$；

(2) $x\leqslant-10$ 或 $x\geqslant10$，即 $x\notin(-10,10)$；

(3) $x\in(-3,6]$ 且 $y\notin[-2,7)$。

In[∘]:=
```
s < m < t
x ≤ -10 || x ≥ 10
-3 < x ≤ 6 && (y < -2 || y ≥ 7)
```

Out[∘]=
```
s < m < t
```

Out[∘]=
```
x ≤ -10 || x ≥ 10
```

Out[∘]=
```
-3 < x ≤ 6 && (y < -2 || y ≥ 7)
```

T15

15. 给定平面上三个圆心分别为(x_1, y_1),(x_2, y_2),(x_3, y_3),半径分别为r_1,r_2,r_3的圆盘。判断平面上一点(x, y)是否被这三个圆盘所覆盖。

In[∘]:=

```
x = 1; y = 1;
x1 = 0; y1 = 0; r1 = Sqrt[2];
x2 = 1; y2 = 0; r2 = 1;
x3 = 0; y3 = 1; r3 = Sqrt[3];
p = {x, y};
ℛ =
  RegionIntersection[Disk[{x1, y1}, r1], Disk[{x2, y2}, r2], Disk[{x3, y3}, r3]];
RegionMember[ℛ, p]
Show[Region[ℛ], Graphics[Point[p]], ImageSize → Small]

Clear["Global`*"]
```

Out[∘]=

```
True
```

Out[∘]=

T_1

1. 展开多项式：

(1) $(x+1)(x^2-2x+3)$

(2) $(3a-2)(a-1)+(a+1)(a+2)$

(3) $x(y-z)+y(z-x)+z(x-y)$

(4) $(2x^2-1)(x-4)-(x^2+3)(2x-5)$

In[∘]:=
```
Expand[(x + 1) (x^2 - 2 x + 3)]

Expand[(3 a - 2) (a - 1) + (a + 1) (a + 2)]

Expand[x (y - z) + y (z - x) + z (x - y)]

Expand[(2 x^2 - 1) (x - 4) - (x^2 + 3) (2 x - 5)]
```

Out[∘]= $3 + x - x^2 + x^3$

Out[∘]= $4 - 2a + 4a^2$

Out[∘]= 0

Out[∘]= $19 - 7x - 3x^2$

T_2

2. 先化简，再求值：

(1) $(x-2)(x^2+2x+4)+(x+5)(x^2-5x+25)$，其中 $x=-4$。

(2) $(y-2)(y^2-6y-9)-y(y^2-2y-15)$，其中 $y=1/2$。

In[∘]:=
```
Simplify[(x - 2) (x^2 + 2 x + 4) + (x + 5) (x^2 - 5 x + 25)]
% /. x → -4

Simplify[(y - 2) (y^2 - 6 y - 9) - y (y^2 - 2 y - 15)]
% /. y → 1/2
```

Out[∘]= $117 + 2 x^3$

Out[∘]= -11

Out[∘]= $-6 (-3 - 3 y + y^2)$

Out[∘]= $\frac{51}{2}$

T3

3. 因式分解：

(1) $x^5 - x^3$

(2) $x^4 - y^4$

(3) $16 - x^4$

(4) $x^3 - 6x^2 + 11x - 6$

(5) $(x + y)^2 - 10(x + y) + 25$

(6) $\frac{x^2}{4} + xy + y^2$

(7) $3ax + 4by + 4ay + 3bx$

(8) $x^4 + 4x^3 - 19x^2 - 46x + 120$

In[∘]:=
```
Factor[x^5 - x^3]

Factor[x^4 - y^4]

Factor[16 - x^4]

Factor[x^3 - 6 x^2 + 11 x - 6]

Factor[(x + y)^2 - 10 (x + y) + 25]

Factor[x^2/4 + x y + y^2]

Factor[3 a x + 4 b y + 4 a y + 3 b x]

Factor[x^4 + 4 x^3 - 19 x^2 - 46 x + 120]
```

Out[∘]= $(-1+x)\,x^3\,(1+x)$

Out[∘]= $(x-y)\,(x+y)\,(x^2+y^2)$

Out[∘]= $-(-2+x)\,(2+x)\,(4+x^2)$

Out[∘]= $(-3+x)\,(-2+x)\,(-1+x)$

Out[∘]= $(-5+x+y)^2$

Out[∘]= $\frac{1}{4}(x+2y)^2$

Out[∘]= $(a+b)\,(3x+4y)$

Out[∘]= $(-3+x)\,(-2+x)\,(4+x)\,(5+x)$

T4

4. 约分：

(1) $\dfrac{x^2+y^2-z^2+2xy}{x^2-y^2+z^2-2xz}$

(2) $\dfrac{ax^3-ay^3}{x^2-y^2}$

In[ ]:=
```
Cancel[(x^2 + y^2 - z^2 + 2 x y) / (x^2 - y^2 + z^2 - 2 x z)]

Cancel[(a x^3 - a y^3) / (x^2 - y^2)]
```

Out[ ]= $\frac{x + y + z}{x - y - z}$

Out[ ]= $\frac{a x^2 + a x y + a y^2}{x + y}$

T5

5. 化简分式：

(1) $\frac{x^2+2x+4}{x^2+4x+4} \div \frac{x^3-8}{3x+6} \div \frac{1}{x^2-4}$

(2) $\frac{1}{x+1} - \frac{x+3}{x^2-1} \cdot \frac{x^2-2x+1}{x^2+4x+3}$

(3) $\frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)}$

(4) $\frac{2\sqrt{2}+3\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

In[ ]:=
```
Simplify[(x^2 + 2 x + 4)/(x^2 + 4 x + 4) / (x^3 - 8)/(3 x + 6) / 1/(x^2 - 4)]

Simplify[1/(x + 1) - (x + 3)/(x^2 - 1) * (x^2 - 2 x + 1)/(x^2 + 4 x + 3)]

Simplify[a/((a - b) (a - c)) + b/((b - c) (b - a)) + c/((c - a) (c - b))]

FullSimplify[(2 Sqrt[2] + 3 Sqrt[3])/(3 Sqrt[2] - 2 Sqrt[3])]
```

Out[ ]= 3

Out[ ]= $\frac{2}{(1+x)^2}$

Out[ ]= 0

Out[ ]= $5+\frac{13}{\sqrt{6}}$

T6

6. 求解方程或方程组：

(1) $(y-3)^3-(y+3)^3=9y(1-2y)$

(2) $3x^2+5(2x+1)=0$

(3) $abx^2+(a^4+b^4)x+a^3b^3=0\ (ab\neq 0)$

(4) $x^2-(2m+1)x+m^2+m=0$

(5) $\begin{cases}4x^2-9y^2=15\\2x-3y=5\end{cases}$

(6) $\begin{cases}x^2+2xy+y^2=9\\(x-y)^2-3(x-y)-10=0\end{cases}$

(7) $\begin{cases}\sqrt{3}x+\sqrt{3}y=\sqrt{7}\\\sqrt{6}x-\sqrt{7}y=\sqrt{5}\end{cases}$

```
Solve[(y - 3)^2 - (y + 3)^2 == 9 y (1 - 2 y), y]

Solve[3 x^2 + 5 (2 x + 1) == 0, x]

Assuming[a b ≠ 0, Solve[a b x^2 + (a^4 + b^4) x + a^3 b^3 == 0, x]]

Solve[x^2 - (2 m + 1) x + m^2 + m == 0, x]

Solve[{4 x^2 - 9 y^2 == 15, 2 x - 3 y == 5}, {x, y}]

Solve[{x^2 + 2 x y + y^2 == 9, (x - y)^2 - 3 (x - y) - 10 == 0}, {x, y}]

Solve[{√3 x + √3 y == √7, √6 x - √7 y == √5}, {x, y}]
```

Out[•]= $\left\{\{y \to 0\}, \left\{y \to \frac{7}{6}\right\}\right\}$

Out[•]= $\left\{\left\{x \to \frac{1}{3}\left(-5 - \sqrt{10}\right)\right\}, \left\{x \to \frac{1}{3}\left(-5 + \sqrt{10}\right)\right\}\right\}$

Out[•]= $\left\{\left\{x \to -\frac{a^3}{b}\right\}, \left\{x \to -\frac{b^3}{a}\right\}\right\}$

Out[•]= $\{\{x \to m\}, \{x \to 1 + m\}\}$

Out[•]= $\left\{\left\{x \to 2, y \to -\frac{1}{3}\right\}\right\}$

Out[•]= $\left\{\left\{x \to -\frac{5}{2}, y \to -\frac{1}{2}\right\}, \left\{x \to \frac{1}{2}, y \to \frac{5}{2}\right\}, \{x \to 1, y \to -4\}, \{x \to 4, y \to -1\}\right\}$

Out[•]= $\left\{\left\{x \to -\frac{-7 - \sqrt{15}}{3\sqrt{2} + \sqrt{21}}, y \to -\frac{\sqrt{15} - \sqrt{42}}{3\sqrt{2} + \sqrt{21}}\right\}\right\}$

T_7

7. 用形式求和获取数列求和计算公式：

(1) $\sum_{k=1}^{n} k^3$ (2) $\sum_{k=m}^{n} k^3$ (3) $\sum_{k=1}^{n} k^5$ (4) $\sum_{k=m}^{n} k^5$

In[∘]:=
```
Sum[k^3, {k, 1, n}]

Sum[k^3, {k, m, n}]

Sum[k^5, {k, 1, n}]

Sum[k^5, {k, m, n}]
```

Out[∘]= $\frac{1}{4} n^2 (1+n)^2$

Out[∘]= $-\frac{1}{4} (-1+m-n) (m+n) (-m+m^2+n+n^2)$

Out[∘]= $\frac{1}{12} n^2 (1+n)^2 (-1+2n+2n^2)$

Out[∘]= $-\frac{1}{12} (-1+m-n) (m+n) (m+m^2-4m^3+2m^4-n-2mn+2m^2n+n^2-2mn^2+2m^2n^2+4n^3+2n^4)$

T8

8. 求下列数列的通项公式：

(1) $a_{n+1} = x + a_n - y$

(2) $a_{n+1} = \frac{a_n}{x} - y$

(3) $a_{n+1} = \frac{2a_n + 3}{a_n + 4}$, $a_0 = 0$

(4) $a_{n+1} - a_n = n^2, a_0 = 1$

In[∘]:=
```
Clear["Global`*"]

RSolve[a[n + 1] == x + a[n] - y, a[n], n]

RSolve[a[n + 1] == a[n] / x - y, a[n], n]

RSolve[{a[n + 1] == (2 a[n] + 3) / (a[n] + 4), a[0] == 0}, a[n], n]

RSolve[{a[n + 1] - a[n] == n^2, a[0] == 1}, a[n], n]
```

Out[∘]= $\{\{a[n] \to n\,(x - y) + c_1\}\}$

Out[∘]= $\left\{\left\{a[n] \to -\frac{\left(1 - \left(\frac{1}{x}\right)^n\right) x\, y}{-1 + x} + \left(\frac{1}{x}\right)^{-1+n} c_1\right\}\right\}$

Out[∘]= $\left\{\left\{a[n] \to \frac{3\left((-1)^n + (-1)^{1+n}\, 5^{-n}\right)}{3\,(-1)^n + \left(-\frac{1}{5}\right)^n}\right\}\right\}$

Out[∘]= $\left\{\left\{a[n] \to \frac{1}{6}\left(6 + n - 3\,n^2 + 2\,n^3\right)\right\}\right\}$

T_1

1. 计算极限：

(1) $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{\sin x}$

(2) $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\ln x}\right)$

(3) $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$

(4) $\lim\limits_{x\to \infty}\left(1+\dfrac{1}{x^2}\right)^x$

(5) $\lim\limits_{x\to \infty}(\sqrt{n-\sqrt{n}}-\sqrt{n})$

In[∘]:=
```
Limit[(E^x - E^(-x)) / Sin[x], x → 0]

Limit[x / (x - 1) - 1 / Log[x], x → 1]

Limit[(1 - Cos[x]) / x^2, x → 0]

Limit[(1 + 1 / x^2) ^x, x -> Infinity]

Limit[Sqrt[n - Sqrt[n]] - Sqrt[n], n → Infinity]
```

Out[∘]= 2

Out[∘]= $\frac{1}{2}$

Out[∘]= $\frac{1}{2}$

Out[∘]= 1

Out[∘]= $-\frac{1}{2}$

T_2

2. 求下列函数的微商：

(1) $y = a^x \ln x$

(2) $y = \dfrac{1 - \ln x}{1 + \ln x}$

(3) $y = \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{x}}}$

(4) $y = \arctan \dfrac{1 + x}{1 - x}$

(5) $e^x + e^{e^x}$

(6) $y = x^{x^x}$

(7) $y = (\sin x)^{\cos x}$

(8) $y = \ln \cos \arctan \dfrac{e^x - e^{-x}}{2}$

In[∘]:=
```
D[a^x * Log[x], x]
D[(1 - Log[x]) / (1 + Log[x]), x]
D[(1 + (1 + x^(1/3))^(1/3))^(1/3), x]
D[ArcTan[(1 + x) / (1 - x)], x]
D[E^x + E^(E^x), x]
D[x^(x^x), x]
D[(Sin[x])^Cos[x], x]
D[Log[Cos[ArcTan[(E^x - E^(-x)) / 2]]], x]
```

Out[∘]= $\dfrac{a^x}{x} + a^x \operatorname{Log}[a] \operatorname{Log}[x]$

Out[∘]= $-\dfrac{1 - \operatorname{Log}[x]}{x (1 + \operatorname{Log}[x])^2} - \dfrac{1}{x (1 + \operatorname{Log}[x])}$

Out[∘]= $\dfrac{1}{27 \left(1 + \left(1 + x^{1/3}\right)^{1/3}\right)^{2/3} \left(1 + x^{1/3}\right)^{2/3} x^{2/3}}$

Out[∘]= $\dfrac{\frac{1}{1-x} + \frac{1+x}{(1-x)^2}}{1 + \frac{(1+x)^2}{(1-x)^2}}$

Out[∘]= $e^x + e^{e^x + x}$

Out[∘]= $x^{x^x} \left(x^{-1+x} + x^x \operatorname{Log}[x] (1 + \operatorname{Log}[x])\right)$

Out[∘]= $\operatorname{Sin}[x]^{\operatorname{Cos}[x]} \left(\operatorname{Cos}[x] \operatorname{Cot}[x] - \operatorname{Log}[\operatorname{Sin}[x]] \operatorname{Sin}[x]\right)$

Out[∘]= $-\dfrac{\left(-e^{-x} + e^x\right) \left(e^{-x} + e^x\right)}{4 \left(1 + \frac{1}{4} \left(-e^{-x} + e^x\right)^2\right)}$

T3

3. (1) 已知 $y=\sin x\sin 2x\sin 3x$，计算高阶导数 $y^{(20)}$；

(2) 已知 $y=\arctan x$，计算高阶导数 $y^{(20)}$；

(3) 已知 $y=\dfrac{1}{1-x^2}$，计算 $y^{(60)}$；

(4) 已知 $y=\dfrac{1+x}{\sqrt{1-x}}$，计算 $y^{(60)}$。

In[∘]:=
```
D[Sin[x] Sin[2 x] Sin[3 x], {x, 20}]

D[ArcTan[x], {x, 20}]

D[1 / (1 - x^2), {x, 60}]

D[(1 + x) / Sqrt[1 - x], {x, 60}]
```

Out[∘]=
```
60 Cos[3 x] (-581 130 734 Cos[2 x] Sin[x] - 581 130 733 Cos[x] Sin[2 x]) +
 3 767 472 Cos[3 x] (-7 174 454 Cos[2 x] Sin[x] - 7 174 453 Cos[x] Sin[2 x]) +
 3 305 956 680 Cos[3 x] (-88 574 Cos[2 x] Sin[x] - 88 573 Cos[x] Sin[2 x]) +
 123 591 918 960 Cos[3 x] (-1094 Cos[2 x] Sin[x] - 1093 Cos[x] Sin[2 x]) +
 147 219 785 820 Cos[3 x] (-14 Cos[2 x] Sin[x] - 13 Cos[x] Sin[2 x]) -
 23 245 229 340 Cos[3 x] (2 Cos[2 x] Sin[x] + Cos[x] Sin[2 x]) -
 222 465 454 128 Cos[3 x] (122 Cos[2 x] Sin[x] + 121 Cos[x] Sin[2 x]) -
 29 753 610 120 Cos[3 x] (9842 Cos[2 x] Sin[x] + 9841 Cos[x] Sin[2 x]) -
 169 536 240 Cos[3 x] (797 162 Cos[2 x] Sin[x] + 797 161 Cos[x] Sin[2 x]) -
 30 780 Cos[3 x] (64 570 082 Cos[2 x] Sin[x] + 64 570 081 Cos[x] Sin[2 x]) +
 3 486 784 401 Sin[x] Sin[2 x] Sin[3 x] -
 1710 (193 710 244 Cos[x] Cos[2 x] - 193 710 245 Sin[x] Sin[2 x]) Sin[3 x] -
 28 256 040 (2 391 484 Cos[x] Cos[2 x] - 2 391 485 Sin[x] Sin[2 x]) Sin[3 x] -
 10 909 657 044 (29 524 Cos[x] Cos[2 x] - 29 525 Sin[x] Sin[2 x]) Sin[3 x] -
 185 387 878 440 (364 Cos[x] Cos[2 x] - 365 Sin[x] Sin[2 x]) Sin[3 x] -
 73 609 892 910 (4 Cos[x] Cos[2 x] - 5 Sin[x] Sin[2 x]) Sin[3 x] +
 208 561 363 245 (-40 Cos[x] Cos[2 x] + 41 Sin[x] Sin[2 x]) Sin[3 x] +
 66 945 622 770 (-3280 Cos[x] Cos[2 x] + 3281 Sin[x] Sin[2 x]) Sin[3 x] +
 826 489 170 (-265 720 Cos[x] Cos[2 x] + 265 721 Sin[x] Sin[2 x]) Sin[3 x] +
 392 445 (-21 523 360 Cos[x] Cos[2 x] + 21 523 361 Sin[x] Sin[2 x]) Sin[3 x] +
 (-1 743 392 200 Cos[x] Cos[2 x] + 1 743 392 201 Sin[x] Sin[2 x]) Sin[3 x]
```

Out[•]=

$$\frac{1}{\left(1+x^2\right)^{20}} 2\,432\,902\,008\,176\,640\,000\, x \left(1-57\,x^2+\frac{3876\,x^4}{5}-3876\,x^6+8398\,x^8-8398\,x^{10}+3876\,x^{12}-\frac{3876\,x^{14}}{5}+57\,x^{16}-x^{18}\right)$$

Out[•]=

$$\frac{1}{\left(1-x^2\right)^{61}} 9\,593\,444\,981\,835\,986\,954\,891\,939\,947\,669\,322\,185\,182\,489\,942\,608\,389\,896\,364\,094\,195\,294\,295\,395\,488\,811\,817\,369\,600\,000\,000\,000\,000\, x^{60} + \frac{1}{\left(1-x^2\right)^{60}} 141\,503\,313\,482\,080\,807\,584\,656\,114\,228\,122\,502\,231\,441\,726\,653\,473\,750\,971\,370\,389\,380\,590\,857\,083\,459\,974\,306\,201\,600\,000\,000\,000\,000\, x^{58} + \frac{1}{\left(1-x^2\right)^{59}} 991\,122\,784\,685\,930\,402\,277\,273\,545\,843\,586\,848\,256\,665\,992\,195\,729\,281\,168\,115\,481\,551\,341\,893\,046\,437\,870\,881\,996\,800\,000\,000\,000\,000\, x^{56} + \frac{1}{\left(1-x^2\right)^{58}} 4\,386\,003\,127\,633\,140\,285\,939\,658\,794\,825\,068\,236\,538\,119\,620\,636\,273\,255\,743\,959\,314\,911\,110\,676\,125\,041\,152\,753\,664\,000\,000\,000\,000\,000\, x^{54} + \frac{1}{\left(1-x^2\right)^{57}} 13\,763\,970\,341\,322\,420\,502\,586\,955\,560\,076\,036\,505\,451\,862\,230\,549\,357\,519\,670\,188\,113\,240\,788\,108\,629\,240\,985\,944\,064\,000\,000\,000\,000\,000\, x^{52} + \frac{1}{\left(1-x^2\right)^{56}} 32\,591\,115\,486\,774\,159\,975\,768\,398\,344\,037\,186\,439\,694\,945\,210\,193\,657\,269\,790\,481\,139\,566\,580\,414\,361\,381\,334\,574\,694\,400\,000\,000\,000\,000\,000\, x^{50} + \frac{1}{\left(1-x^2\right)^{55}} 60\,491\,085\,562\,573\,251\,470\,176\,193\,896\,129\,626\,346\,403\,496\,791\,647\,318\,417\,414\,150\,599\,953\,122\,738\,776\,806\,264\,930\,304\,000\,000\,000\,000\,000\, x^{48} + \frac{1}{\left(1-x^2\right)^{54}} 90\,256\,540\,363\,204\,533\,939\,627\,971\,845\,018\,807\,564\,475\,058\,704\,997\,586\,210\,110\,002\,482\,469\,738\,689\,603\,488\,712\,753\,152\,000\,000\,000\,000\,000\, x^{46} + \frac{1}{\left(1-x^2\right)^{53}} 110\,159\,810\,466\,882\,892\,249\,428\,008\,089\,144\,417\,251\,452\,459\,622\,255\,308\,640\,877\,184\,633\,674\,740\,027\,994\,824\,077\,475\,840\,000\,000\,000\,000\,000\, x^{44} + \frac{1}{\left(1-x^2\right)^{52}} 111\,336\,731\,518\,879\,504\,346\,109\,931\,252\,489\,977\,264\,822\,678\,207\,963\,164\,502\,425\,017\,802\,837\,931\,694\,960\,580\,744\,970\,240\,000\,000\,000\,000\,000\, x^{42} + \frac{1}{\left(1-x^2\right)^{51}} 93\,981\,299\,840\,936\,522\,786\,275\,147\,851\,366\,539\,632\,365\,025\,428\,486\,553\,565\,282\,294\,439\,454\,371\,754\,275\,549\,040\,607\,232\,000\,000\,000\,000\,000\, x^{40} + \frac{1}{\left(1-x^2\right)^{50}} 66\,641\,285\,341\,754\,988\,884\,813\,286\,658\,241\,728\,102\,949\,745\,303\,835\,919\,800\,836\,536\,057\,067\,645\,425\,759\,025\,683\,339\,673\,600\,000\,000\,000\,000\,000\, x^{38} + \frac{1}{\left(1-x^2\right)^{49}} 39\,837\,435\,029\,977\,684\,681\,992\,976\,633\,285\,658\,891\,474\,209\,990\,303\,275\,187\,064\,698\,000\,100\,811$

$$848\,901\,866\,543\,697\,100\,800\,000\,000\,000\,000\,x^{36} + \frac{1}{(1-x^2)^{48}}$$

$$20\,110\,243\,644\,940\,658\,132\,736\,839\,165\,841\,318\,190\,407\,654\,081\,643\,480\,262\,700\,929\,278\,897\,044 \backslash$$

$$442\,955\,269\,168\,693\,248\,000\,000\,000\,000\,000\,x^{34} + \frac{1}{(1-x^2)^{47}}$$

$$8\,572\,831\,827\,364\,520\,678\,165\,172\,319\,177\,036\,097\,886\,545\,546\,962\,000\,324\,753\,207\,694\,119\,484 \backslash$$

$$751\,138\,226\,446\,532\,608\,000\,000\,000\,000\,000\,x^{32} + \frac{1}{(1-x^2)^{46}}$$

$$3\,081\,249\,700\,270\,146\,562\,586\,902\,514\,718\,702\,829\,385\,309\,124\,125\,472\,580\,491\,007\,982\,813\,959 \backslash$$

$$736\,640\,985\,737\,304\,473\,600\,000\,000\,000\,000\,x^{30} + \frac{1}{(1-x^2)^{45}}$$

$$930\,794\,180\,289\,940\,107\,448\,126\,801\,321\,274\,813\,043\,478\,797\,912\,903\,175\,356\,658\,661\,475\,050\,337 \backslash$$

$$110\,297\,774\,810\,726\,400\,000\,000\,000\,000\,x^{28} + \frac{1}{(1-x^2)^{44}}$$

$$235\,187\,299\,565\,238\,877\,416\,705\,836\,162\,728\,528\,964\,194\,509\,098\,313\,770\,243\,861\,613\,661\,476\,622 \backslash$$

$$612\,093\,956\,469\,555\,200\,000\,000\,000\,000\,x^{26} + \frac{1}{(1-x^2)^{43}}$$

$$49\,377\,178\,526\,293\,691\,964\,101\,677\,488\,944\,943\,096\,487\,865\,282\,268\,717\,912\,955\,442\,144\,689\,859 \backslash$$

$$398\,533\,937\,889\,280\,000\,000\,000\,000\,000\,x^{24} + \frac{1}{(1-x^2)^{42}}$$

$$8\,538\,910\,572\,216\,202\,369\,731\,869\,039\,441\,606\,700\,896\,397\,755\,580\,304\,601\,488\,535\,107\,728\,321\,550 \backslash$$

$$122\,410\,311\,680\,000\,000\,000\,000\,000\,x^{22} + \frac{1}{(1-x^2)^{41}}$$

$$1\,202\,736\,794\,013\,379\,724\,029\,305\,943\,970\,128\,748\,723\,821\,878\,987\,225\,831\,063\,324\,152\,369\,050\,169 \backslash$$

$$559\,924\,867\,072\,000\,000\,000\,000\,000\,x^{20} + \frac{1}{(1-x^2)^{40}}$$

$$136\,023\,804\,084\,846\,516\,408\,076\,267\,472\,812\,179\,915\,194\,141\,075\,936\,254\,703\,590\,231\,517\,928\,292 \backslash$$

$$985\,943\,883\,776\,000\,000\,000\,000\,000\,x^{18} + \frac{1}{(1-x^2)^{39}}$$

$$12\,127\,996\,518\,054\,497\,092\,328\,478\,393\,554\,932\,125\,305\,771\,319\,707\,603\,129\,166\,261\,901\,074\,026 \backslash$$

$$122\,872\,619\,008\,000\,000\,000\,000\,000\,x^{16} + \frac{1}{(1-x^2)^{38}}$$

$$832\,585\,573\,321\,819\,022\,356\,646\,113\,974\,022\,800\,364\,240\,594\,030\,270\,237\,700\,201\,045\,840\,322\,159 \backslash$$

$$464\,939\,520\,000\,000\,000\,000\,000\,x^{14} + \frac{1}{(1-x^2)^{37}}$$

$$42\,660\,634\,669\,079\,690\,897\,778\,601\,560\,605\,897\,991\,636\,201\,608\,532\,990\,783\,062\,103\,137\,088\,579 \backslash$$

$$116\,728\,320\,000\,000\,000\,000\,000\,x^{12} + \frac{1}{(1-x^2)^{36}}$$

$$1\,564\,223\,271\,199\,588\,666\,251\,882\,057\,222\,216\,259\,693\,327\,392\,312\,876\,328\,712\,277\,115\,026\,581\,234 \backslash$$

$$280\,038\,400\,000\,000\,000\,000\,x^{10} + \frac{1}{(1-x^2)^{35}}$$

$$38\,675\,850\,112\,077\,741\,747\,986\,094\,821\,428\,424\,003\,406\,446\,513\,230\,458\,676\,951\,906\,690\,217\,667 \backslash$$

$$880\,550\,400\,000\,000\,000\,000\,x^{8} + \frac{1}{(1-x^2)^{34}}$$

$$589\,827\,779\,487\,024\,383\,956\,214\,953\,703\,701\,455\,389\,640\,796\,498\,067\,997\,251\,989\,862\,378\,047\,222 \backslash$$

```
    579 200 000 000 000 000 x^6 + 1/(1 - x^2)^33
4 787 563 145 186 886 233 410 835 663 179 394 930 110 720 750 796 006 471 201 216 415 406 227 456 \
    000 000 000 000 000 x^4 + 1/(1 - x^2)^32
15 477 036 029 698 985 668 353 994 600 795 457 748 202 761 047 831 917 471 555 656 515 321 856 \
    000 000 000 000 000 x^2 +
8 320 987 112 741 390 144 276 341 183 223 364 380 754 172 606 361 245 952 449 277 696 409 600 000 \
    000 000 000 / (1 - x^2)^31
```

Out[ ]=

```
87 894 875 568 921 902 884 253 090 598 307 649 451 923 547 315 094 294 787 079 697 172 739 171 813 \
    012 930 524 808 746 337 890 625 / (144 115 188 075 855 872 (1 - x)^(119/2)) +
 (697 299 346 180 113 762 881 741 185 413 240 685 651 926 808 699 748 071 977 498 930 903 730 763 \
      049 902 582 163 482 720 947 265 625 (1 + x)) / (1 152 921 504 606 846 976 (1 - x)^(121/2))
```

T4

4. 计算下列不定积分：

(1) $\int (2x-3)^{100}\mathrm{d}x$

(2) $\int \frac{x+1}{\sqrt{x}}\mathrm{d}x$

(3) $\int x^2 a^x \mathrm{d}x$

(4) $\int \frac{2x^2-5}{x^4-5x^2+6}\mathrm{d}x$

(5) $\int \ln(x+\sqrt{1+x^2})\mathrm{d}x$

(6) $\int \frac{\mathrm{e}^{2x}+1}{\mathrm{e}^x+1}\mathrm{d}x$

(7) $\iint \arctan\frac{y}{x}\mathrm{d}x\mathrm{d}y$

(8) $\iiint xyz(1-x-y)\mathrm{d}x\mathrm{d}y\mathrm{d}z$

In[∘]:=
```
Integrate[(2 x - 3)^100, x]

Integrate[(x + 1) / √x, x]

Integrate[x^2 * a^x, x]

Integrate[(2 x^2 - 5) / (x^4 - 5 x^2 + 6), x]

Integrate[Log[x + √(1 + x^2)], x]

Integrate[(E^(2 x) + 1) / (E^x + 1), x]

Integrate[ArcTan[y / x], y, x]

Integrate[x y z (1 - x - y), z, y, x]
```

Out[∘]= $-\frac{1}{202}(3-2x)^{101}$

Out[∘]= $\frac{2}{3}\sqrt{x}\,(3+x)$

Out[∘]= $\frac{a^x\left(2-2x\,\mathrm{Log}[a]+x^2\,\mathrm{Log}[a]^2\right)}{\mathrm{Log}[a]^3}$

Out[∘]= $\frac{1}{12}\left(3\sqrt{2}\,\mathrm{Log}\left[\sqrt{2}-x\right]+2\sqrt{3}\,\mathrm{Log}\left[\sqrt{3}-x\right]-3\sqrt{2}\,\mathrm{Log}\left[\sqrt{2}+x\right]-2\sqrt{3}\,\mathrm{Log}\left[\sqrt{3}+x\right]\right)$

Out[∘]= $-\sqrt{1+x^2}+x\,\mathrm{Log}\left[x+\sqrt{1+x^2}\right]$

Out[∘]= $e^x+x-2\,\mathrm{Log}\left[1+e^x\right]$

Out[∘]= $\frac{1}{4}\left(-y^2+4\,x\,y\,\mathrm{ArcTan}\left[\frac{y}{x}\right]+\left(-x^2+y^2\right)\mathrm{Log}\left[x^2+y^2\right]\right)$

Out[∘]= $-\frac{1}{24}x^2y^2\left(-3+2x+2y\right)z^2$

T_5

5. 计算下列定积分：

(1) $\int_0^1 \sin^2 x \cos^2 x \mathrm{d}x$

(2) $\int_0^{\ln 2} \sqrt{\mathrm{e}^x - 1}\mathrm{d}x$

(3) $\int_0^1 \frac{\sqrt{\mathrm{e}^x}}{\sqrt{\mathrm{e}^x + \mathrm{e}^{-x}}}\mathrm{d}x$

(4) $\int_0^a \frac{x^2}{\sqrt{x^2 + a^2}}\mathrm{d}x$

(5) $\int_0^\infty \frac{\prod_{k=0}^{8} \sin \frac{x}{2k+1}}{x^9}\mathrm{d}x$

(6) $\int_0^1 \frac{\left(\frac{1}{2}\sqrt{4x+1}+1\right)}{x}\mathrm{d}x$

(7) $\int_1^2 \int_1^{1-x} (x^2 + y^3)\mathrm{d}y\mathrm{d}x$

(8) $\int_0^1 \int_{x^2}^{x} xy^2 \mathrm{d}x\mathrm{d}y$

(9) $\int_0^{2\pi} \mathrm{d}\varphi \int_0^a r^2 \sin^2 \varphi \mathrm{d}r$

(10) $\int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y \int_0^{x+y} xyz\mathrm{d}z$

(11) $\iint\limits_{x^2+y^2\leqslant 1} x^2 y^4 \mathrm{d}x\mathrm{d}y$

(12) $\iint\limits_{x^2\leqslant y\leqslant\sqrt{x}} x\sqrt{y}\mathrm{d}x\mathrm{d}y$

In[∘]:=
```
Integrate[Sin[x]^2 * Cos[x]^2, {x, 0, 1}]

Integrate[√(E^x - 1), {x, 0, Log[2]}]

Integrate[√(E^x) / √(E^x + E^(-x)), {x, 0, 1}]

Integrate[x^2 / √(x^2 + a^2), {x, 0, a}]

Integrate[Product[Sin[x / (2 k + 1)], {k, 0, 8}] / x^9, {x, 0, Infinity}]

Integrate[(√(4 x + 1) / 2 + 1) / x, {x, 0, 1}]

Integrate[x^2 + y^3, {x, 1, 2}, {y, 1, 1 - x}]

Integrate[x y^2, {x, 0, 1}, {y, x^2, x}]

Integrate[r^2 * Sin[φ]^2, {φ, 0, 2 π}, {r, 0, a}]

Integrate[x y z, {x, 0, 1}, {y, 0, x}, {z, 0, x + y}]

Integrate[x^2 * y^4 * Boole[x^2 + y^2 ≤ 1], {x, -1, 1}, {y, -1, 1}]

Integrate[x √y * Boole[x^2 ≤ y ≤ √x], {x, 0, Infinity}, {y, 0, Infinity}]
```

Out[∘]= $\frac{1}{32}(4 - \mathrm{Sin}[4])$

Out[∘]= $2 - \frac{\pi}{2}$

Out[•]= $-\text{ArcSinh}[1]+\text{ArcSinh}[e]$

Out[•]= $\frac{1}{2}a\sqrt{a^2}\left(\sqrt{2}-\text{ArcCoth}\left[\sqrt{2}\right]\right)$

Out[•]= $\frac{17\,708\,695\,183\,056\,190\,642\,497\,315\,530\,628\,422\,295\,569\,865\,119\,\pi}{1\,220\,462\,921\,565\,155\,916\,674\,902\,677\,397\,230\,198\,502\,690\,752\,000\,000\,000}$

... Integrate: $\frac{1}{x}+\frac{\sqrt{1+4x}}{2x}$ 的积分在 {0, 1} 上不收敛.

Out[•]= $\int_0^1 \frac{1+\frac{1}{2}\sqrt{1+4x}}{x}\,dx$

Out[•]= $-\frac{79}{20}$

Out[•]= $\frac{1}{40}$

Out[•]= $\frac{a^3\pi}{3}$

Out[•]= $\frac{17}{144}$

Out[•]= $\frac{\pi}{64}$

Out[•]= $\frac{6}{55}$

T6

6. 求悬链线 $y(x)=a\cosh\frac{x}{a}(a\leqslant x\leqslant a)$的弧长。

In[•]:=
```
y[x_] := a * Cosh[x / a]
Integrate[√(1 + y'[x]^2), {x, -a, a}]
```

Out[•]= $\boxed{2\,a\,\text{Sinh}[1]\ \text{if}\ a\in\mathbb{R}}$

T7

7. 求下列幂级数的 5 阶展开式：

(1) e^{x^2}，在 $x=0$ 处　　(2) $\frac{x^2}{1-x}$，在 $x=0$ 处

(3) $\ln\sqrt{\frac{1+x}{1-x}}$，在 $x=0$ 处　　(4) $(1+x)e^{-x}$，在 $x=0$ 处

(5) $\cos x\cos y$，在 $\{0,0\}$ 处

In[∘]:=
```
Clear["Global`*"]
Series[E^(x^2), {x, 0, 5}]
Series[x^2/(1-x), {x, 0, 5}]
Series[Log[Sqrt[(1+x)/(1-x)]], {x, 0, 5}]
Series[(1+x)*E^(-x), {x, 0, 5}]
Series[Cos[x]*Cos[y], {x, 0, 5}, {y, 0, 5}]
```

Out[∘]= $1+x^2+\frac{x^4}{2}+O[x]^6$

Out[∘]= $x^2+x^3+x^4+x^5+O[x]^6$

Out[∘]= $x+\frac{x^3}{3}+\frac{x^5}{5}+O[x]^6$

Out[∘]= $1-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{8}+\frac{x^5}{30}+O[x]^6$

Out[∘]= $\left(1-\frac{y^2}{2}+\frac{y^4}{24}+O[y]^6\right)+\left(-\frac{1}{2}+\frac{y^2}{4}-\frac{y^4}{48}+O[y]^6\right)x^2+\left(\frac{1}{24}-\frac{y^2}{48}+\frac{y^4}{576}+O[y]^6\right)x^4+O[x]^6$

T8

8. 解下列常微分方程或常微分方程组：

(1) $xy'(x)+y(x)=y^2(x)$　　(2) $\frac{dy}{dx}=y^2(x)-\frac{2}{x^2}$

(3) $(1+y^2(x))\mathrm{d}x = x\mathrm{d}y$

(4) $y^{(4)}(x) = y(x)$

(5) $\begin{cases} y'(x)+y(x)=a\sin(x) \\ y(0)=1 \end{cases}$

(6) $\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = y+x \\ y(0)=1 \end{cases}$

(7) $\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} + y = \cos t \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} + x = \sin t \end{cases}$

(8) $\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}x(t) = 2x - y + z \\ \dfrac{\mathrm{d}}{\mathrm{d}t}y(t) = 2x + 2y - z \\ \dfrac{\mathrm{d}}{\mathrm{d}t}z(t) = x + 2y - z \end{cases}$

In[∘]:=
```
DSolve[x * y'[x] + y[x] == y[x]^2, y[x], x]

DSolve[y'[x] == y[x]^2 - 2/x^2, y[x], x]

DSolve[1 + y[x]^2 == x * y'[x], y[x], x]

DSolve[D[y[x], {x, 4}] == y[x], y[x], x]

DSolve[{y'[x] + y[x] == a Sin[x], y[0] == 1}, y[x], x]

DSolve[{y'[x] == y[x] + x, y[0] == 1}, y[x], x]

DSolve[{x'[t] + y[t] == Cos[t], y'[t] + x[t] == Sin[t]}, {x[t], y[t]}, t]

DSolve[{x'[t] == 2 x[t] - y[t] + z[t], y'[t] == 2 x[t] + 2 y[t] - z[t],
  z'[t] == x[t] + 2 y[t] - z[t]}, {x[t], y[t], z[t]}, t]
```

Out[∘]= $\left\{\left\{y[x] \to \frac{1}{1+e^{c_1}x}\right\}\right\}$

Out[∘]= $\left\{\left\{y[x] \to \frac{-1+3\left(-1+\frac{2c_1}{x^3+c_1}\right)}{2x}\right\}\right\}$

Out[∘]= $\{\{y[x] \to \mathrm{Tan}[c_1+\mathrm{Log}[x]]\}\}$

Out[∘]= $\left\{\left\{y[x] \to e^x c_1 + e^{-x} c_3 + c_2 \mathrm{Cos}[x] + c_4 \mathrm{Sin}[x]\right\}\right\}$

Out[∘]= $\left\{\left\{y[x] \to -\frac{1}{2} e^{-x}\left(-2-a+a e^x \mathrm{Cos}[x] - a e^x \mathrm{Sin}[x]\right)\right\}\right\}$

Out[∘]= $\left\{\left\{y[x] \to -1+2e^x - x\right\}\right\}$

Out[•]=

$$\left\{\left\{x[t]\to \frac{1}{2}e^{-t}\left(1+e^{2t}\right)c_1-\frac{1}{2}e^{-t}\left(-1+e^{2t}\right)c_2-\frac{1}{4}e^{-2t}\left(-1+e^{2t}\right)^2\mathrm{Sin}[t]+\frac{1}{4}\left(1+e^{-2t}\right)\left(1+e^{2t}\right)\mathrm{Sin}[t],\right.\right.$$
$$y[t]\to -\frac{1}{2}e^{-t}\left(-1+e^{2t}\right)c_1+\frac{1}{2}e^{-t}\left(1+e^{2t}\right)c_2-\frac{1}{4}\left(1+e^{-2t}\right)\left(-1+e^{2t}\right)\mathrm{Sin}[t]+\left.\left.\frac{1}{4}e^{-2t}\left(-1+e^{2t}\right)\left(1+e^{2t}\right)\mathrm{Sin}[t]\right\}\right\}$$

Out[•]=

$$\left\{\left\{x[t]\to e^{t}\left(1+t\right)c_1-e^{t}t\,c_2+e^{t}t\,c_3,\right.\right.$$
$$y[t]\to \frac{1}{2}e^{t}t\left(4+3t\right)c_1-\frac{1}{2}e^{t}\left(-2-2t+3t^2\right)c_2+\frac{1}{2}e^{t}t\left(-2+3t\right)c_3,$$
$$\left.\left.z[t]\to \frac{1}{2}e^{t}t\left(2+3t\right)c_1-\frac{1}{2}e^{t}t\left(-4+3t\right)c_2+\frac{1}{2}e^{t}\left(2-4t+3t^2\right)c_3\right\}\right\}$$

T9

9. 求解一阶偏微分方程：

(1) $(y+z)u_x+(z+x)u_y+(x+y)u_z=0$

(2) $(x^2+y^2)u_x+6xyu_y=0$

(3) $(xy^3-2x^4)u_x+(3y^4-x^3y)u_y=9u(x^3-y^3)$

(4) $x^2u_x-y^2u_y=u$

In[•]:=

```
DSolve[(y + z) ∂x u[x, y, z] + (z + x) ∂y u[x, y, z] + (x + y) ∂z u[x, y, z] == 0,
 u[x, y, z], {x, y, z}]

DSolve[(x^2 + y^2) ∂x u[x, y] + 6 x y ∂y u[x, y] == 0, u[x, y], {x, y}]

DSolve[(x y^3 - 2 x^4) ∂x u[x, y] + (3 y^4 - x^3 y) ∂y u[x, y] == 9 u[x, y] (x^3 - y^3),
 u[x, y], {x, y}]

DSolve[x^2 * ∂x u[x, y] - y^2 * ∂y u[x, y] == u[x, y], u[x, y], {x, y}]
```

Out[•]=

$$\mathrm{DSolve}\left[(x+y)\,u^{(0,0,1)}[x,y,z]+(x+z)\,u^{(0,1,0)}[x,y,z]+(y+z)\,u^{(1,0,0)}[x,y,z]==0,\ u[x,y,z],\{x,y,z\}\right]$$

... Solve: Solve 正在使用反函数，因此可能无法找到某些解；请使用 Reduce 来获取完整的解信息.

Out[•]=

$$\left\{\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[\frac{\left(5x^2-y^2\right)^{3/5}}{y^{1/5}}\right]\right]\right\},\ \left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{(-1)^{1/5}\left(5x^2-y^2\right)^{3/5}}{y^{1/5}}\right]\right]\right\},\right.$$
$$\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[\frac{(-1)^{2/5}\left(5x^2-y^2\right)^{3/5}}{y^{1/5}}\right]\right]\right\},$$
$$\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{(-1)^{3/5}\left(5x^2-y^2\right)^{3/5}}{y^{1/5}}\right]\right]\right\},$$
$$\left.\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[\frac{(-1)^{4/5}\left(5x^2-y^2\right)^{3/5}}{y^{1/5}}\right]\right]\right\}\right\}$$

... Solve: Solve 正在使用反函数，因此可能无法找到某些解；请使用 Reduce 来获取完整的解信息.

Out[•]=

$$\left\{\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\right\},\ \left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\right\},\right.$$
$$\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\right\},$$
$$\left.\left\{u[x,y]\to c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\right\}\right\}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[1]^3-9\,\mathrm{Root}[K[1]^{15}+10K[1]^{12}\#1+40K[1]^9\#1^2+80K[1]^6\#1^3+\left(80K[1]^3-\frac{80K[1]^9}{x^6}-\frac{x^6K[1]^9}{y^{12}}-\frac{10x^3K[1]^9}{y^9}-\frac{40K[1]^9}{y^6}-\frac{80K[1]^9}{x^3y^3}-\frac{32y^3K[1]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}{-2K[1]^4+K[1]\,\mathrm{Root}[K[1]^{15}+10K[1]^{12}\#1+40K[1]^9\#1^2+80K[1]^6\#1^3+\left(80K[1]^3-\frac{80K[1]^9}{x^6}-\frac{x^6K[1]^9}{y^{12}}-\frac{10x^3K[1]^9}{y^9}-\frac{40K[1]^9}{y^6}-\frac{80K[1]^9}{x^3y^3}-\frac{32y^3K[1]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}\,dK[1]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[2]^3-9\,\mathrm{Root}[K[2]^{15}+10K[2]^{12}\#1+40K[2]^9\#1^2+80K[2]^6\#1^3+\left(80K[2]^3-\frac{80K[2]^9}{x^6}-\frac{x^6K[2]^9}{y^{12}}-\frac{10x^3K[2]^9}{y^9}-\frac{40K[2]^9}{y^6}-\frac{80K[2]^9}{x^3y^3}-\frac{32y^3K[2]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}{-2K[2]^4+K[2]\,\mathrm{Root}[K[2]^{15}+10K[2]^{12}\#1+40K[2]^9\#1^2+80K[2]^6\#1^3+\left(80K[2]^3-\frac{80K[2]^9}{x^6}-\frac{x^6K[2]^9}{y^{12}}-\frac{10x^3K[2]^9}{y^9}-\frac{40K[2]^9}{y^6}-\frac{80K[2]^9}{x^3y^3}-\frac{32y^3K[2]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}\,dK[2]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[3]^3-9\,\mathrm{Root}[K[3]^{15}+10K[3]^{12}\#1+40K[3]^9\#1^2+80K[3]^6\#1^3+\left(80K[3]^3-\frac{80K[3]^9}{x^6}-\frac{x^6K[3]^9}{y^{12}}-\frac{10x^3K[3]^9}{y^9}-\frac{40K[3]^9}{y^6}-\frac{80K[3]^9}{x^3y^3}-\frac{32y^3K[3]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}{-2K[3]^4+K[3]\,\mathrm{Root}[K[3]^{15}+10K[3]^{12}\#1+40K[3]^9\#1^2+80K[3]^6\#1^3+\left(80K[3]^3-\frac{80K[3]^9}{x^6}-\frac{x^6K[3]^9}{y^{12}}-\frac{10x^3K[3]^9}{y^9}-\frac{40K[3]^9}{y^6}-\frac{80K[3]^9}{x^3y^3}-\frac{32y^3K[3]^9}{x^9}\right)\#1^4+32\#1^5\&,1]}\,dK[3]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[4]^3-9\,\mathrm{Root}[K[4]^{15}+10K[4]^{12}\#1+40K[4]^9\#1^2+80K[4]^6\#1^3+\left(80K[4]^3-\frac{80K[4]^9}{x^6}-\frac{x^6K[4]^9}{y^{12}}-\frac{10x^3K[4]^9}{y^9}-\frac{40K[4]^9}{y^6}-\frac{80K[4]^9}{x^3y^3}-\frac{32y^3K[4]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}{-2K[4]^4+K[4]\,\mathrm{Root}[K[4]^{15}+10K[4]^{12}\#1+40K[4]^9\#1^2+80K[4]^6\#1^3+\left(80K[4]^3-\frac{80K[4]^9}{x^6}-\frac{x^6K[4]^9}{y^{12}}-\frac{10x^3K[4]^9}{y^9}-\frac{40K[4]^9}{y^6}-\frac{80K[4]^9}{x^3y^3}-\frac{32y^3K[4]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}\,dK[4]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[5]^3-9\,\mathrm{Root}[K[5]^{15}+10K[5]^{12}\#1+40K[5]^9\#1^2+80K[5]^6\#1^3+\left(80K[5]^3-\frac{80K[5]^9}{x^6}-\frac{x^6K[5]^9}{y^{12}}-\frac{10x^3K[5]^9}{y^9}-\frac{40K[5]^9}{y^6}-\frac{80K[5]^9}{x^3y^3}-\frac{32y^3K[5]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}{-2K[5]^4+K[5]\,\mathrm{Root}[K[5]^{15}+10K[5]^{12}\#1+40K[5]^9\#1^2+80K[5]^6\#1^3+\left(80K[5]^3-\frac{80K[5]^9}{x^6}-\frac{x^6K[5]^9}{y^{12}}-\frac{10x^3K[5]^9}{y^9}-\frac{40K[5]^9}{y^6}-\frac{80K[5]^9}{x^3y^3}-\frac{32y^3K[5]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}\,dK[5]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[6]^3-9\,\mathrm{Root}[K[6]^{15}+10K[6]^{12}\#1+40K[6]^9\#1^2+80K[6]^6\#1^3+\left(80K[6]^3-\frac{80K[6]^9}{x^6}-\frac{x^6K[6]^9}{y^{12}}-\frac{10x^3K[6]^9}{y^9}-\frac{40K[6]^9}{y^6}-\frac{80K[6]^9}{x^3y^3}-\frac{32y^3K[6]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}{-2K[6]^4+K[6]\,\mathrm{Root}[K[6]^{15}+10K[6]^{12}\#1+40K[6]^9\#1^2+80K[6]^6\#1^3+\left(80K[6]^3-\frac{80K[6]^9}{x^6}-\frac{x^6K[6]^9}{y^{12}}-\frac{10x^3K[6]^9}{y^9}-\frac{40K[6]^9}{y^6}-\frac{80K[6]^9}{x^3y^3}-\frac{32y^3K[6]^9}{x^9}\right)\#1^4+32\#1^5\&,2]}\,dK[6]}$$

$$u[x,y]\to e^{\int_1^x \frac{9K[7]^3-9\,\mathrm{Root}[K[7]^{15}+10K[7]^{12}\#1+40K[7]^9\#1^2+80K[7]^6\#1^3+\left(80K[7]^3-\frac{80K[7]^9}{x^6}-\frac{x^6K[7]^9}{y^{12}}-\frac{10x^3K[7]^9}{y^9}-\frac{40K[7]^9}{y^6}-\frac{80K[7]^9}{x^3y^3}-\frac{32y^3K[7]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}{-2K[7]^4+K[7]\,\mathrm{Root}[K[7]^{15}+10K[7]^{12}\#1+40K[7]^9\#1^2+80K[7]^6\#1^3+\left(80K[7]^3-\frac{80K[7]^9}{x^6}-\frac{x^6K[7]^9}{y^{12}}-\frac{10x^3K[7]^9}{y^9}-\frac{40K[7]^9}{y^6}-\frac{80K[7]^9}{x^3y^3}-\frac{32y^3K[7]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}\,dK[7]}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[8]^3-9\,\mathrm{Root}[K[8]^{15}+10K[8]^{12}\#1+40K[8]^9\#1^2+80K[8]^6\#1^3+\left(80K[8]^3-\frac{80K[8]^9}{x^6}-\frac{x^6K[8]^9}{y^{12}}-\frac{10x^3K[8]^9}{y^9}-\frac{40K[8]^9}{y^6}-\frac{80K[8]^9}{x^3y^3}-\frac{32y^3K[8]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}{-2K[8]^4+K[8]\,\mathrm{Root}[K[8]^{15}+10K[8]^{12}\#1+40K[8]^9\#1^2+80K[8]^6\#1^3+\left(80K[8]^3-\frac{80K[8]^9}{x^6}-\frac{x^6K[8]^9}{y^{12}}-\frac{10x^3K[8]^9}{y^9}-\frac{40K[8]^9}{y^6}-\frac{80K[8]^9}{x^3y^3}-\frac{32y^3K[8]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}\,dK[8]}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[9]^3-9\,\mathrm{Root}[K[9]^{15}+10K[9]^{12}\#1+40K[9]^9\#1^2+80K[9]^6\#1^3+\left(80K[9]^3-\frac{80K[9]^9}{x^6}-\frac{x^6K[9]^9}{y^{12}}-\frac{10x^3K[9]^9}{y^9}-\frac{40K[9]^9}{y^6}-\frac{80K[9]^9}{x^3y^3}-\frac{32y^3K[9]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}{-2K[9]^4+K[9]\,\mathrm{Root}[K[9]^{15}+10K[9]^{12}\#1+40K[9]^9\#1^2+80K[9]^6\#1^3+\left(80K[9]^3-\frac{80K[9]^9}{x^6}-\frac{x^6K[9]^9}{y^{12}}-\frac{10x^3K[9]^9}{y^9}-\frac{40K[9]^9}{y^6}-\frac{80K[9]^9}{x^3y^3}-\frac{32y^3K[9]^9}{x^9}\right)\#1^4+32\#1^5\&,3]}\,dK[9]}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[10]^3-9\,\mathrm{Root}[K[10]^{15}+10K[10]^{12}\#1+40K[10]^9\#1^2+80K[10]^6\#1^3+\left(80K[10]^3-\frac{80K[10]^9}{x^6}-\frac{x^6K[10]^9}{y^{12}}-\frac{10x^3K[10]^9}{y^9}-\frac{40K[10]^9}{y^6}-\frac{80K[10]^9}{x^3y^3}-\frac{32y^3K[10]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}{-2K[10]^4+K[10]\,\mathrm{Root}[K[10]^{15}+10K[10]^{12}\#1+40K[10]^9\#1^2+80K[10]^6\#1^3+\left(80K[10]^3-\frac{80K[10]^9}{x^6}-\frac{x^6K[10]^9}{y^{12}}-\frac{10x^3K[10]^9}{y^9}-\frac{40K[10]^9}{y^6}-\frac{80K[10]^9}{x^3y^3}-\frac{32y^3K[10]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}\,dK[10}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[11]^3-9\,\mathrm{Root}[K[11]^{15}+10K[11]^{12}\#1+40K[11]^9\#1^2+80K[11]^6\#1^3+\left(80K[11]^3-\frac{80K[11]^9}{x^6}-\frac{x^6K[11]^9}{y^{12}}-\frac{10x^3K[11]^9}{y^9}-\frac{40K[11]^9}{y^6}-\frac{80K[11]^9}{x^3y^3}-\frac{32y^3K[11]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}{-2K[11]^4+K[11]\,\mathrm{Root}[K[11]^{15}+10K[11]^{12}\#1+40K[11]^9\#1^2+80K[11]^6\#1^3+\left(80K[11]^3-\frac{80K[11]^9}{x^6}-\frac{x^6K[11]^9}{y^{12}}-\frac{10x^3K[11]^9}{y^9}-\frac{40K[11]^9}{y^6}-\frac{80K[11]^9}{x^3y^3}-\frac{32y^3K[11]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}\,dK[11}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[12]^3-9\,\mathrm{Root}[K[12]^{15}+10K[12]^{12}\#1+40K[12]^9\#1^2+80K[12]^6\#1^3+\left(80K[12]^3-\frac{80K[12]^9}{x^6}-\frac{x^6K[12]^9}{y^{12}}-\frac{10x^3K[12]^9}{y^9}-\frac{40K[12]^9}{y^6}-\frac{80K[12]^9}{x^3y^3}-\frac{32y^3K[12]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}{-2K[12]^4+K[12]\,\mathrm{Root}[K[12]^{15}+10K[12]^{12}\#1+40K[12]^9\#1^2+80K[12]^6\#1^3+\left(80K[12]^3-\frac{80K[12]^9}{x^6}-\frac{x^6K[12]^9}{y^{12}}-\frac{10x^3K[12]^9}{y^9}-\frac{40K[12]^9}{y^6}-\frac{80K[12]^9}{x^3y^3}-\frac{32y^3K[12]^9}{x^9}\right)\#1^4+32\#1^5\&,4]}\,dK[12}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[13]^3-9\,\mathrm{Root}[K[13]^{15}+10K[13]^{12}\#1+40K[13]^9\#1^2+80K[13]^6\#1^3+\left(80K[13]^3-\frac{80K[13]^9}{x^6}-\frac{x^6K[13]^9}{y^{12}}-\frac{10x^3K[13]^9}{y^9}-\frac{40K[13]^9}{y^6}-\frac{80K[13]^9}{x^3y^3}-\frac{32y^3K[13]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}{-2K[13]^4+K[13]\,\mathrm{Root}[K[13]^{15}+10K[13]^{12}\#1+40K[13]^9\#1^2+80K[13]^6\#1^3+\left(80K[13]^3-\frac{80K[13]^9}{x^6}-\frac{x^6K[13]^9}{y^{12}}-\frac{10x^3K[13]^9}{y^9}-\frac{40K[13]^9}{y^6}-\frac{80K[13]^9}{x^3y^3}-\frac{32y^3K[13]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}\,dK[13}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[14]^3-9\,\mathrm{Root}[K[14]^{15}+10K[14]^{12}\#1+40K[14]^9\#1^2+80K[14]^6\#1^3+\left(80K[14]^3-\frac{80K[14]^9}{x^6}-\frac{x^6K[14]^9}{y^{12}}-\frac{10x^3K[14]^9}{y^9}-\frac{40K[14]^9}{y^6}-\frac{80K[14]^9}{x^3y^3}-\frac{32y^3K[14]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}{-2K[14]^4+K[14]\,\mathrm{Root}[K[14]^{15}+10K[14]^{12}\#1+40K[14]^9\#1^2+80K[14]^6\#1^3+\left(80K[14]^3-\frac{80K[14]^9}{x^6}-\frac{x^6K[14]^9}{y^{12}}-\frac{10x^3K[14]^9}{y^9}-\frac{40K[14]^9}{y^6}-\frac{80K[14]^9}{x^3y^3}-\frac{32y^3K[14]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}\,dK[14}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\},\ \Big\{u[x,y]\to$$

$$e^{\int_1^x \frac{9K[15]^3-9\,\mathrm{Root}[K[15]^{15}+10K[15]^{12}\#1+40K[15]^9\#1^2+80K[15]^6\#1^3+\left(80K[15]^3-\frac{80K[15]^9}{x^6}-\frac{x^6K[15]^9}{y^{12}}-\frac{10x^3K[15]^9}{y^9}-\frac{40K[15]^9}{y^6}-\frac{80K[15]^9}{x^3y^3}-\frac{32y^3K[15]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}{-2K[15]^4+K[15]\,\mathrm{Root}[K[15]^{15}+10K[15]^{12}\#1+40K[15]^9\#1^2+80K[15]^6\#1^3+\left(80K[15]^3-\frac{80K[15]^9}{x^6}-\frac{x^6K[15]^9}{y^{12}}-\frac{10x^3K[15]^9}{y^9}-\frac{40K[15]^9}{y^6}-\frac{80K[15]^9}{x^3y^3}-\frac{32y^3K[15]^9}{x^9}\right)\#1^4+32\#1^5\&,5]}\,dK[15}$$

$$c_1\left[\mathrm{Log}\left[-\frac{\left(x^3+2y^3\right)^{5/6}}{x^{3/2}y^2}\right]\right]\Big\}\Big\}$$

Out[•]=

$$\left\{\left\{u[x,y]\to e^{-1/x}\,c_1\left[\frac{-x-y}{x\,y}\right]\right\}\right\}$$

第 4 章 线性代数

T_1

(1) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$ (2) $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1-a & 1 & 1 \\ 1 & 1 & 1+b & 1 \\ 1 & 1 & 1 & 1-b \end{vmatrix}$

(3) $\begin{vmatrix} a & x & \cdots & x \\ y & a & \ddots & \vdots \\ \vdots & \ddots & \ddots & x \\ y & \cdots & y & a \end{vmatrix}_{n\times n}$ (4) $\begin{vmatrix} 1^{n-2} & 2^{n-2} & \cdots & n^{n-2} \\ 2^{n-2} & 3^{n-2} & \cdots & (n+1)^{n-2} \\ \vdots & \vdots & \iddots & \vdots \\ n^{n-2} & (n+1)^{n-2} & \cdots & (2n-1)^{n-2} \end{vmatrix}_{n\times n}$

In[∘]:=
```
Clear["Global`*"]
A = Table[If[i + j ≤ 5, i + j - 1, i + j - 5], {i, 4}, {j, 4}];
Det[A]

B = ConstantArray[1, {4, 4}] + DiagonalMatrix[{a, -a, b, -b}];
Det[B]

n = 5;
C1 = UpperTriangularize[ConstantArray[x, {n, n}], 1] + LowerTriangularize[
   ConstantArray[y, {n, n}], -1] + DiagonalMatrix[Table[a, {i, n}]];
Det[C1]

n = 5;
D1 = Table[(i + j - 1) ^ (n - 2), {i, n}, {j, n}];
Det[D1]
```

Out[∘]= 160

Out[∘]= $a^2 b^2$

Out[∘]= $a^5 - 10 a^3 x y + 10 a^2 x^2 y - 5 a x^3 y + x^4 y + 10 a^2 x y^2 - 5 a x^2 y^2 + x^3 y^2 - 5 a x y^3 + x^2 y^3 + x y^4$

Out[∘]= 0

T_2

2. 计算多项式 $p(x)=\begin{vmatrix} 2x & x & 1 & 2 \\ 1 & x & -2 & -1 \\ 3 & 2 & x & -1 \\ 1 & 1 & 0 & x \end{vmatrix}$。

```
Clear["Global`*"]
p[x] = Det[(2 x  x  1  2
            1  x -2 -1
            3  2  x -1
            1  1  0  x)]
```

Out[∘]= $4+5x-2x^2-x^3+2x^4$

T3

3. 设 $A=\begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 3 \end{pmatrix}$, $B=\begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & -2 \\ 0 & 3 & 1 \end{pmatrix}$, $C=\begin{pmatrix} 0 & 2 \\ -1 & 0 \\ 3 & 1 \end{pmatrix}$, 计算 AB, AC, CA, B^2。

In[∘]:=
```
Clear["Global`*"]
A = (1 0 -1
     0 2  3);
B = (2 -1  4
     1  0 -2
     0  3  1);
C1 = (0  2
      -1 0
      3  1);
MatrixForm /@ {A.B, A.C1, C1.A, B.B}
```

Out[∘]= $\left\{\begin{pmatrix} 2 & -4 & 3 \\ 2 & 9 & -1 \end{pmatrix}, \begin{pmatrix} -3 & 1 \\ 7 & 3 \end{pmatrix}, \begin{pmatrix} 0 & 4 & 6 \\ -1 & 0 & 1 \\ 3 & 2 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 10 & 14 \\ 2 & -7 & 2 \\ 3 & 3 & -5 \end{pmatrix}\right\}$

T4

4. 判断下列向量组是否线性相关。

(1) $a_1=\begin{pmatrix}1\\-2\\1\end{pmatrix}, a_2=\begin{pmatrix}0\\3\\-1\end{pmatrix}, a_3=\begin{pmatrix}2\\-1\\3\end{pmatrix}$;

(2) $a_1=\begin{pmatrix}-2\\1\\1\end{pmatrix}, a_2=\begin{pmatrix}1\\-1\\1\end{pmatrix}, a_3=\begin{pmatrix}-5\\3\\1\end{pmatrix}$。

```
Det[{{1, 0, 2}, {-2, 3, -1}, {1, -1, 3}}] (*线性无关*)

Det[{{-2, 1, -5}, {1, -1, 3}, {1, 1, 1}}] (*线性相关*)
```

Out[•]= 6

Out[•]= 0

T5

5. 计算向量 $\beta=(1,1,1,1)$ 在基 $\alpha_1=(1,-1,1,-1)$, $\alpha_2=(0,1,-1,1)$, $\alpha_3=(001,-1)$, $\alpha_4=(0,0,0,1)$ 下的坐标。

```
Clear["Global`*"]
Solve[{{1, 0, 0, 0}, {-1, 1, 0, 0}, {1, -1, 1, 0}, {-1, 1, -1, 1}}.{{a}, {b}, {c}, {d}} == {{1}, {1}, {1}, {1}}]
(*β=α1+2α2+2α3+2α4*)
```

Out[•]= {{a → 1, b → 2, c → 2, d → 2}}

T6

6. 在由不超过 3 次的实系数多项式组成的实线性空间 $P_4[x]$ 中，求从基 $\alpha_1=1, \alpha_2=x, \alpha_3=x^2, \alpha_4=x^3$ 到基 $\beta_1=1, \beta_2=x-\lambda, \beta_3=(x-\lambda)^2, \beta_4=(x-\lambda)^3$ 的坐标变换矩阵。

In[2]:=

```
Inverse[{{1, -λ, λ^2, -λ^3}, {0, 1, -2λ, 3λ^2}, {0, 0, 1, -3λ}, {0, 0, 0, 1}}] // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & \lambda & \lambda^2 & \lambda^3 \\ 0 & 1 & 2\lambda & 3\lambda^2 \\ 0 & 0 & 1 & 3\lambda \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T7

7. 求实线性空间 R^4 中从基 $\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}$ 到基 $\{\beta_1,\beta_2,\beta_3,\beta_4\}$ 的坐标变换矩阵，其中

$$\alpha_1=\begin{pmatrix}1\\0\\0\\0\end{pmatrix},\alpha_2=\begin{pmatrix}0\\1\\0\\0\end{pmatrix},\alpha_3=\begin{pmatrix}0\\0\\1\\0\end{pmatrix},\alpha_4=\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\beta_1=\begin{pmatrix}1\\1\\1\\1\end{pmatrix},\beta_2=\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix},\beta_3=\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix},\beta_4=\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}$$

记A = $(\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4)$, $B=(\beta_1,\ \beta_2,\ \beta_3,\ \beta_4)$, 设向量x在两组基下的坐标分别为$\alpha$, β则x = $A\alpha = B\beta \Longrightarrow \beta = B^{-1}A\alpha$, 坐标变换矩阵T = $B^{-1}A$

In[∘]:=

```
Inverse[{{1, 1, 1, 1}, {1, 1, -1, -1}, {1, -1, 1, -1}, {1, -1, -1, 1}}].IdentityMatrix[4] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

T8

8. 计算下列矩阵的秩：

(1) $\begin{pmatrix} 2 & 1 & -1 & 1 & 1 \\ 3 & -2 & 1 & -3 & 4 \\ 1 & 4 & -3 & 5 & -2 \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 1 & -3 & -4 & 1 \\ 3 & -1 & 1 & 4 & 3 \\ 1 & 5 & -9 & -8 & 1 \end{pmatrix}$

In[ ]:=
```
MatrixRank[{{2, 1, -1, 1, 1}, {3, -2, 1, -3, 4}, {1, 4, -3, 5, -2}}]

MatrixRank[{{1, 1, -3, -4, 1}, {3, -1, 1, 4, 3}, {1, 5, -9, -8, 1}}]
```

Out[ ]= 2

Out[ ]= 3

T9

9. 计算下列矩阵的逆矩阵：

(1) $\begin{pmatrix} 3 & 3 & -4 & -3 \\ 0 & 6 & 1 & 1 \\ 5 & 4 & 2 & 1 \\ 2 & 3 & 3 & 2 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 5 & 7 & 1 \\ 6 & 3 & 4 & 0 \\ 5 & -2 & -3 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}$

(3) $\begin{pmatrix} x & 1 & 0 & 0 \\ 1 & x & \ddots & 0 \\ 0 & \ddots & \ddots & 1 \\ 0 & 0 & 1 & x \end{pmatrix}_{n\times n}$ (4) $\begin{pmatrix} 1 & 2 & \cdots & n \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 2 \\ 0 & \cdots & 0 & 1 \end{pmatrix}_{n\times n}$

In[ ]:=

```
Clear["Global`*"]
```

$$\text{Inverse}\left[\begin{pmatrix} 3 & 3 & -4 & -3 \\ 0 & 6 & 1 & 1 \\ 5 & 4 & 2 & 1 \\ 2 & 3 & 3 & 2 \end{pmatrix}\right] \text{ // MatrixForm}$$

$$\text{Inverse}\left[\begin{pmatrix} 2 & 5 & 7 & 1 \\ 6 & 3 & 4 & 0 \\ 5 & -2 & -3 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix}\right] \text{ // MatrixForm}$$

```
n = 5;
A = SparseArray[{Band[{1, 1}] → x, Band[{1, 2}] → 1, Band[{2, 1}] → 1}, {n, n}];
Inverse[A] // MatrixForm

n = 5;
B = Sum[SparseArray[{Band[{1, i}] → i}, {n, n}], {i, 1, n}];
Inverse[B] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -7 & 5 & 12 & -19 \\ 3 & -2 & -5 & 8 \\ 41 & -30 & -69 & 111 \\ -59 & 43 & 99 & -159 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{15} & \frac{3}{20} & \frac{1}{20} & -\frac{1}{60} \\ \frac{2}{5} & -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ -\frac{1}{5} & \frac{13}{40} & -\frac{9}{40} & -\frac{17}{40} \\ \frac{8}{15} & -\frac{23}{40} & \frac{19}{40} & \frac{1}{120} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{1-3x^2+x^4}{3x-4x^3+x^5} & \frac{2x-x^3}{3x-4x^3+x^5} & \frac{-1+x^2}{3x-4x^3+x^5} & -\frac{x}{3x-4x^3+x^5} & \frac{1}{3x-4x^3+x^5} \\ \frac{2x-x^3}{3x-4x^3+x^5} & \frac{-2x^2+x^4}{3x-4x^3+x^5} & \frac{x-x^3}{3x-4x^3+x^5} & \frac{x^2}{3x-4x^3+x^5} & -\frac{x}{3x-4x^3+x^5} \\ \frac{-1+x^2}{3x-4x^3+x^5} & \frac{x-x^3}{3x-4x^3+x^5} & \frac{1-2x^2+x^4}{3x-4x^3+x^5} & \frac{x-x^3}{3x-4x^3+x^5} & \frac{-1+x^2}{3x-4x^3+x^5} \\ -\frac{x}{3x-4x^3+x^5} & \frac{x^2}{3x-4x^3+x^5} & \frac{x-x^3}{3x-4x^3+x^5} & \frac{-2x^2+x^4}{3x-4x^3+x^5} & \frac{2x-x^3}{3x-4x^3+x^5} \\ \frac{1}{3x-4x^3+x^5} & -\frac{x}{3x-4x^3+x^5} & \frac{-1+x^2}{3x-4x^3+x^5} & \frac{2x-x^3}{3x-4x^3+x^5} & \frac{1-3x^2+x^4}{3x-4x^3+x^5} \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

T_{10}

10. 求解下列线性方程组：

$$(1)\begin{pmatrix}1 & -2 & 1 & 1\\ 1 & -2 & 1 & -1\\ 1 & -2 & 1 & 5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}=\begin{pmatrix}1\\ -1\\ 5\end{pmatrix}$$

$$(2)\begin{pmatrix}1 & -2 & -1 & -2\\ 4 & 1 & 2 & 1\\ 2 & 5 & 4 & -1\\ 1 & 1 & 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix}=\begin{pmatrix}2\\ 3\\ 0\\ \frac{1}{3}\end{pmatrix}$$

In[∘]:=
```
Solve[{{1, -2, 1, 1}, {1, -2, 1, -1}, {1, -2, 1, 5}} . {x1, x2, x3, x4} == {1, -1, 5}]

Solve[{{1, -2, -1, -2}, {4, 1, 2, 1}, {2, 5, 4, -1}, {1, 1, 1, 1}} . {x1, x2, x3, x4} == {2, 3, 0, 1/3}]
```

Out[∘]= $\{\{x3 \to -x1 + 2\,x2,\ x4 \to 1\}\}$

Out[∘]= $\left\{\left\{x2 \to -\frac{13}{6} + 2\,x1,\ x3 \to \frac{8}{3} - 3\,x1,\ x4 \to -\frac{1}{6}\right\}\right\}$

T11

11. 设 R^2 上的线性变换 $\mathscr{A}$ 在基 $\alpha_1=(1,-1),\alpha_2=(1,1)$ 下的矩阵是 $\begin{pmatrix}2 & 3\\ 0 & 1\end{pmatrix}$，

求 $\mathscr{A}$ 在基 $\beta_1=(2,0),\beta_2=(-1,1)$ 下的矩阵。

$\mathscr{A}(\alpha_1{}^T,\ \alpha_2{}^T)=(\alpha_1{}^T,\ \alpha_2{}^T)A$

$\mathscr{A}(\beta_1{}^T,\ \beta_2{}^T)=(\beta_1{}^T,\ \beta_2{}^T)B$

设基 $\{\alpha_1,\ \alpha_2\}$ 到基 $\{\beta_1,\ \beta_2\}$ 的过渡矩阵为T，即 $(\beta_1{}^T,\ \beta_2{}^T)=(\alpha_1{}^T,\ \alpha_2{}^T)T$

$\mathscr{A}(\beta_1{}^T,\ \beta_2{}^T)=\mathscr{A}(\alpha_1{}^T,\ \alpha_2{}^T)T=(\beta_1{}^T,\ \beta_2{}^T)B \Longrightarrow (\beta_1{}^T,\ \beta_2{}^T)B\,T^{-1}=(\alpha_1{}^T,\ \alpha_2{}^T)A$

$\Longrightarrow B=(\beta_1^T,\ \beta_2^T)^{-1}(\alpha_1^T,\ \alpha_2^T)A(\alpha_1^T,\ \alpha_2^T)^{-1}(\beta_1{}^T,\ \beta_2{}^T)$

记 $\alpha=(\alpha_1{}^T,\ \alpha_2{}^T)$, $\beta=(\beta_1{}^T,\ \beta_2{}^T)$

In[∘]:=
```
Clear["Global`*"]
α = (1 1; -1 1); β = (2 -1; 0 1);
Inverse[β].α.(2 3; 0 1).Inverse[α].β // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ -4 & 2 \end{pmatrix}$$

T12

12. 计算下列复方阵的全部特征值和特征向量：

(1) $\begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$ (2) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$

In[∘]:=
```
Eigensystem[(0 a; -a 0)]

Eigensystem[(0 0 1; 0 1 0; 1 0 0)]

Eigensystem[(1 1 1 1; 1 1 -1 -1; 1 -1 1 -1; 1 1 -1 1)]
```

Out[∘]= {{i a, -i a}, {{-i, 1}, {i, 1}}}

Out[∘]= {{-1, 1, 1}, {{-1, 0, 1}, {1, 0, 1}, {0, 1, 0}}}

Out[∘]= $\left\{\left\{2, 2, -\sqrt{2}, \sqrt{2}\right\}, \left\{\{1, 0, 0, 1\}, \{1, 0, 1, 0\}, \left\{-\frac{3+2\sqrt{2}}{1+\sqrt{2}}, 1+\sqrt{2}, -\frac{-3-2\sqrt{2}}{1+\sqrt{2}}, 1\right\}, \left\{-\frac{-3+2\sqrt{2}}{-1+\sqrt{2}}, 1-\sqrt{2}, -\frac{3-2\sqrt{2}}{-1+\sqrt{2}}, 1\right\}\right\}\right\}$

T13

13. 判断下列实二次型是否定正：

(1) $Q(x_1, x_2, x_3) = 2x_1x_2 + 2x_1x_3 + 2x_2x_3$；

(2) $Q(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + 6x_3^2 + 2x_1x_2 + 2x_1x_3 + 6x_2x_3$。

In[ ]:=
```
Eigenvalues[{{0, 1, 1}, {1, 0, 1}, {1, 1, 0}}] (*非正定*)

Eigenvalues[{{1, 1, 1}, {1, 2, 3}, {1, 3, 6}}] (*正定*)
```

Out[ ]= $\{2, -1, -1\}$

Out[ ]= $\{4+\sqrt{15}, 1, 4-\sqrt{15}\}$

T14

14. 计算下列矩阵 A 的 Jordan 标准形 J，并写出过渡矩阵 P 使 $A = PJP^{-1}$。

(1) $\begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{pmatrix}$ (2) $\begin{pmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$

In[ ]:=

```
{P, J} = JordanDecomposition[{{2, -1, 1}, {2, 2, -1}, {1, 2, -1}}];
MatrixForm /@ {P, J}
P.J.Inverse[P] // MatrixForm(*examination*)

{P, J} = JordanDecomposition[{{3, -4, 0, 2}, {4, -5, -2, 4}, {0, 0, 3, -2}, {0, 0, 2, -1}}];
MatrixForm /@ {P, J}
P.J.Inverse[P] // MatrixForm(*examination*)
```

Out[ ]=

$$\left\{\begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{9} \\ 1 & \frac{1}{3} & -\frac{1}{9} \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\right\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 1 \\ 2 & 2 & -1 \\ 1 & 2 & -1 \end{pmatrix}$$

Out[ ]=

$$\left\{\begin{pmatrix} 1 & \frac{1}{4} & 1 & \frac{1}{2} \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right\}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$$

第 5 章 数值计算方法

T_1

1. 对以下插值点，作出拉格朗日(Lagrange)插值多项式。

(1) $(-1,3),(0,0.5),(0.5,0),(1,1)$，计算 $f(0.25),f(0.75)$。

(2) $(-1,1.5),(0,0),(0.5,0),(1,0.5)$，计算 $f(-0.25),f(0.25)$。

```
Clear["Global`*"]
清除

data1 = {{-1, 3}, {0, 0.5}, {0.5, 0}, {1, 1}};
f1[x_] := InterpolatingPolynomial[data1, x]
          插值多项式
{f1[0.25], f1[0.75]}

data2 = {{-1, 1.5}, {0, 0}, {0.5, 0}, {1, 0.5}};
f2[x_] := InterpolatingPolynomial[data1, x]
          插值多项式
{f2[-0.25], f2[0.25]}
```

Out[•]= {0.109375, 0.265625}

Out[•]= {1.07813, 0.109375}

T_2

2. 给出离散数据：

x_i	-1.00	-0.50	0	0.25	0.75	1.00
y_i	0.22	0.80	2.0	2.5	3.8	4.2

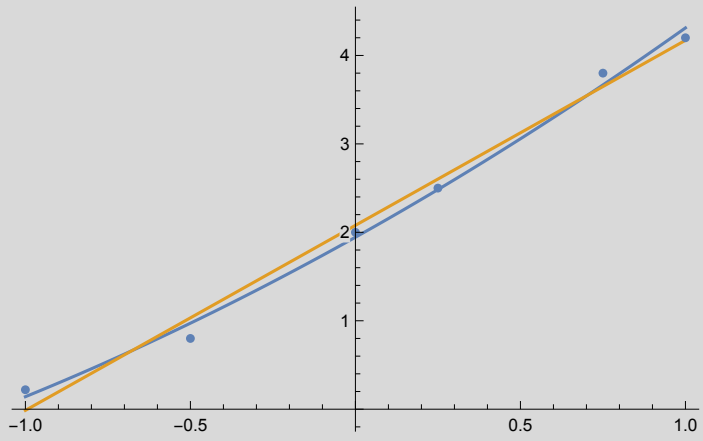
试对以上数据分别作出线性、二次曲线拟合。

```
Clear["Global`*"]
清除

data = {{-1, 0.22}, {-0.5, 0.8}, {0, 2}, {0.25, 2.5}, {0.75, 3.8}, {1, 4.2}};
(*线性拟合*)
Fit[data, {1, x}, x]
拟合
(*二次曲线拟合*)
Fit[data, {1, x, x^2}, x]
拟合

Show[Plot[{%, %%}, {x, -1, 1}], ListPlot[data]]
显示  绘图                          绘制点集
```

Out[•]= $2.07897 + 2.09235\,x$

Out[•]= $1.94449 + 2.0851\,x + 0.28191\,x^2$

Out[•]=



T3

3. 给出离散数据：

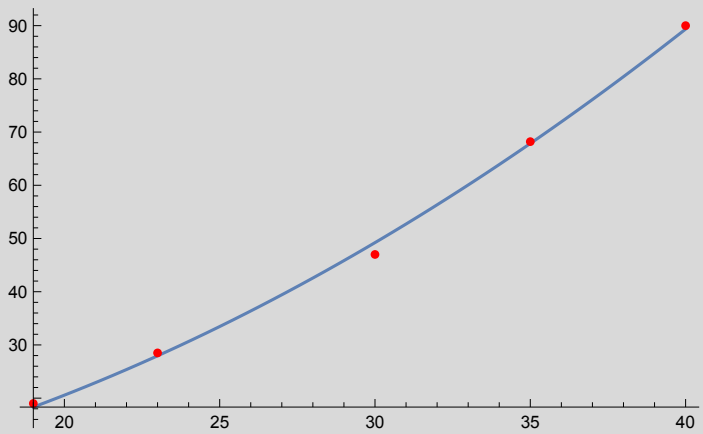
x_i	19	23	30	35	40
y_i	19.00	28.50	47.00	68.20	90.00

试对以上数据作出形如 $a + bx^2$ 的拟合曲线。

In[•]:=

```
Clear["Global`*"]
清除

data = {{19, 19}, {23, 28.5}, {30, 47}, {35, 68.2}, {40, 90}};
model = a + b * x^2;
sol = FindFit[data, model, {a, b}, x];
      求拟合
f[x_] := model /. sol
fig1 = Plot[f[x], {x, 19, 40}];
       绘图
fig2 = ListPlot[data, PlotStyle → Red];
       绘制点集          绘图样式      红色
Show[fig1, fig2]
显示
```

Out[•]=



T4

4. 给出离散数据：

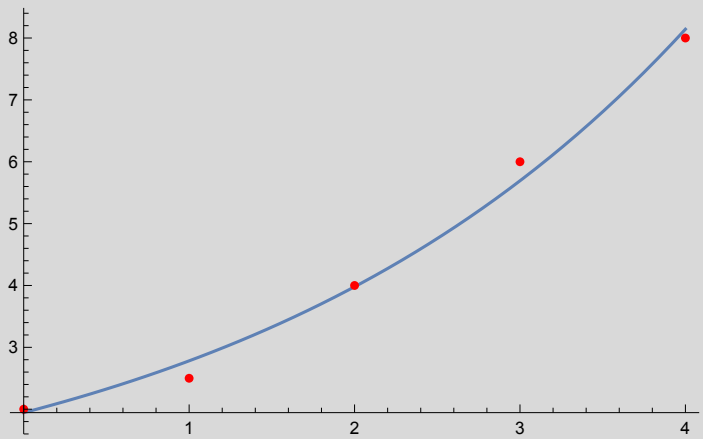
x_i	0	1	2	3	4
y_i	2.00	2.50	4.00	6.00	8.00

试对以上数据作出形如 $a\mathrm{e}^{bx}$ 的拟合曲线。

In[•]:=

```
Clear["Global`*"]
清除

data = {{0, 2}, {1, 2.5}, {2, 4}, {3, 6}, {4, 8}};
model = a * Exp[b * x];
            指数形式
sol = FindFit[data, model, {a, b}, x];
      求拟合
f[x_] := model /. sol
fig1 = Plot[f[x], {x, 0, 4}];
       绘图
fig2 = ListPlot[data, PlotStyle → Red];
       绘制点集          绘图样式      红色
Show[fig1, fig2]
显示
```

Out[•]=



T5

5. 证明数值积分公式

$$\int_{-1}^{1} f(x)\,\mathrm{d}x \approx \frac{5}{9} f\left(2-\sqrt{\frac{3}{5}}\right)+\frac{8}{9} f(2)+\frac{5}{9} f\left(2-\sqrt{\frac{3}{5}}\right)$$

具有 5 阶代数精度。

```
Clear["Global`*"]
清除

f[x_] := Insert[Table[x^k, {k, 1, 6}], 1, 1];
         插入    表格
Integrate[f[x], {x, -1, 1}] -
积分
  (5/9 * f[-Sqrt[3/5]] + 8/9 * f[0] + 5/9 * f[Sqrt[3/5]]) (*原题数值积分公式错误*)
            平方根                                 平方根
```

Out[•]= $\left\{0, 0, 0, 0, 0, 0, \frac{8}{175}\right\}$

T6

6. (1) 找出 $y=\sin x\cos x$ 在 $x=0.5$ 邻近的极小解；

(2) 找出 $z=\sin xye^{x^2}$ 在 $\{0.2,0.3\}$ 邻近的极小解。

In[•]:=
```
FindMinimum[Sin[x] Cos[x], {x, 0.5}]
求极小值和其坐标 正弦    余弦

FindMinimum[Sin[x y E^(x^2)] , {{x, 0.2}, {y, 0.3}}]
求极小值和其坐标 正弦    自然常数
```

Out[•]= {-0.5, {x → -0.785398}}

Out[•]= {-1., {x → -1.35561, y → 0.184453}}

T7

7. 用 FindRoot 解下列三角函数方程：

(1) $\cos 3x+2\cos x=0$

(2) $\sin^4 x-\cos^4 x=\cos x+\sin x$

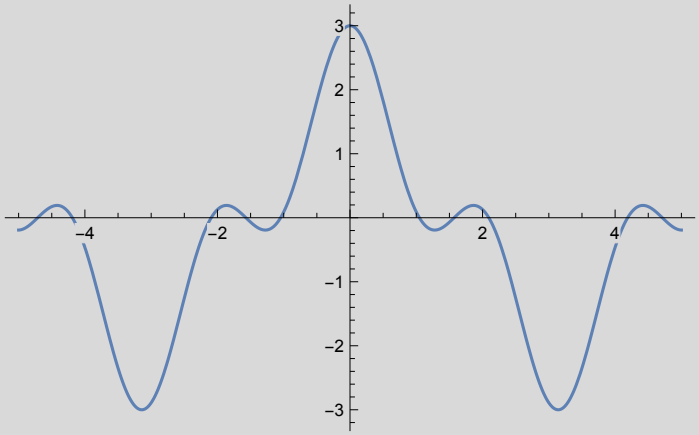
(3) $\sin\left(x+\frac{\pi}{4}\right)\sin\left(x-\frac{\pi}{12}\right)=\frac{1}{2}$

(4) $6\sin^2 x+3\sin x\cos x-5\cos^2 x=2$

In[•]:=

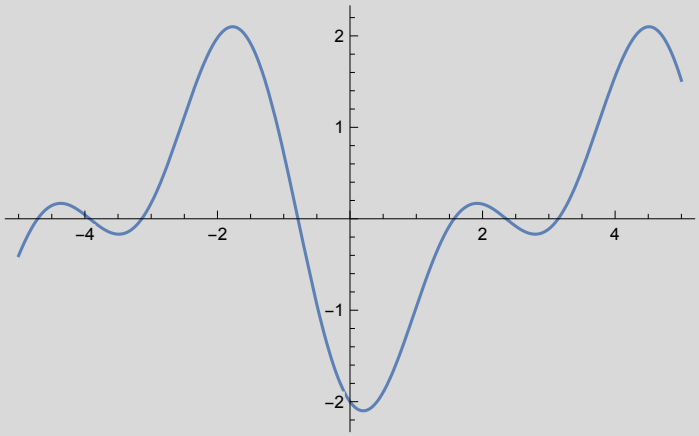
```
(*每个三角函数方程都有无穷多解,不一一列举*)

Plot[Cos[3 x] + 2 Cos[x], {x, -5, 5}]
绘图 余弦 余弦
FindRoot[Cos[3 x] + 2 Cos[x] == 0, {x, 0.1}]
求根 余弦 余弦

Plot[Sin[x]^4 - Cos[x]^4 - Cos[x] - Sin[x], {x, -5, 5}]
绘图 正弦 余弦 余弦 正弦
FindRoot[Sin[x]^4 - Cos[x]^4 == Cos[x] - Sin[x], {x, 0}]
求根 正弦 余弦 余弦 正弦

Plot[{Sin[x + π / 4] Sin[x - π / 12], 1 / 2}, {x, -5, 5}]
绘图 正弦 正弦
FindRoot[Sin[x + π / 4] Sin[x - π / 12] == 1 / 2, {x, 0}]
求根 正弦 正弦

Plot[{6 Sin[x]^2 + 3 Sin[x] Cos[x] - 5 Cos[x]^2, 2}, {x, -5, 5}]
绘图 正弦 正弦 余弦 余弦
FindRoot[6 Sin[x]^2 + 3 Sin[x] Cos[x] - 5 Cos[x]^2 == 2, {x, 0}]
求根 正弦 正弦 余弦 余弦
```

Out[•]=

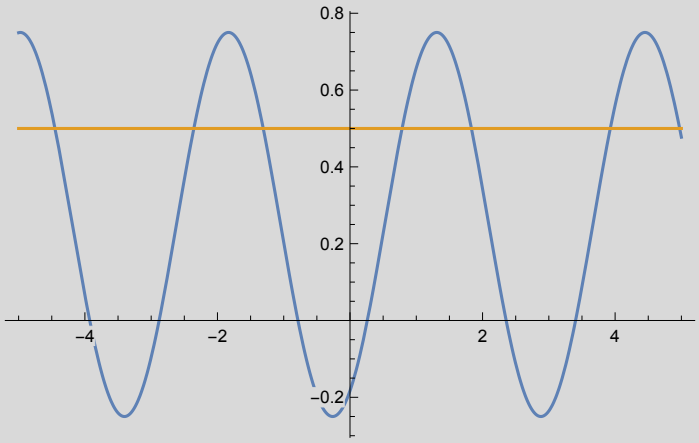


Out[•]= {x → 2.0944}

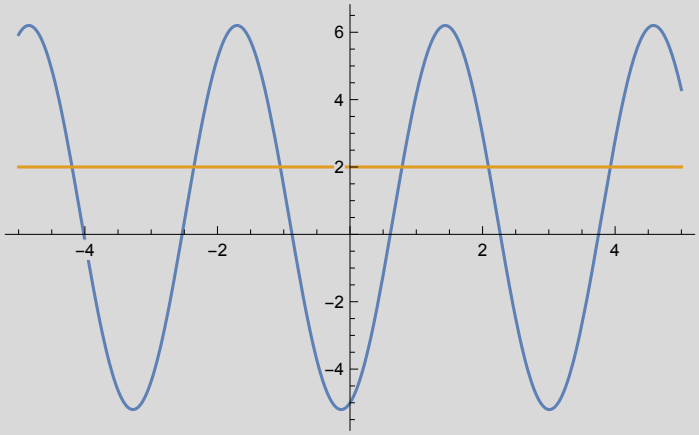
Out[•]=



Out[•]=
```
{x → 3.92699}
```

Out[•]=


Out[•]=
```
{x → 1.8326}
```

Out[•]=


Out[•]=
```
{x → 2.08994}
```

T8

8. 解下列方程组 $AX = b$。

$$(1)\ A=\begin{pmatrix} 3.0 & -2.0 & 5.3 & -2.1 & 1.0 \\ 1.0 & 4.0 & -6.0 & 4.5 & -6.0 \\ 3.0 & 6.0 & -7.3 & -9.0 & 3.4 \\ -2.0 & -3.0 & 1.0 & -4.0 & 6.0 \\ 1.0 & -4.0 & 6.5 & 1.0 & -3.0 \end{pmatrix},\ b=\begin{pmatrix} 28.3 \\ -36.2 \\ 24.5 \\ 16.2 \\ 4.3 \end{pmatrix}$$

$$(2)\ \begin{pmatrix} 2 & -1 & 4 & -3 & 1 \\ -1 & 1 & 2 & 1 & 3 \\ 4 & 2 & 3 & 3 & -1 \\ -3 & 1 & 3 & 2 & 4 \\ 1 & 3 & 1 & 4 & 4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}=\begin{pmatrix} 11 \\ 14 \\ 4 \\ 16 \\ 18 \end{pmatrix}$$

In[•]:=
```
Clear["Global`*"]
清除

A1 = {{3, -2, 5.3, -2.1, 1}, {1, 4, -6, 4.5, -6},
   {3, 6, -7.3, -9, 3.4}, {-2, -3, 1, -4, 6}, {1, -4, 6.5, 1, -3}};
B1 = {28.3, -36.2, 24.5, 16.2, 4.3};
LinearSolve[A1, B1]
线性求解

A2 = {{2, -1, 4, -3, 1}, {-1, 1, 2, 1, 3},
   {4, 2, 3, 3, -1}, {-3, 1, 3, 2, 4}, {1, 3, 1, 4, 4}};
B2 = {11, 14, 4, 16, 18};
LinearSolve[A2, B2]
线性求解
```

Out[•]=
```
{2.06093, 3.24831, 4.03635, -2.01121, 2.9976}
```

Out[•]= $\left\{\frac{1}{27}, \frac{14}{3}, \frac{13}{9}, -\frac{62}{27}, \frac{79}{27}\right\}$

T9

9. 方程组

$$\begin{cases} 10x_1 - x_2 = 1 \\ -x_1 + 10x_2 - x_3 = 0 \\ -x_2 + 10x_3 - x_4 = 1 \\ -x_3 + 10x_4 = 2 \end{cases}$$

写出 Jacobi 迭代计算式，并对 $x^{(0)}=(0,0,0)^{\mathrm{T}}$ 迭代求出 $x^{(1)}, x^{(2)}, x^{(3)}$。

In[•]:=

```
A = SparseArray[{Band[{1, 1}] → 10, Band[{1, 2}] → -1, Band[{2, 1}] → -1}, 4];
    稀疏数组        带                  带                  带
b = {{1}, {0}, {1}, {2}};
x = {{0, 0, 0, 0}};
T = Input["请输入迭代次数"]; m = Dimensions[A][[1]];(*矩阵维度*)
    输入                      维数
M1 = Table[If[i ≠ j, -A[[i, j]]/A[[i, i]], 0], {i, m}, {j, m}];(*构造迭代矩阵M1*)
     表格   如果
g1 = Table[b[[i]]/A[[i, i]], {i, m}];(*构造迭代余项g1*)
     表格
Print["Jacobi迭代矩阵M1: ",
打印
 M1 // MatrixForm, "\t迭代余项g1: ", g1 // MatrixForm]
       矩阵格式                          矩阵格式
For[i = 1, i ≤ T, i++, AppendTo[x, N[M1.x[[i]] + (Transpose[g1])[[1]], 6]]]
For循环                 附加        数值运算          转置
Print["迭代结果Xn+1=M1.Xn+g1: \n"]
打印
Print@TableForm[Delete[x, 1],
打印    表格形式    删除
  TableHeadings -> {Automatic, {"x1", "x2", "x3", "x4"}}]
  表格标头            自动
```

Jacobi迭代矩阵M1: $\begin{pmatrix} 0 & \frac{1}{10} & 0 & 0 \\ \frac{1}{10} & 0 & \frac{1}{10} & 0 \\ 0 & \frac{1}{10} & 0 & \frac{1}{10} \\ 0 & 0 & \frac{1}{10} & 0 \end{pmatrix}$ 迭代余项g1: $\begin{pmatrix} \frac{1}{10} \\ 0 \\ \frac{1}{10} \\ \frac{1}{5} \end{pmatrix}$

迭代结果Xn+1=M1.Xn+g1:

	x1	x2	x3	x4
1	0.100000	0	0.100000	0.200000
2	0.100000	0.0200000	0.120000	0.210000
3	0.102000	0.0220000	0.123000	0.212000

T_{10}

10. 计算方程 $x^3+x^2-5x+3=0$ 的实根数、正实根数，$[a,b]=[0,5]$上实根数。

In[•]:=
```
Clear["Global`*"]
清除

f = x^3 + x^2 - 5 x + 3;
CountRoots[f, x]
计算根数
CountRoots[f, {x, 0, Infinity}]
计算根数                无穷大
CountRoots[f, {x, 0, 5}]
计算根数
```

Out[•]= 3

Out[•]= 2

Out[•]= 2

T11

11. 求解下列线性规划问题。

(1) $\max\ z = 3x_1 + 2x_2 \quad \text{s.t.} \begin{cases} -x_1 + 2x_2 \leqslant 4 \\ 3x_1 + 2x_2 \leqslant 14 \\ x_1 - x_2 \leqslant 3 \\ x_1, x_2 \geqslant 0 \end{cases}$

(2) $\min\ m = 2x + 3y + 4z \quad \text{s.t.} \begin{cases} x + 2y - z > 10 \\ x + y - z \geqslant 60 \\ y + 2z > 12 \\ x > 0, y > 0, z > 1 \end{cases}$

```
Clear["Global`*"]
清除

c1 = {-3, -2};
A1 = {{1, -2}, {-3, -2}, {-1, 1}, {1, 0}, {0, 1}};
b1 = {-4, -14, -3, 0, 0};
{x1 = LinearProgramming[c1, A1, b1], -c1.x1}
      线性规划

c2 = {2, 3, 4};
A2 = {{1, 2, -1}, {1, 1, -1}, {0, 1, 2}, {1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
b2 = {10, 60, 12, 0, 0, 1.000001};
{x2 = LinearProgramming[c2, A2, b2], c2.x2}(*不满足z>1*)
      线性规划
```

Out[•]= {{4, 1}, 14}

Out[•]= {{51., 10., 1.}, 136.}

T12

12. 求解下列微分方程并画出解函数。

$$(1)\begin{cases} y''(x)+y(x)=\cos x \\ y(0)=0 \\ y'(0)=0 \end{cases} \qquad (x\in[0,20])$$

(2) $$\begin{cases} \dfrac{\mathrm{d}u}{\mathrm{d}t} = 0.09u\left(1-\dfrac{u}{20}\right) - 0.45uv \\ \dfrac{\mathrm{d}v}{\mathrm{d}t} = 0.06v\left(1-\dfrac{v}{15}\right) - 0.001uv \\ u(0) = 1.6 \\ v(0) = 1.2 \end{cases}$$

(3) 热方程

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2}$$

$$u(0,x) = 0, \quad u(t,0) = \sin t, \quad u(t,5) = 0$$

$$t \in [0,10], \quad x \in [0,5]$$

(4) 波动方程

$$\frac{\partial^2 u(t,x)}{\partial t^2} = \frac{\partial^2 u(t,x)}{\partial x^2}$$

$$u(0,x) = \mathrm{e}^{-x^2}, \quad u(t,-10) = u(t,10), \quad \left.\frac{\partial u(t,x)}{\partial t}\right|$$

$$t \in [0,40], \quad x \in [-10,10]$$

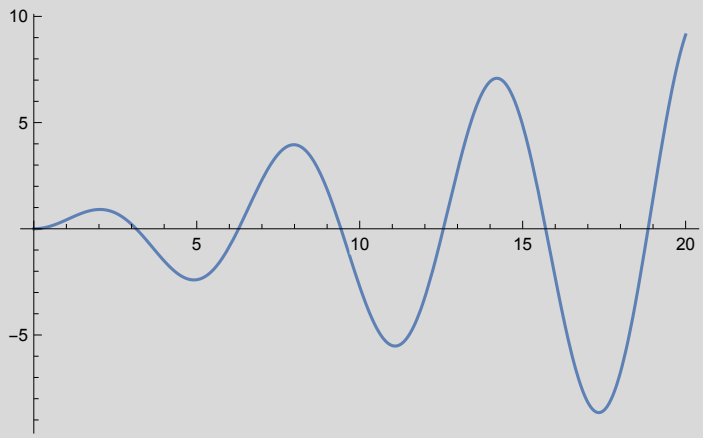
In[•]:=

```
Clear["Global`*"]
清除

sol1 = NDSolve[{y''[x] + y[x] == Cos[x], y[0] == 0, y'[0] == 0}, y[x], {x, 0, 20}]
       数值求解微分方程组                 余弦
Plot[y[x] /. sol1[[1]], {x, 0, 20}]
绘图

tmax = 50;
sol2 = NDSolve[{u'[t] == 0.09 u[t] (1 - u[t] / 20) - 0.45 u[t] × v[t],
       数值求解微分方程组
    v'[t] == 0.06 v[t] (1 - v[t] / 15) - 0.001 u[t] × v[t],
    u[0] == 1.6, v[0] == 1.2}, {u[t], v[t]}, {t, 0, tmax}]
Plot[{u[t], v[t]} /. sol2[[1]], {t, 0, tmax}]
绘图

sol3 = NDSolve[{D[u[t, x], t] == D[u[t, x], {x, 2}], u[0, x] == 0,
       数值求…    偏导              偏导
    u[t, 0] == Sin[t], u[t, 5] == 0}, u[t, x], {t, 0, 10}, {x, 0, 5}]
               正弦
Plot3D[u[t, x] /. sol3[[1]], {t, 0, 10}, {x, 0, 5}, PlotRange → All]
绘制三维图形                                          绘制范围      全部

sol4 = NDSolve[{D[u[t, x], {t, 2}] == D[u[t, x], {x, 2}],
       数值求…    偏导                 偏导
    u[0, x] == Exp[-x^2], u[t, -10] == u[t, 10], D[u[t, x], t] == 0 /. t → 0},
               指数形式                              偏导
   u[t, x], {t, 0, 40}, {x, -10, 10}]
Plot3D[u[t, x] /. sol4[[1]], {t, 0, 40}, {x, -10, 10}]
绘制三维图形
```

Out[•]=

```
{{y[x] → InterpolatingFunction[ Domain: {{0., 20.}} Output: scalar ][x]}}
```

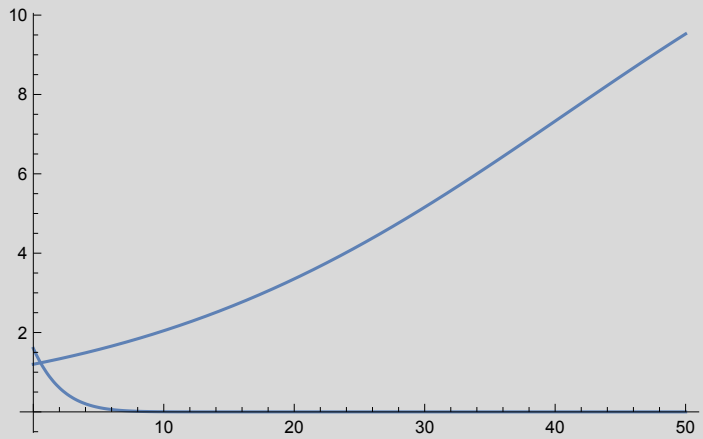
Out[•]=



Out[•]=

```
{{u[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t],
  v[t] → InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar ][t]}}
```

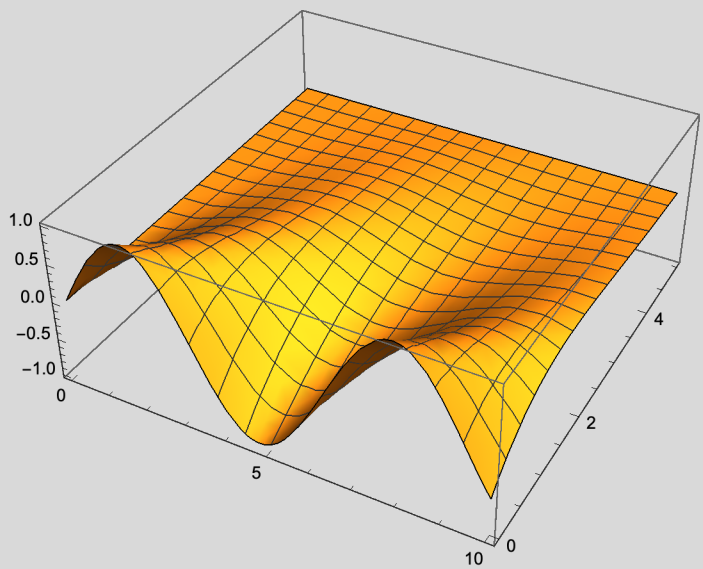
Out[•]=



Out[•]=

```
{{u[t, x] → InterpolatingFunction[ Domain: {{0., 10.}, {0., 5.}} Output: scalar ][t, x]}}
```

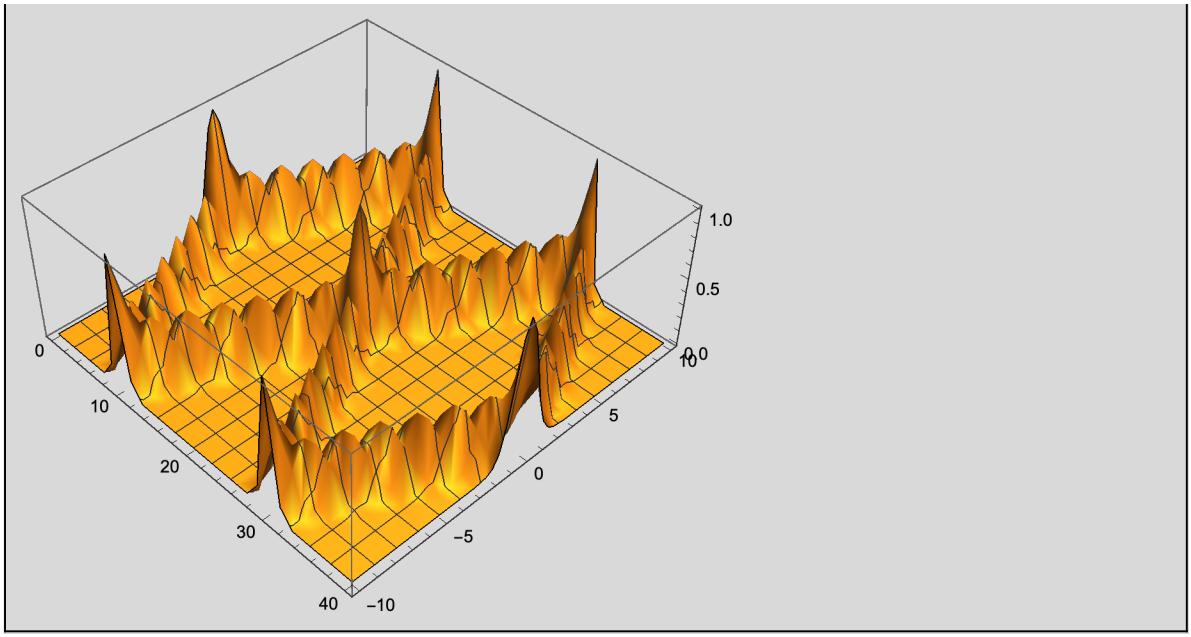
Out[•]=



Out[•]=

```
{{u[t, x] → InterpolatingFunction[ Domain: {{0., 40.}, {−10., 10.}} Output: scalar ][t, x]}}
```

Out[ ]=



第 6 章　在Mathematica中作图

T_1

1. 作出下列函数的图形(每个图形至少含有两个选项,用于设置曲线颜色、坐标标记和点数等)。

(1) $y=1+x+x^3$，$x\in[-100,100]$。

(2) $y=(x-1)(x-2)^2$，$x\in[-70,70]$。

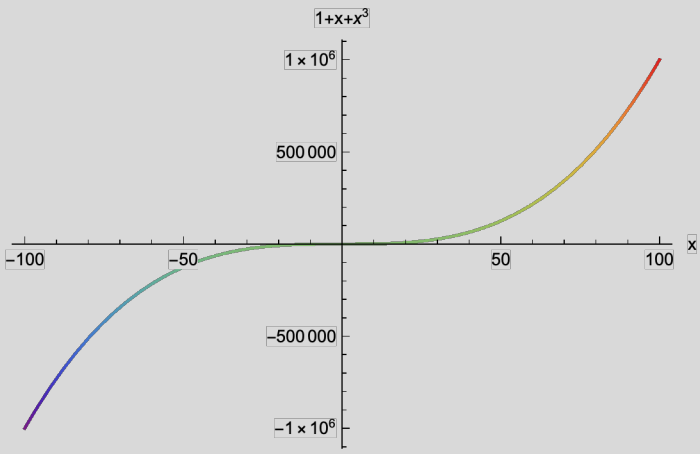
(3) $y=x+\sin x$，$x\in[-10,10]$。

(4) $y=x^2\sin x^2$，$x\in[-60,60]$。

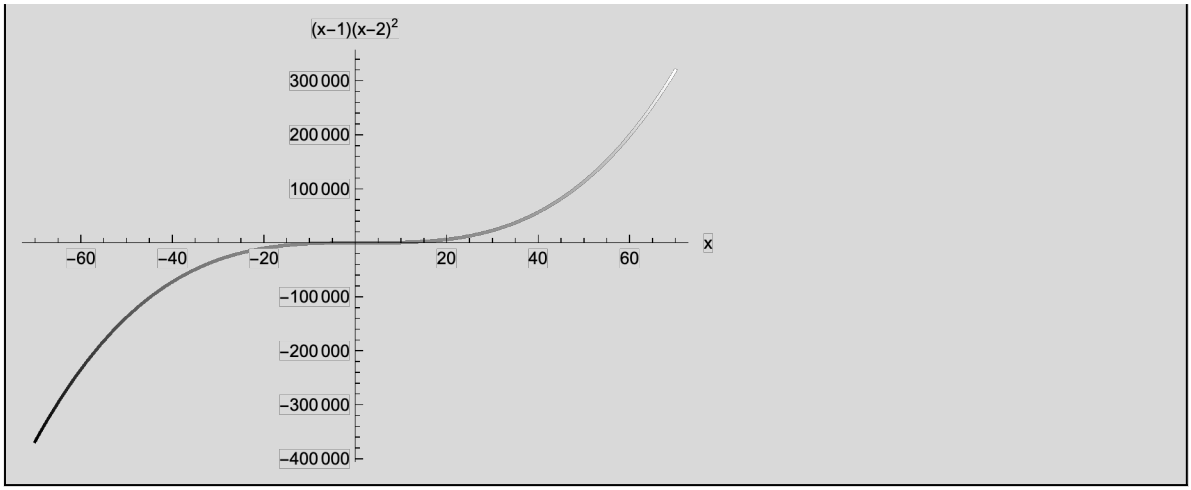
In[ ]:=

```
Plot[1 + x + x^3, {x, -100, 100},
绘图
 AxesLabel → {"x", "1+x+x^3"}, ColorFunction → "Rainbow"]
 坐标轴标签                      颜色函数

Plot[(x - 1) (x - 2)^2, {x, -70, 70},
绘图
 AxesLabel → {"x", "(x-1)(x-2)^2"}, ColorFunction -> GrayLevel]
 坐标轴标签                          颜色函数          灰度级

Plot[x + Sin[x], {x, -10, 10},
绘图     正弦
 AxesLabel → {"x", "x+Sin(x)"}, ColorFunction -> "TemperatureMap"]
 坐标轴标签              正弦       颜色函数

Plot[x^2 * Sin[x^2], {x, -60, 60}, AxesLabel → {"x", "x^2Sin(x^2)"},
绘图        正弦                    坐标轴标签              正弦
 ColorFunction -> "DarkRainbow", PlotPoints → 2000]
                                  绘图点
```

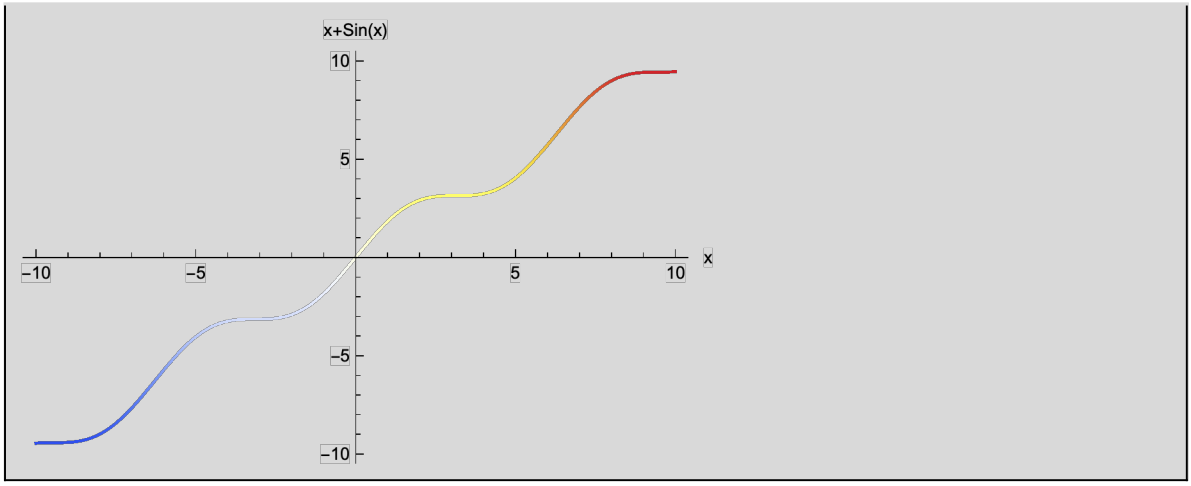
Out[ ]=



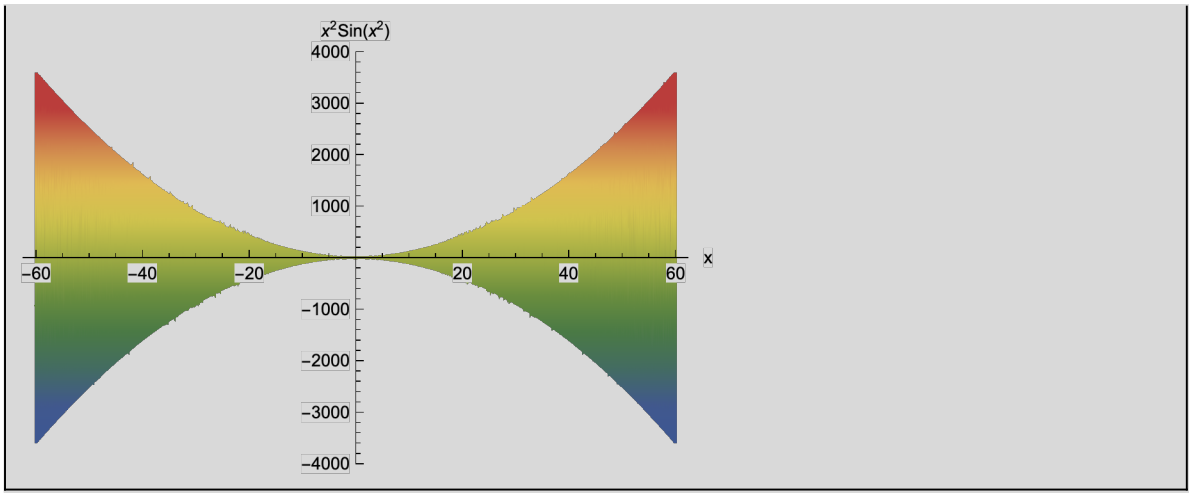
Out[ ]=



Out[ ]=



Out[ ]=



T_2

2. 同时作出 $y(x)$ 和 $y'(x)$ 的图形：

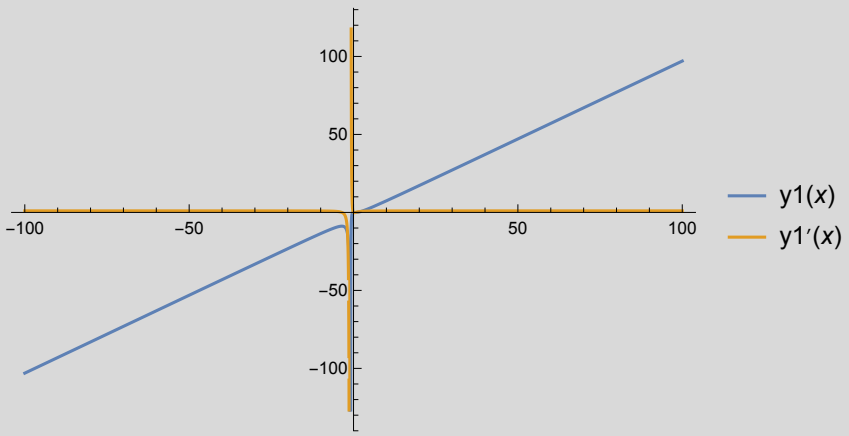
(1) $y(x)=\dfrac{x^2(x-1)}{(x+1)^2}$，$x\in[-100,100]$。

(2) $y(x)=\dfrac{\sin x}{1+x^2}$，$x\in[-90,90]$。

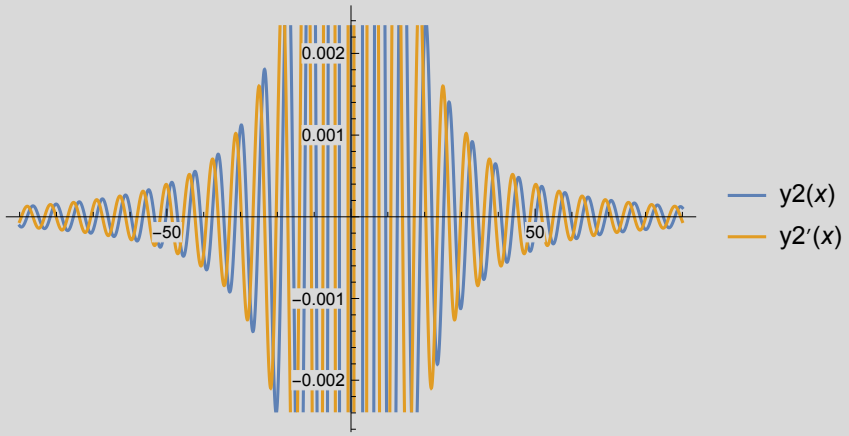
In[∘]:=

```
Clear["Global`*"]
清除
y1[x_] := x^2 (x - 1)/(x + 1)^2
Plot[{y1[x], y1'[x]}, {x, -100, 100}, PlotLegends → "Expressions"]
绘图                                  绘图的图例

y2[x_] := Sin[x]/(1 + x^2)
Plot[{y2[x], y2'[x]}, {x, -90, 90}, PlotLegends → "Expressions"]
绘图                                  绘图的图例
```

Out[∘]=



Out[∘]=



T_3

3. 画出下列参数方程所表示的曲线：

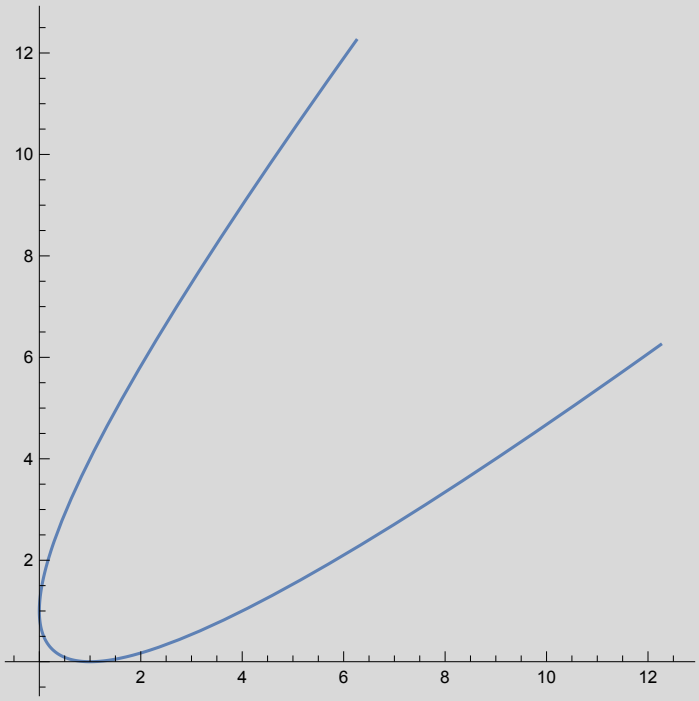
(1) $x=\frac{(t+1)^2}{4}, y=\frac{(t-1)^2}{4}$, $t\in[-6,6]$。

(2) $x=a\cos 2t, y=a\cos 3t$, $t\in[-\pi,\pi]$。

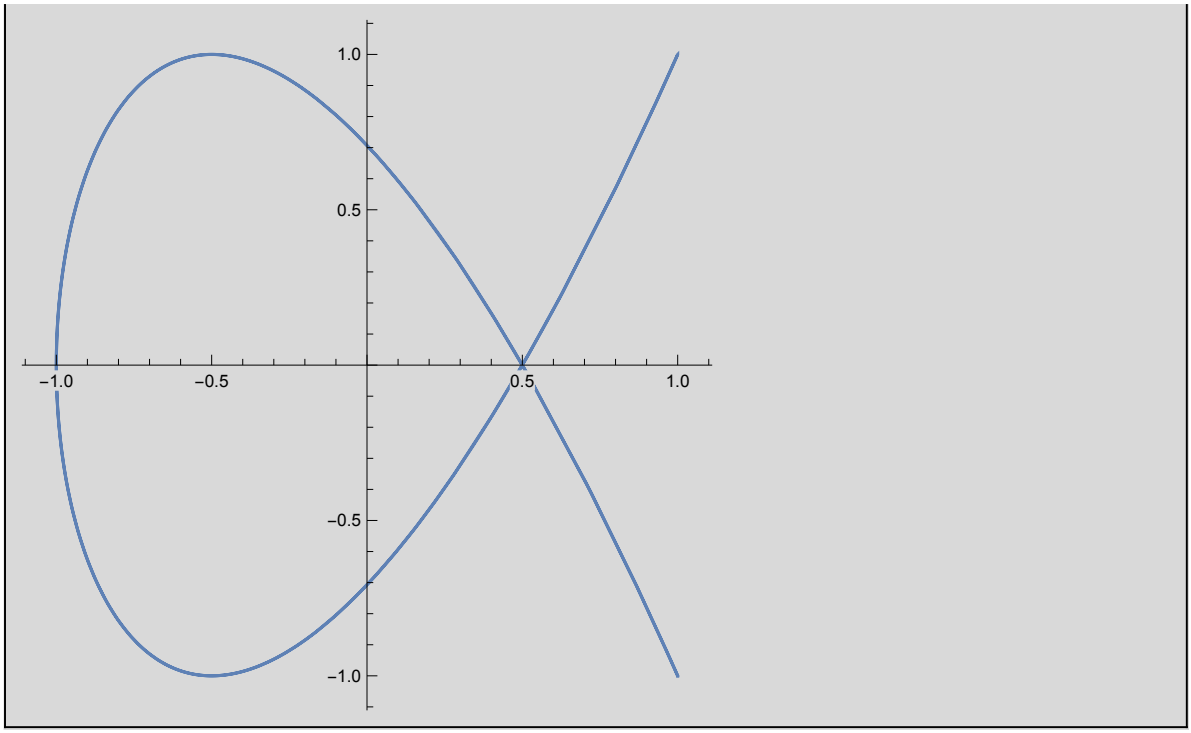
(3) $x=t\ln t, y=\frac{\ln t}{t}$, $t\in[0,6\pi]$。

In[ ]:=
```
Clear["Global`*"]
清除
ParametricPlot[{(t + 1) ^ 2/4, (t - 1) ^ 2/4}, {t, -6, 6}]
绘制参数图

a = 1;
ParametricPlot[{a Cos[2 t], a Cos[3 t]}, {t, -π, π}]
绘制参数图          余弦        余弦

ParametricPlot[{t Log[t], Log[t] / t}, {t, 0, 6 π}, AspectRatio → 1 / 2]
绘制参数图          对数      对数                    宽高比
```

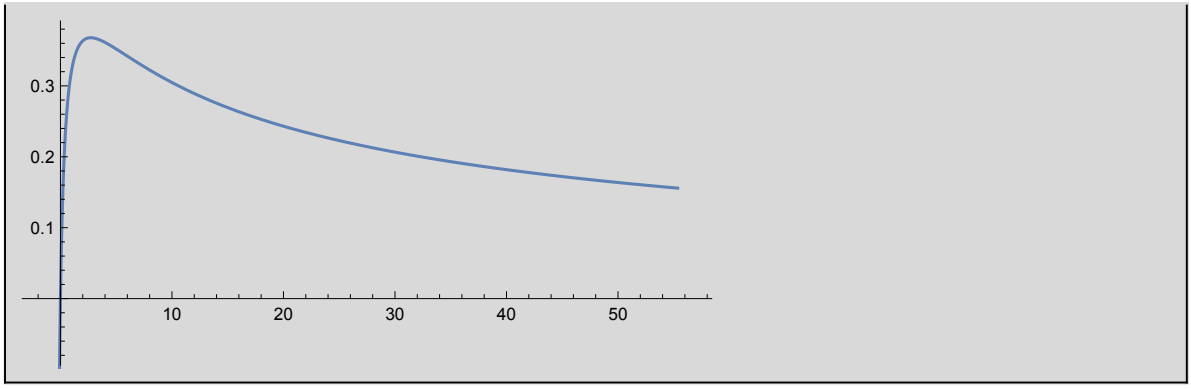
Out[ ]=



Out[•]=



Out[•]=



T_4

4. 画出下列函数的图形，并从不同的方向观察曲面：

(1) $z=e^{-x^2-y^2}$，$-3\leqslant x\leqslant 3$，$-3\leqslant y\leqslant 3$。

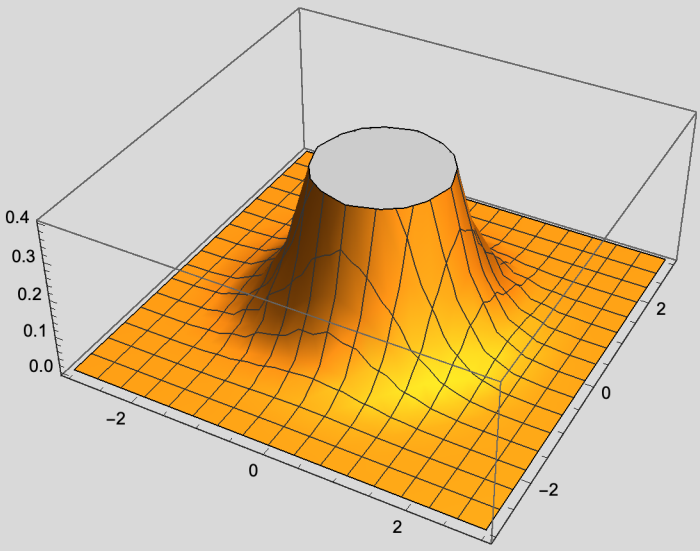
(2) $z=\sin(x+\cos y)$，$-6\leqslant x\leqslant 6$，$-6\leqslant y\leqslant 6$。

(3) $z=\dfrac{x^2-y^2}{x^3+y^3}$，$-10\leqslant x\leqslant 10$，$-10\leqslant y\leqslant 10$。

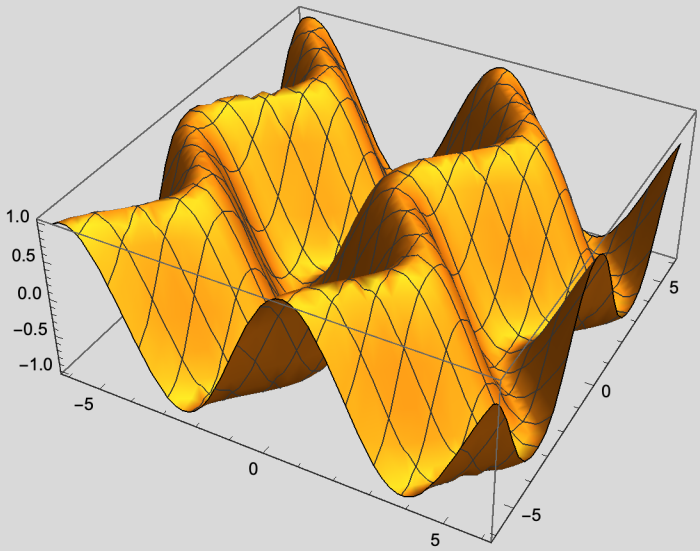
In[•]:=

```
Plot3D[Exp[-x^2 - y^2], {x, -3, 3}, {y, -3, 3}]
绘制…  指数形式

Plot3D[Sin[x + Cos[y]], {x, -6, 6}, {y, -6, 6}]
绘制…  正弦  余弦

Plot3D[(x^2 - y^2)/(x^3 + y^3), {x, -10, 10}, {y, -10, 10}]
绘制三维图形
```

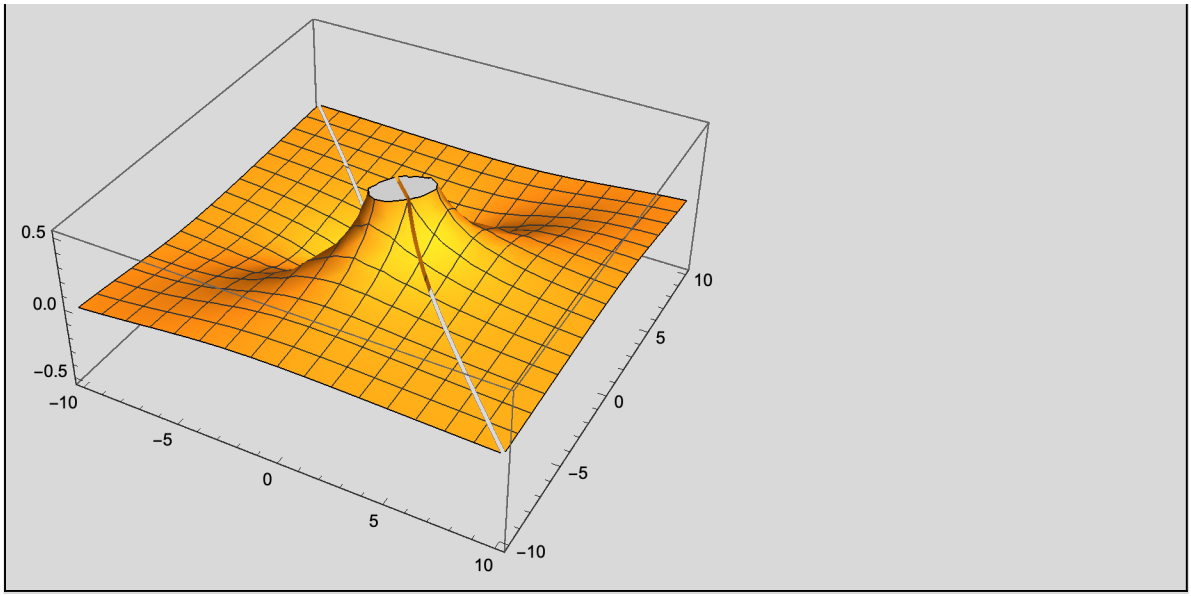
Out[•]=



Out[•]=



Out[ ]=



T_5

5. 画出下列带有限定条件的函数图形：

(1) $f(x,y)=\dfrac{x}{e^{x^2+y^2}}$，$-2\leqslant x\leqslant 2$，$-2\leqslant y\leqslant 2$，限定区域 $2<x^2+y^2<3$。

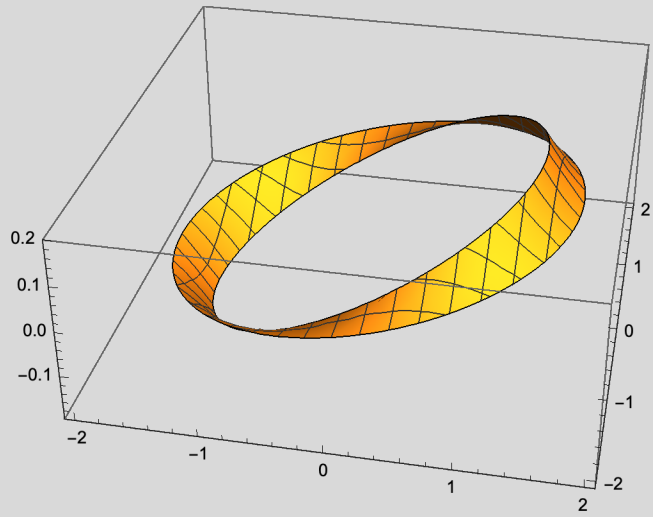
(2) $f(x,y)=\dfrac{1}{y^2-x^3+3x-3}$，$-3\leqslant x\leqslant 3$，$-3\leqslant y\leqslant 3$，限定条件 $0<\mathrm{Mod}(x^2+y^2,2)<1$。

In[•]:=

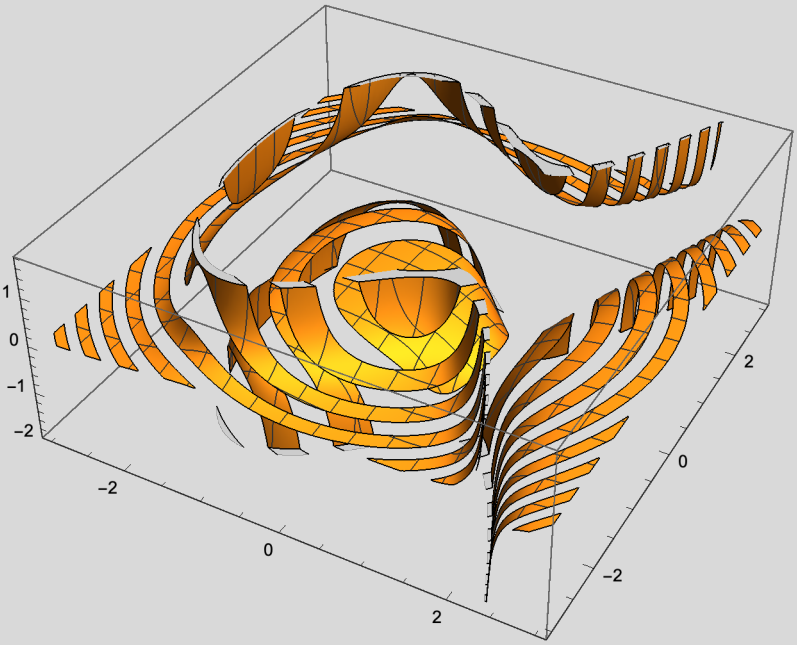
```
Plot3D[x/Exp[x^2 + y^2], {x, -2, 2}, {y, -2, 2},
 RegionFunction → Function[{x, y}, 2 < x^2 + y^2 < 3]]

Plot3D[1/(y^2 - x^3 + 3 x - 3), {x, -3, 3}, {y, -3, 3},
 RegionFunction → Function[{x, y}, 0 < Mod[x^2 + y^2, 2] < 1], PlotPoints → 200]
```

绘制三维图形 区域函数 纯函数 绘制三维图形 区域函数 纯函数 模余 绘图点

Out[•]=



Out[•]=



6. 作出下列参数方程所表示的曲线或曲面：

(1) $x=\sin t, y=\cos t, z=t/3$，$t\in[0,15]$。

(2) $x=u\sin t, y=u\cos t, z=t/3$，$t\in[0,15]$，$u\in[-1,1]$。

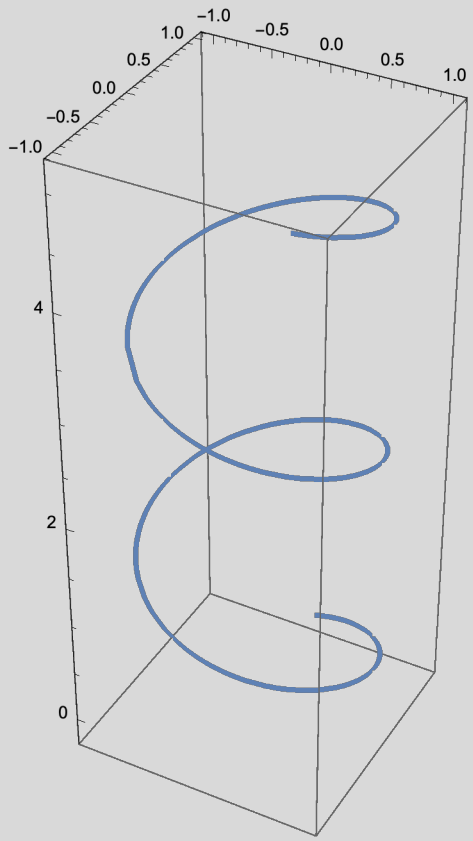
(3) 画出半径为 1 的上半球面。

(4) 画出半径为 2 的左半球面。

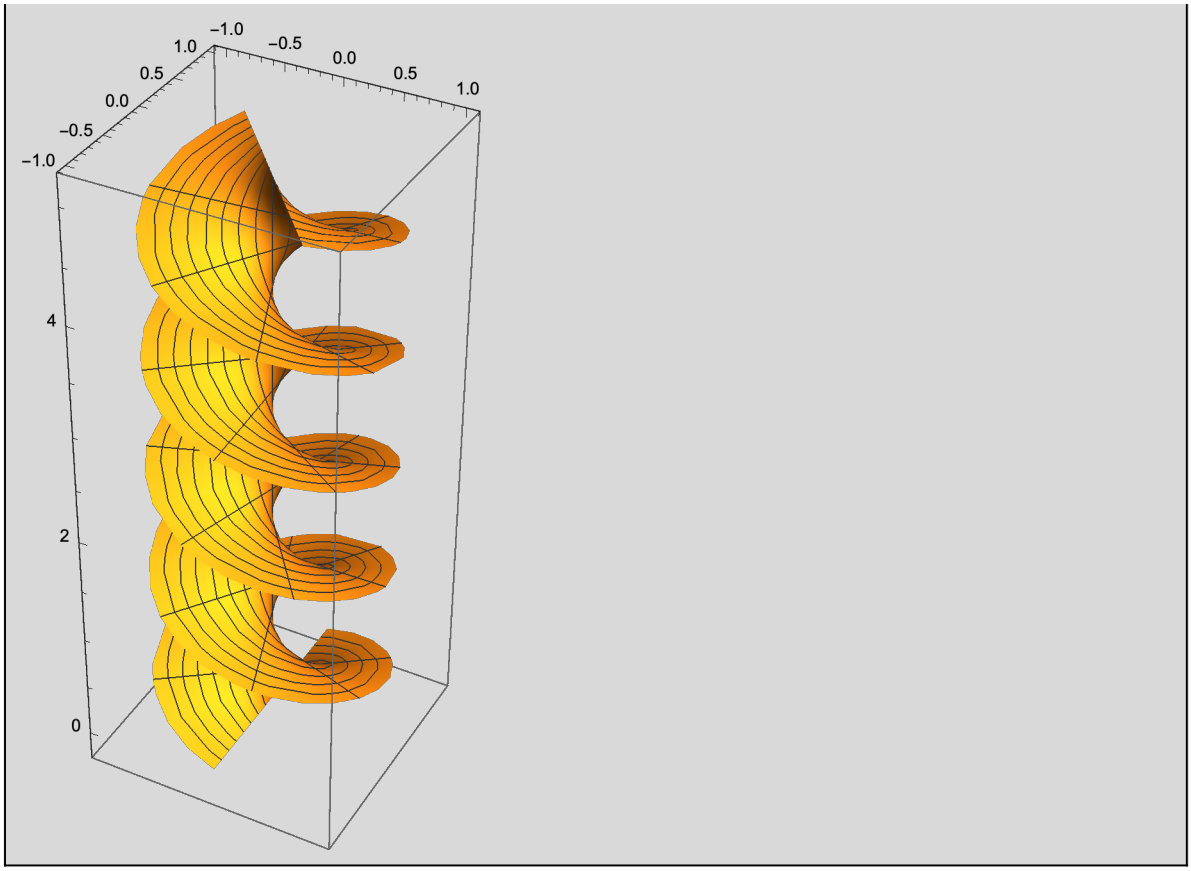
In[ ]:=

```
ParametricPlot3D[{Sin[t], Cos[t], t/3}, {t, 0, 15}]
绘制三维参数图          正弦     余弦

ParametricPlot3D[{u Sin[t], u Cos[t], t/3}, {t, 0, 15}, {u, -1, 1}]
绘制三维参数图          正弦      余弦

ParametricPlot3D[
绘制三维参数图
 {Sin[θ] Cos[φ], Sin[θ] Sin[φ], Cos[θ]}, {θ, 0, π/2}, {φ, 0, 2 π}]
  正弦    余弦     正弦    正弦     余弦

ParametricPlot3D[2 {Sin[θ] Cos[φ], Sin[θ] Sin[φ], Cos[θ]},
绘制三维参数图          正弦    余弦     正弦    正弦     余弦
 {θ, 0, π}, {φ, π/2, 3 π/2}]
```

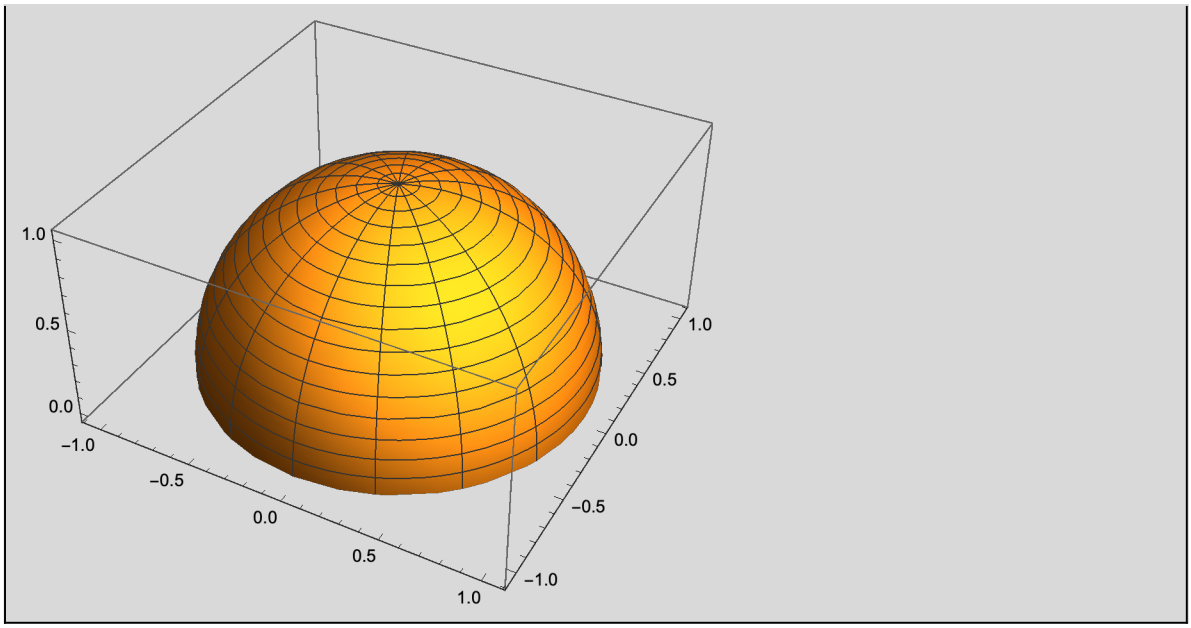
Out[ ]=



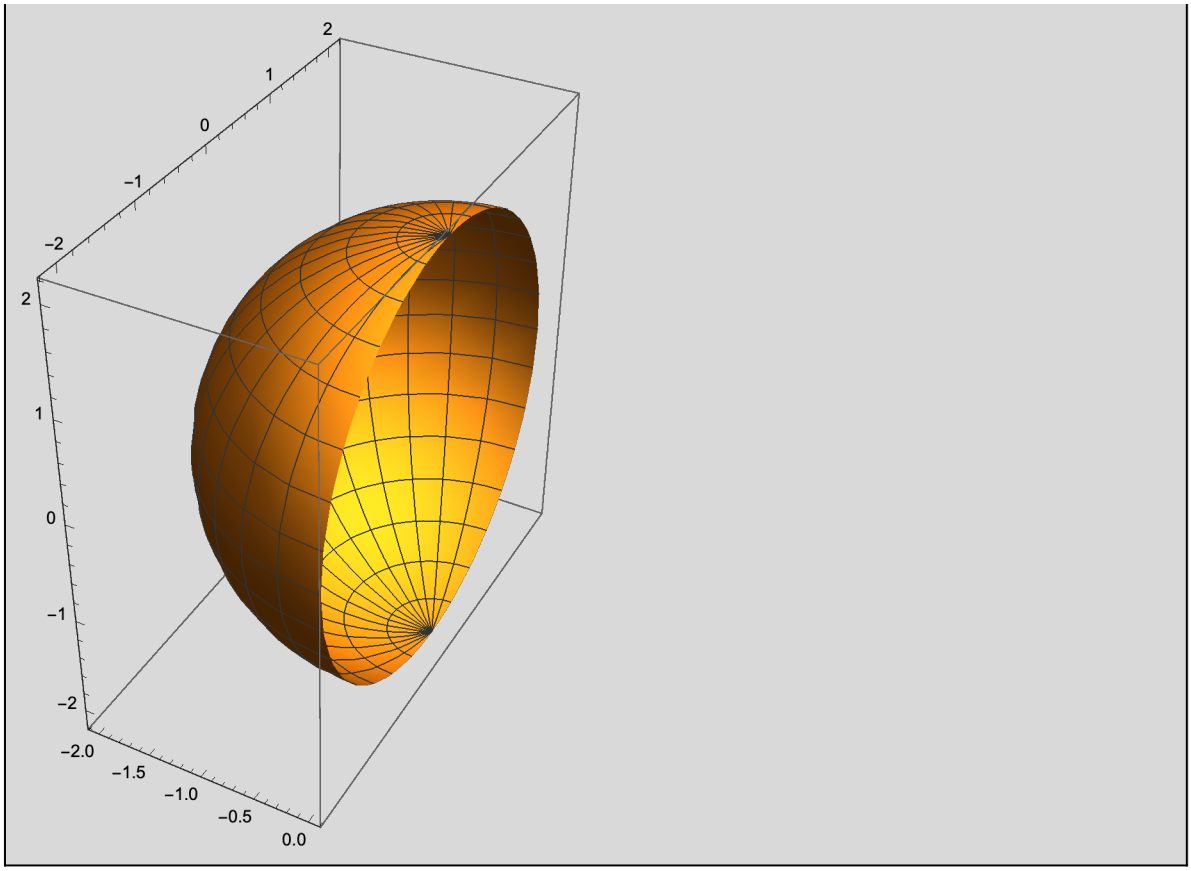
Out[ ]=



Out[ ]=



Out[ ]=

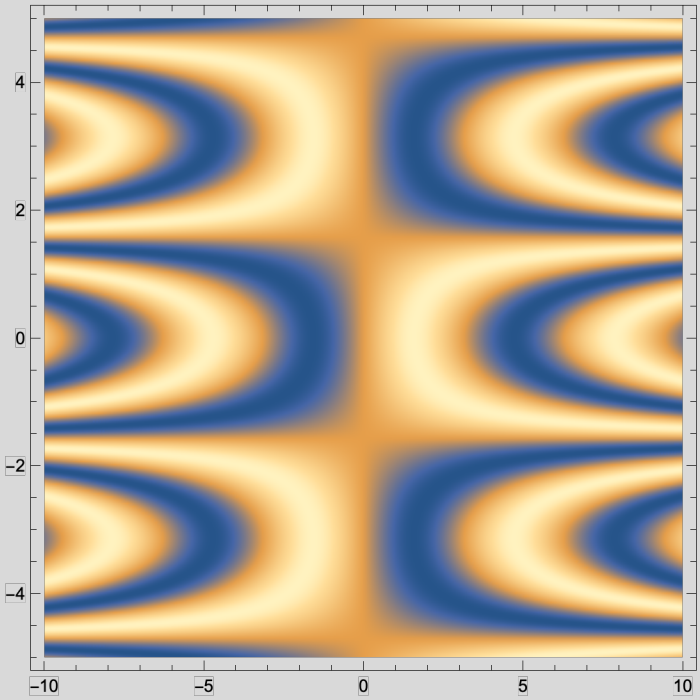


T_7

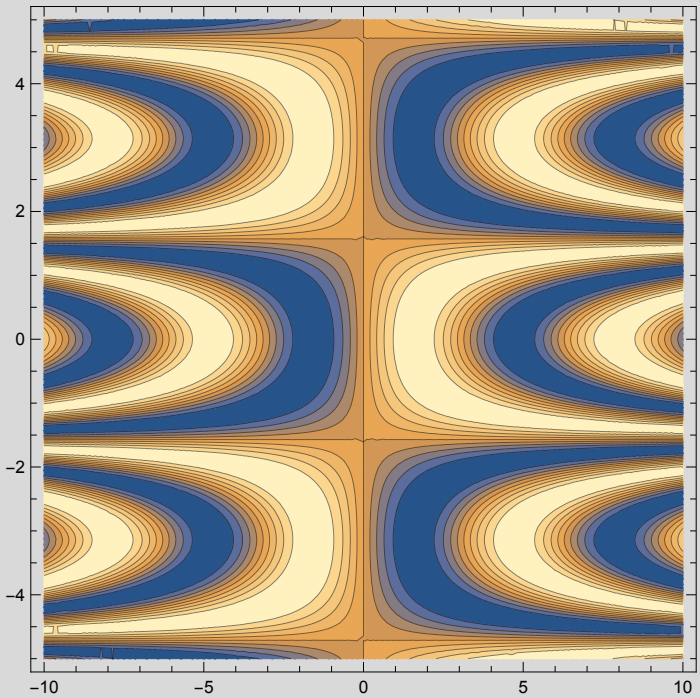
7. 作出函数 $\sin(x\cos y)$ 的密度图和等值线图，$x\in[-10,10]$，$y\in[-5,5]$。

In[ ]:=
```
DensityPlot[Sin[x Cos[y]], {x, -10, 10}, {y, -5, 5}, PlotPoints → 200]
密度图          正弦   余弦                                 绘图点

ContourPlot[Sin[x Cos[y]], {x, -10, 10}, {y, -5, 5}]
绘制等高线       正弦   余弦
```

Out[ ]=



Out[ ]=

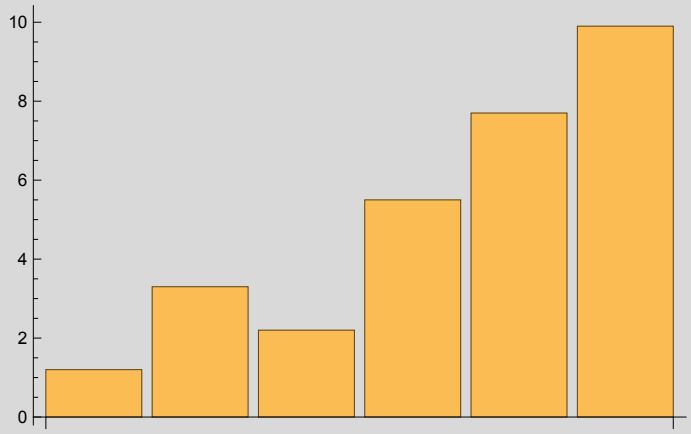


T_8

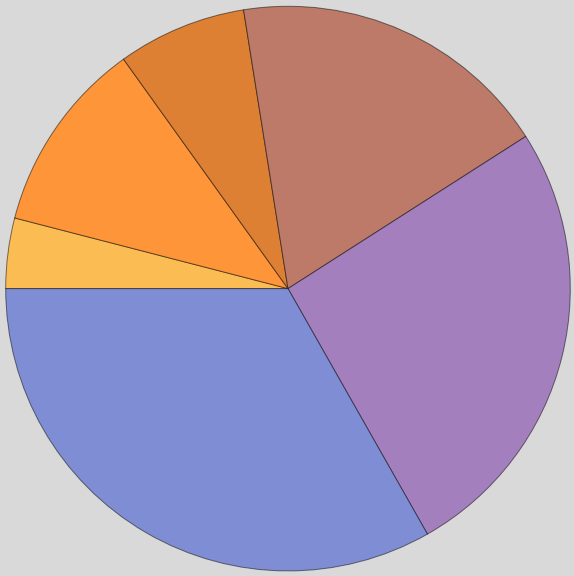
8. 已知 list = {1.2,3.3,2.2,5.5,7.7,9.9},作出 list 的棒图和饼图。

In[ ]:=
```
Clear["Global`*"]
清除
list = {1.2, 3.3, 2.2, 5.5, 7.7, 9.9};
BarChart[list]
条形图
PieChart[list]
饼图
```

Out[ ]=



Out[ ]=



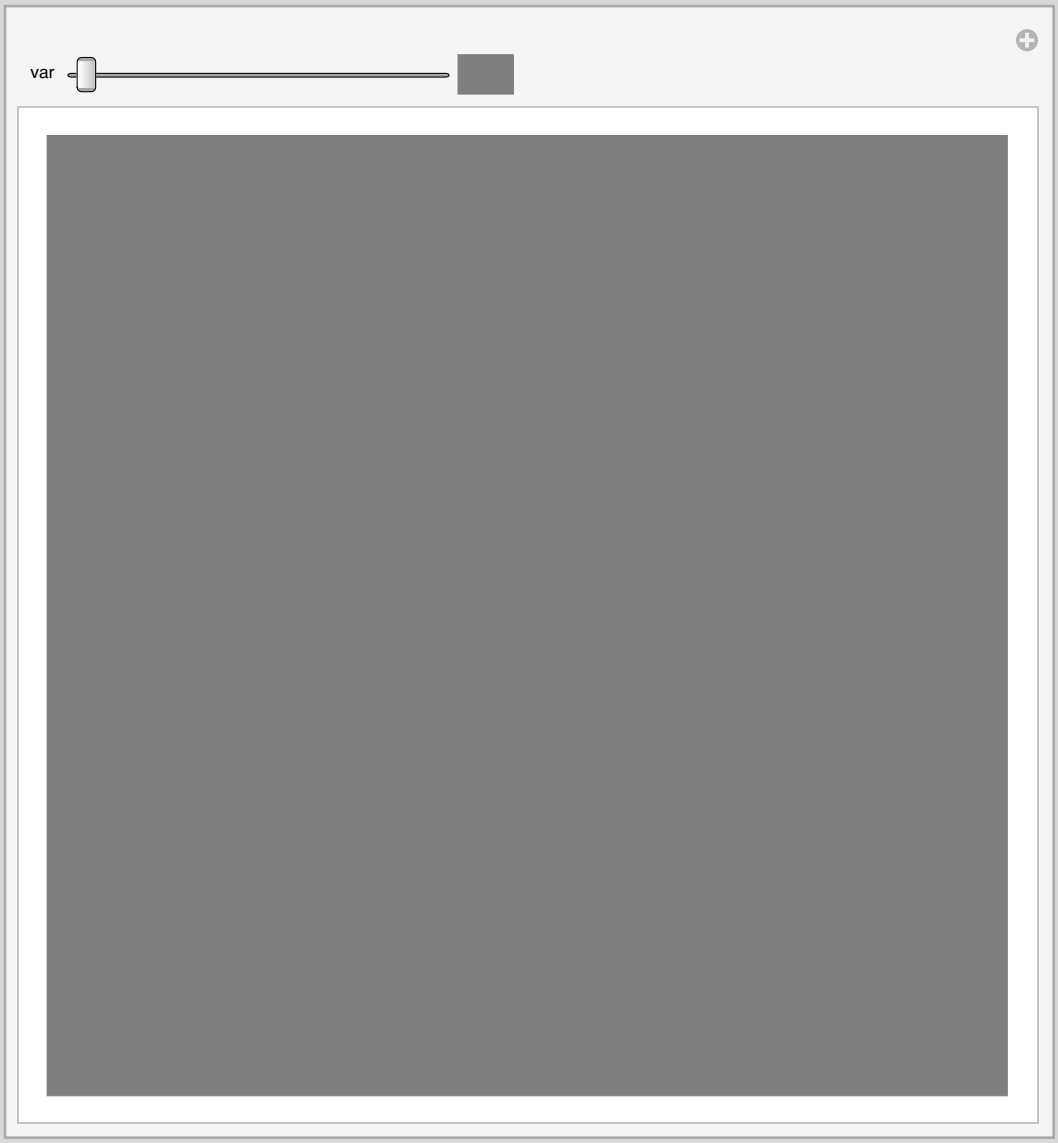
T_9

9. 画出正五边形、正八边形图形,并在打印机上输出图形。

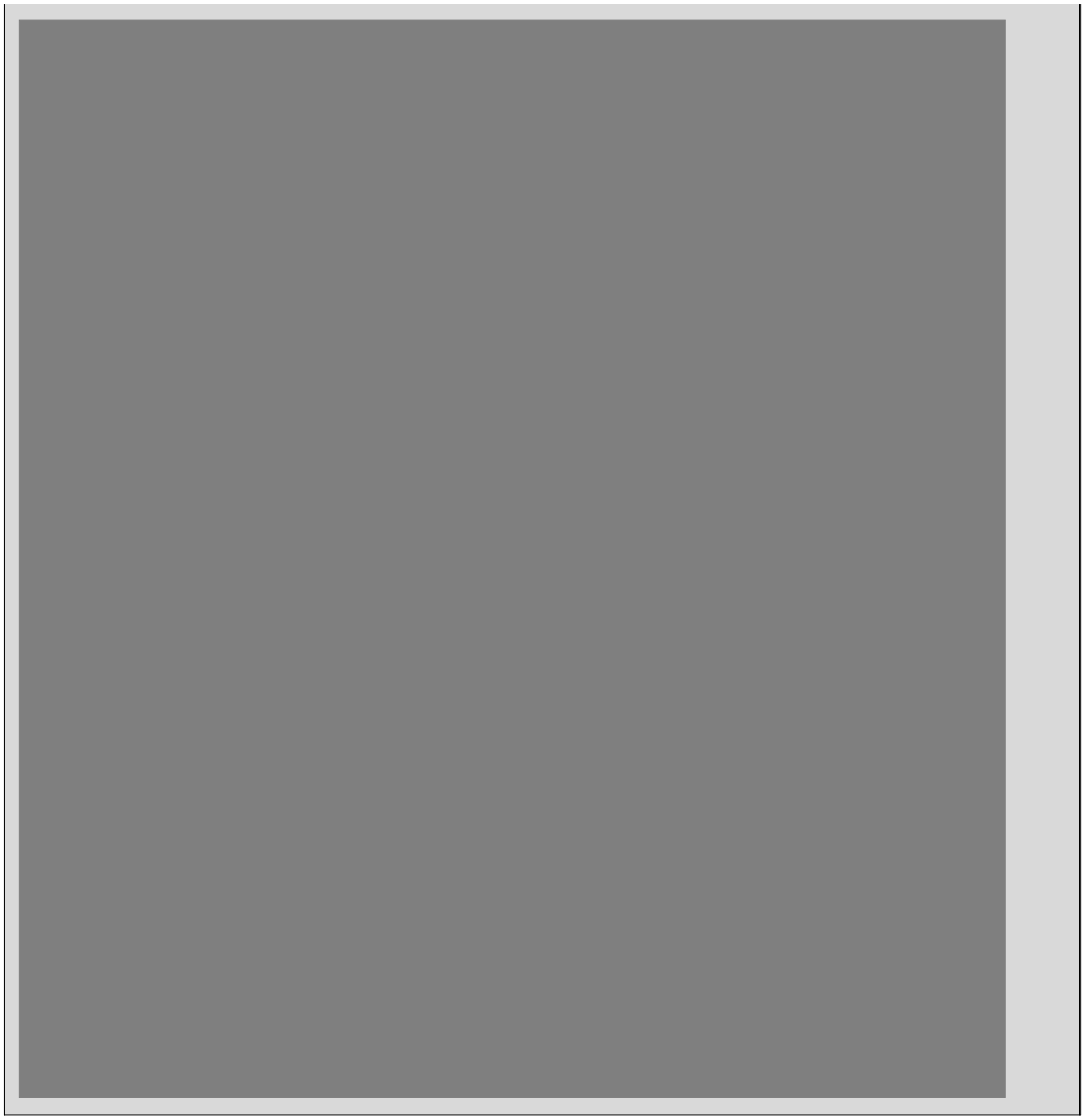
In[ ]:=

```
Clear["Global`*"]
清除

list1 = Table[{Cos[2 k π / 5 + a], Sin[2 k π / 5 + a]}, {k, 0, 5}];
        表格    余弦                正弦
Manipulate[ListLinePlot[list1 /. a → var, Axes → False, AspectRatio → 1,
交互式操作  绘制点集的线条                  坐标轴  假      宽高比
  PerformanceGoal → "Quality"], {var, 0, 2 π}, ControlPlacement → Top]
  性能指标                                     控件位置              顶部

list2 = Table[{Cos[2 k π / 8 + a], Sin[2 k π / 8 + a]}, {k, 0, 8}];
        表格    余弦                正弦
Manipulate[ListLinePlot[list2 /. a → var, Axes → False, AspectRatio → 1,
交互式操作  绘制点集的线条                  坐标轴  假      宽高比
  PerformanceGoal → "Quality"], {var, 0, 2 π}, ControlPlacement → Top]
  性能指标                                     控件位置              顶部
```

Out[ ]=



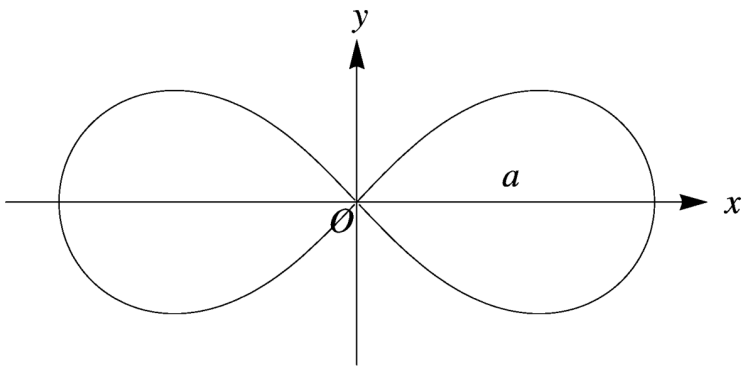
Out[ ]=



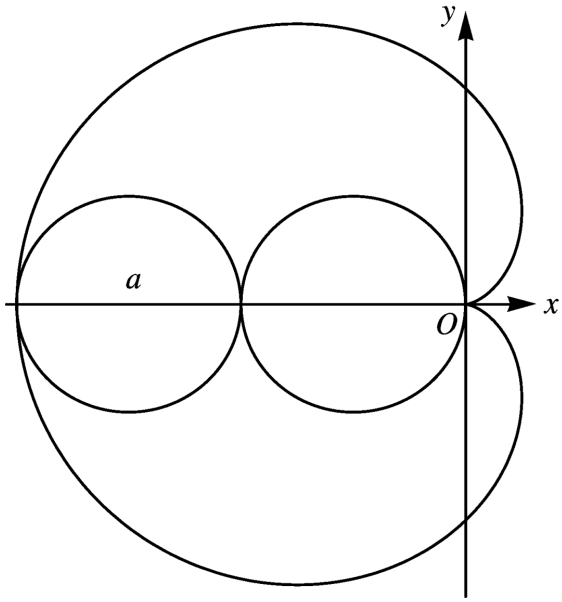
T_{10}

10. 画出下列图形：

（1）双钮线 $\rho^2 = a^2\cos 2\theta$ 或 $(x^2+y^2)^2 = a^2(x^2-y^2)$。



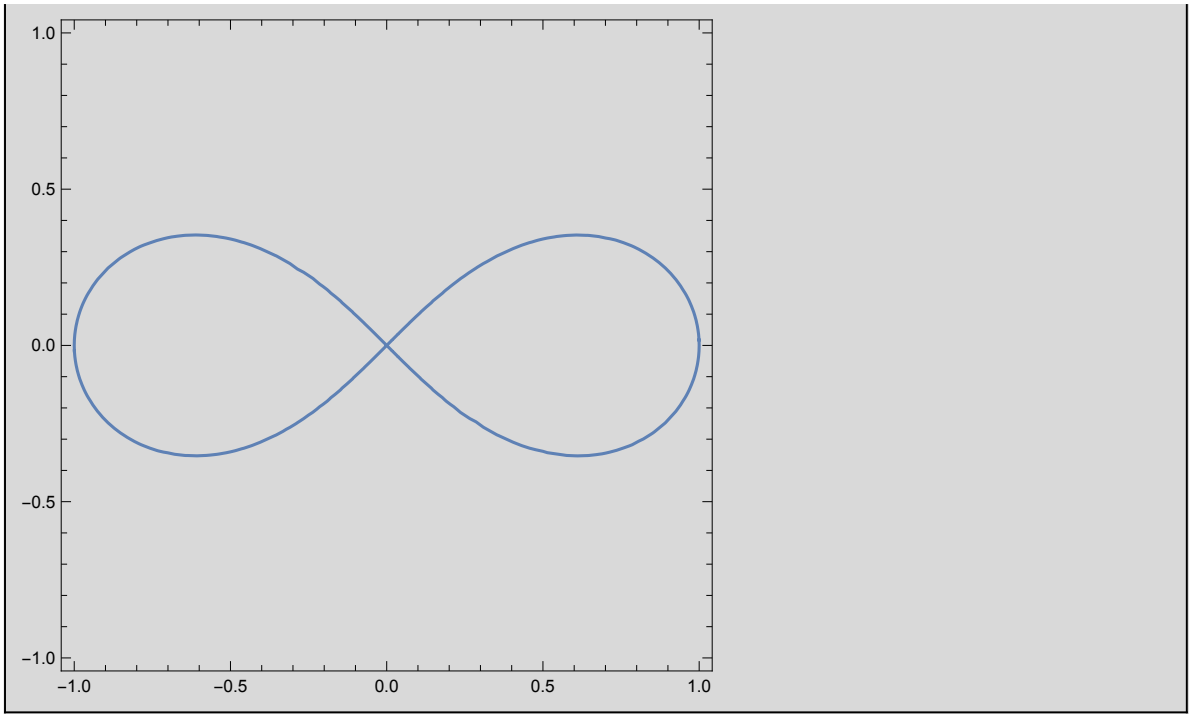
（2）心脏线 $\rho = a(1-\cos\theta)$。



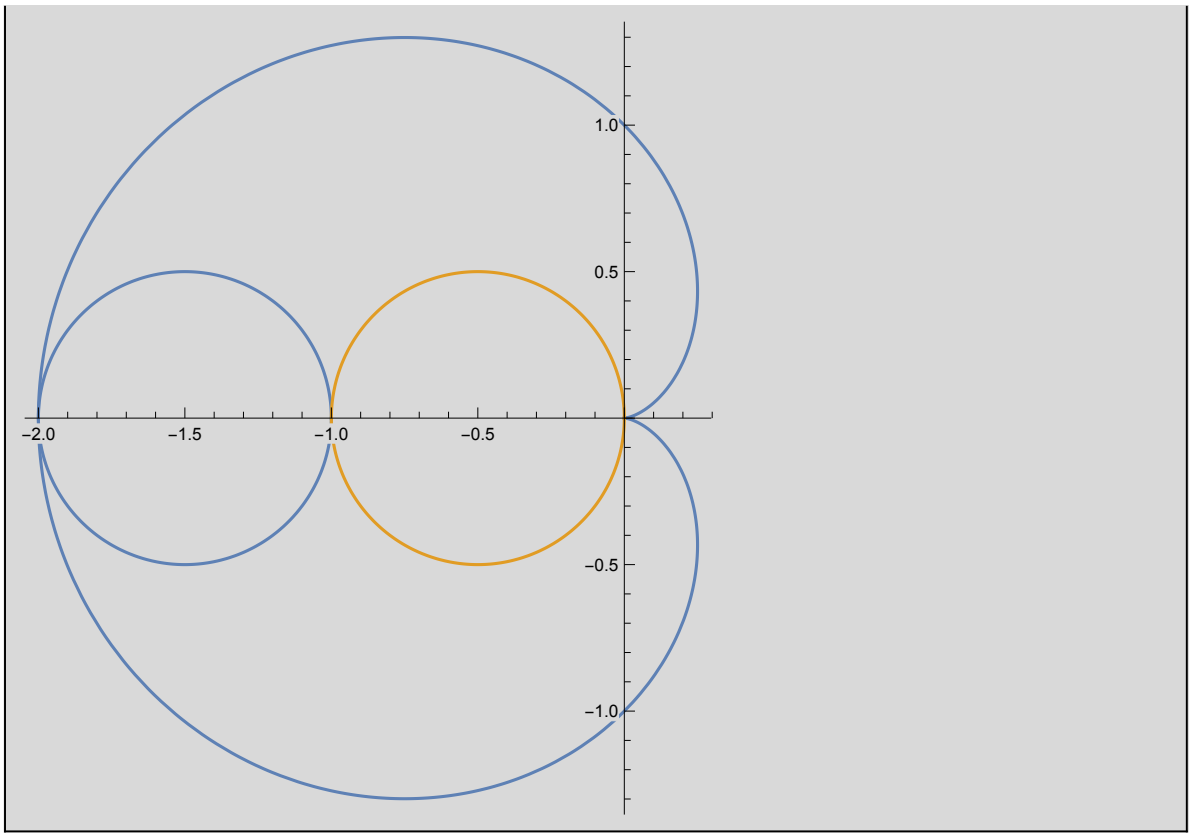
In[ ]:=

```
Clear["Global`*"]
清除
a = 1;
ContourPlot[(x^2 + y^2)^2 == a^2 (x^2 - y^2), {x, -a, a}, {y, -a, a}]
绘制等高线


A = 1;
fig1 = PolarPlot[A (1 - Cos[θ]), {θ, 0, 2 π}];
       极坐标图          余弦
fig2 = ParametricPlot[{{-3/2 A + A/2 * Cos[t], A/2 * Sin[t]},
       绘制参数图                              余弦          正弦
    {-1/2 A + A/2 * Cos[t], A/2 * Sin[t]}}, {t, 0, 2 π}, PlotStyle → Automatic];
                    余弦          正弦                    绘图样式      自动
Show[
显示
 fig1,
 fig2]
```

Out[ ]=



Out[ ]=



T_{11}

11. 画出下列三维图形：

(1) 椭圆球面。

(2) 圆锥面。

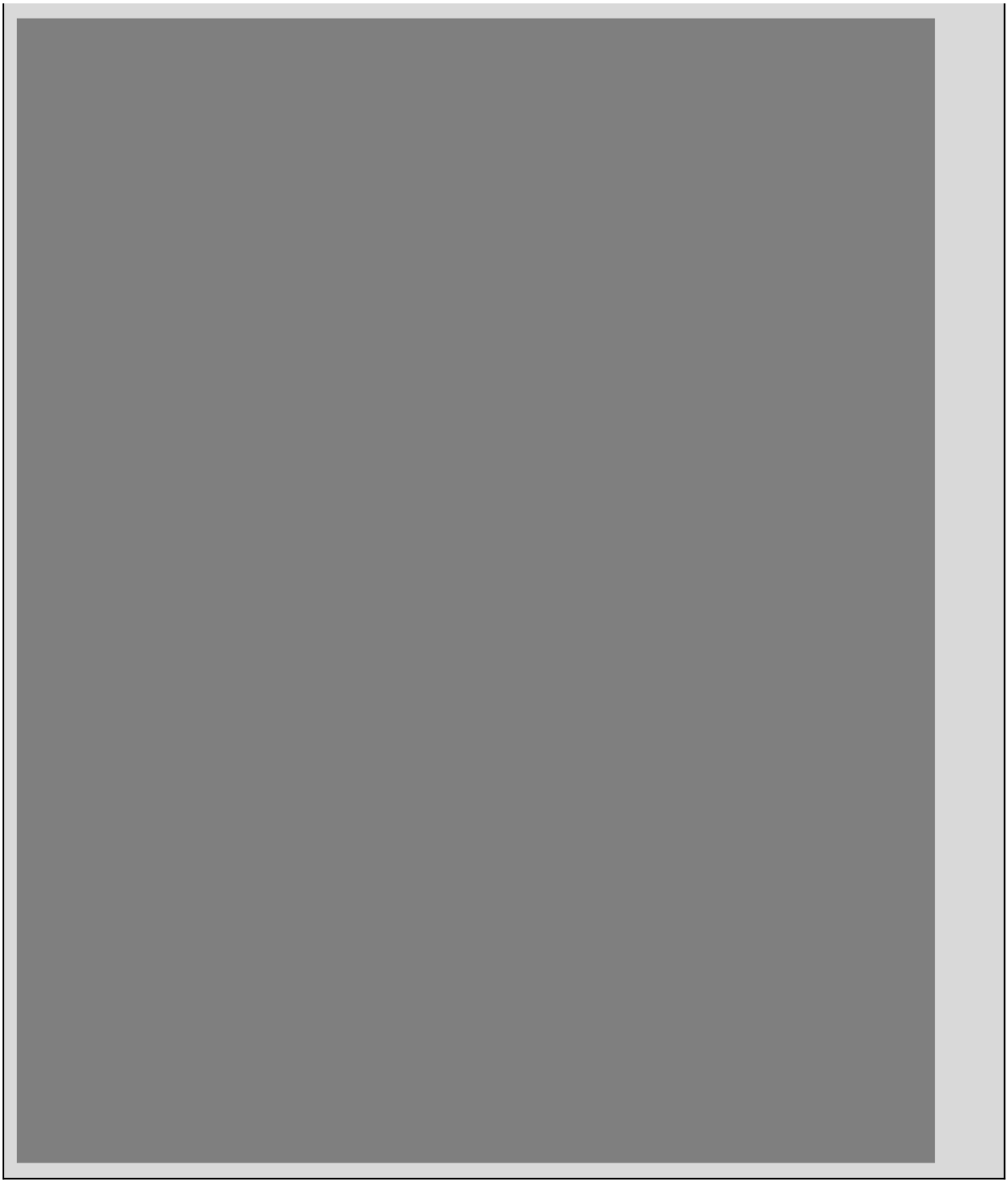
(3) 单叶双曲面。

(4) 双叶双曲面。

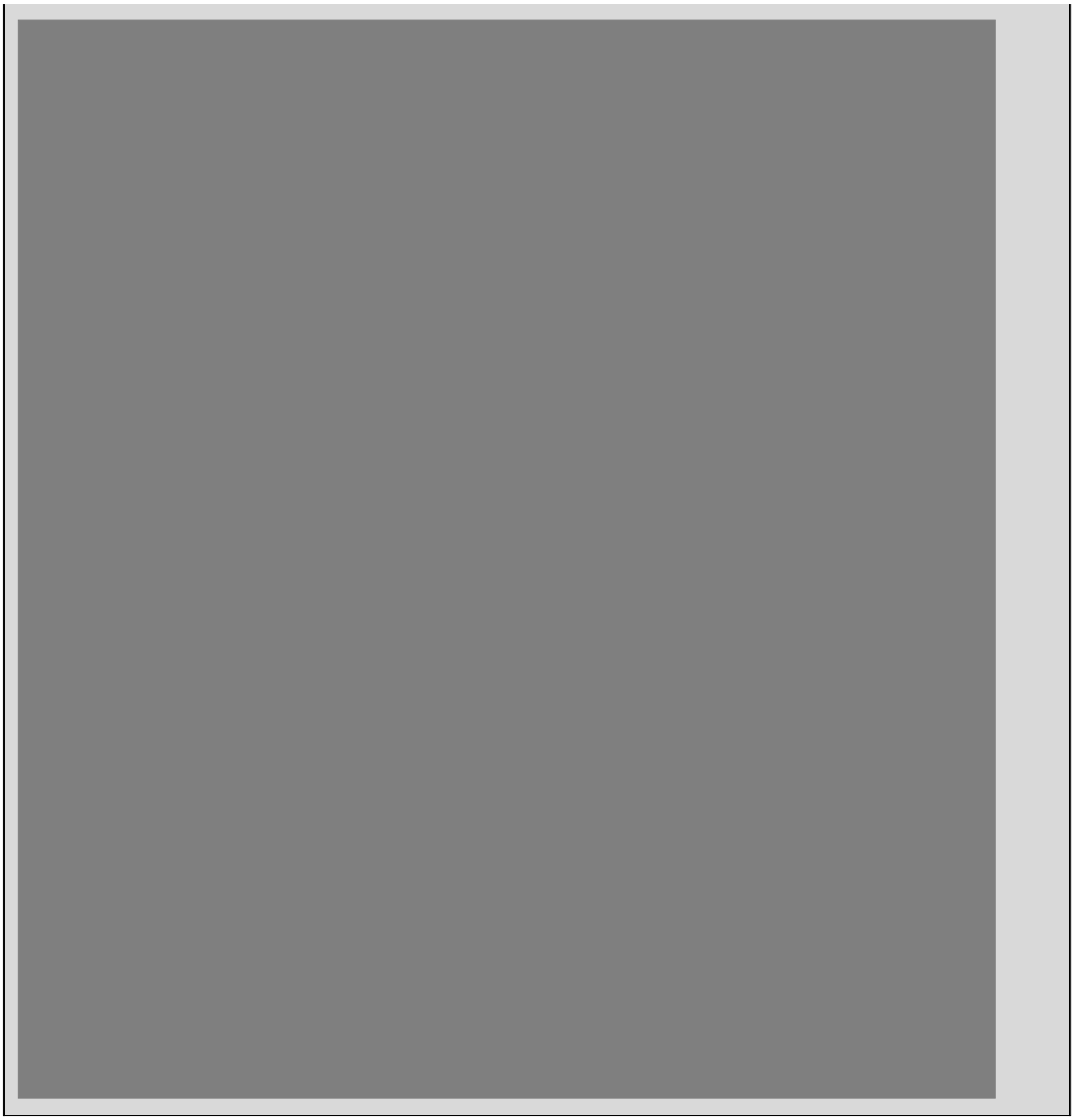
In[•]:=

```
Clear["Global`*"]
清除
Manipulate[ParametricPlot3D[
交互式操作    绘制三维参数图
  {a * Sin[θ] * Cos[φ], b * Sin[θ] * Sin[φ], c * Cos[θ]}, {θ, 0, π}, {φ, 0, 2 π},
     正弦     余弦        正弦     正弦        余弦
  PerformanceGoal → "Quality", PlotRange → {{-2, 2}, {-2, 2}, {-2, 2}}],
  性能指标                     绘制范围
 {{a, 1.2}, 1, 2}, {{b, 1.5}, 1, 2}, {{c, 1.3}, 1, 2}, ControlPlacement → Top]
                                                       控件位置          顶部


Manipulate[
交互式操作
 ParametricPlot3D[{z Tan[θ] Cos[φ], z Tan[θ] Sin[φ], z}, {z, -3, 3}, {φ, 0, 2 π},
 绘制三维参数图      正切    余弦       正切    正弦
  PerformanceGoal → "Quality", PlotRange → {{-3, 3}, {-3, 3}, {-3, 3}}],
  性能指标                     绘制范围
 {{θ, π/3}, 0, 2 π}, ControlPlacement → Top]
                     控件位置          顶部


Manipulate[ParametricPlot3D[{1/a Sec[u] Cos[v], 1/a Sec[u] Sin[v], 1/b Tan[u]},
交互式操作  绘制三维参数图        正割    余弦      正割    正弦      正切
  {u, 0, π}, {v, 0, 2 π}, PerformanceGoal → "Quality",
                          性能指标
  PlotRange → {{-10, 10}, {-10, 10}, {-10, 10}}],
  绘制范围
 {{a, 0.3}, 0, 1}, {{b, 0.4}, 0, 1}, ControlPlacement → Top]
                                     控件位置          顶部


Manipulate[ParametricPlot3D[{1/a Tan[u] Cos[v], 1/a Tan[u] Sin[v], 1/b Sec[u]},
交互式操作  绘制三维参数图        正切    余弦      正切    正弦      正割
  {u, 0, π}, {v, 0, 2 π}, PerformanceGoal → "Quality",
                          性能指标
  PlotRange → {{-10, 10}, {-10, 10}, {-10, 10}}],
  绘制范围
 {{a, 0.3}, 0, 1}, {{b, 0.4}, 0, 1}, ControlPlacement → Top]
                                     控件位置          顶部
```

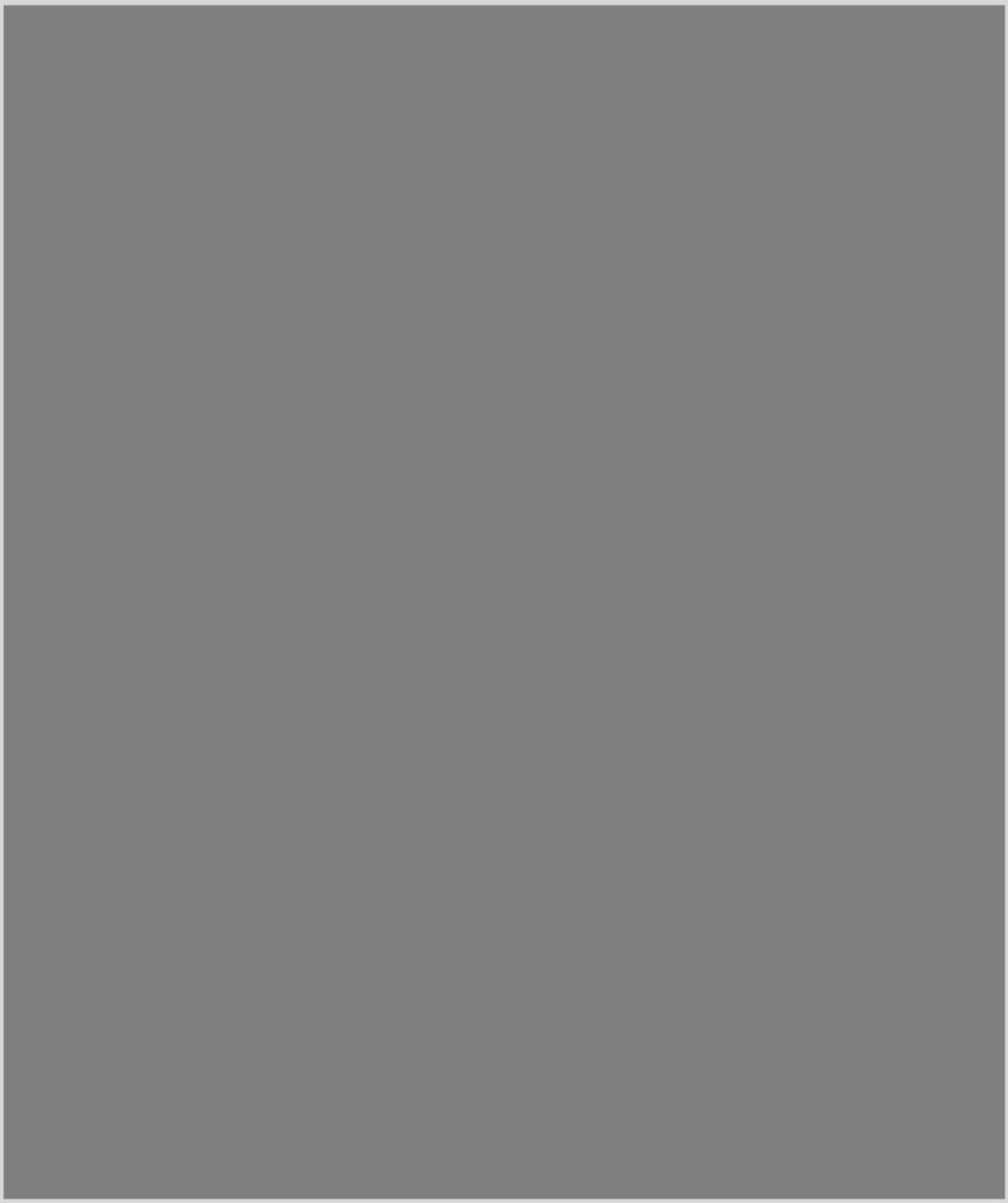
Out[ ]=



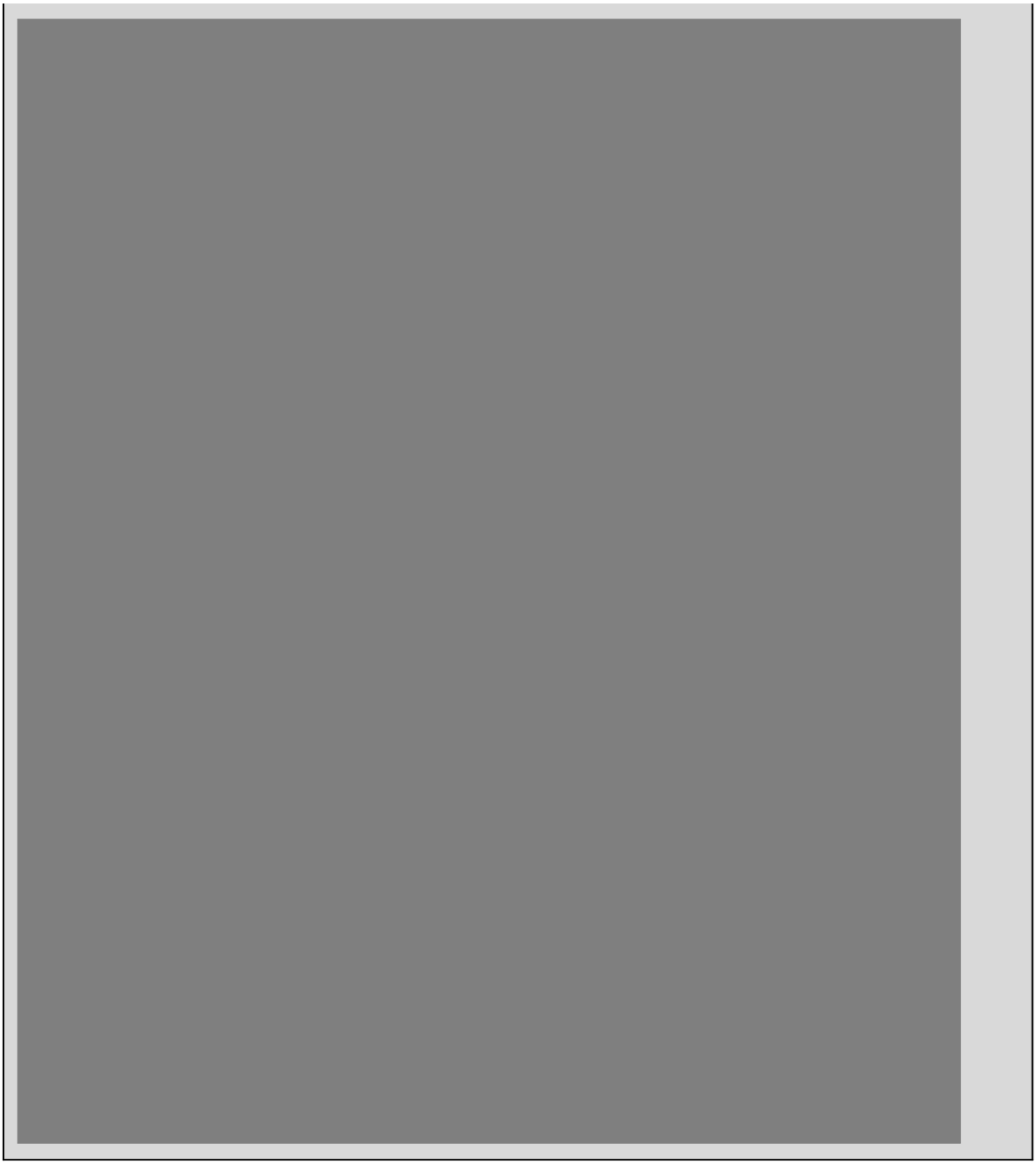
Out[ ]=



Out[ ]=



Out[ ]=



第 7 章　自定义函数和模式替换

T_1

1. 定义函数，输出给定函数及其一阶导数、二阶导数。

```
Clear["Global`*"]
清除
f[x_] := ArcSin[ArcTanh[Log[(1 - x^2) / x]]]
         反正弦  反双曲正切 对数
Simplify[{f[x], f'[x], f''[x]}] // TableForm
化简                              表格形式
```

Out[•]//TableForm=

$$\mathrm{ArcSin}\left[\mathrm{ArcTanh}\left[\mathrm{Log}\left[\frac{1}{x}-x\right]\right]\right]$$

$$-\frac{1+x^2}{x\left(-1+x^2\right)\sqrt{1-\mathrm{ArcTanh}\left[\mathrm{Log}\left[\frac{1}{x}-x\right]\right]^2}\left(-1+\mathrm{Log}\left[\frac{1}{x}-x\right]^2\right)}$$

$$-\frac{-1+4x^2+x^4-\left(1+x^2\right)^2\mathrm{ArcTanh}\left[\mathrm{Log}\left[\frac{1}{x}-x\right]\right]-2\left(1+x^2\right)^2\mathrm{Log}\left[\frac{1}{x}-x\right]-\left(-1+4x^2+x^4\right)\mathrm{Log}\left[\frac{1}{x}-x\right]^2+\mathrm{ArcTanh}\left[\mathrm{Log}\left[\frac{1}{x}-x\right]\right]^2\left(1-4x^2-x^4+2\left(\right.\right.}{x^2\left(-1+x^2\right)^2\left(1-\mathrm{ArcTanh}\left[\mathrm{Log}\left[\frac{1}{x}-x\right]\right]^2\right)^{3/2}\left(-1+\mathrm{Log}\left[\frac{1}{x}-x\right]^2\right)^2}$$

T_2

2. 定义函数 $f(n)$，$f(n)$ 为 n 阶单位矩阵。

In[•]:=

```
Clear["Global`*"]
清除
f[n_] := IdentityMatrix[n]
         单位矩阵
f[5] // MatrixForm
        矩阵格式
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

T_3

3. 定义函数 $g(n)$，$g(n)$ 的对角元素是 $\{1,2,\cdots,n\}$ 的 n 阶对角矩阵。

```
Clear["Global`*"]
清除
g[n_] := Table[Boole[i == j] * i, {i, 1, n}, {j, 1, n}]
         表格    布尔
g[5] // MatrixForm
        矩阵格式
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix}$$

T_4

4. 定义函数，它对参数 n 生成矩阵

$$\begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1-x & 1 & \cdots & 1 \\ 1 & 1 & 2-x & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & n-x \end{pmatrix}$$

对 $n=3,4,5,6,7$ 计算该矩阵的行列式并求逆矩阵。

In[•]:=

```
Clear["Global`*"]
清除
f[n_] := ConstantArray[1, {n + 1, n + 1}] +
         常量数组
  Table[Boole[i == j && i ≠ 1] * (i - 2 - x), {i, 1, n + 1}, {j, 1, n + 1}]
  表格    布尔
Do[Print[Det[f[k]]], {k, 3, 7}]
… 打印    行列式
Do[Print[Inverse[f[k]] // MatrixForm], {k, 3, 7}]
… 打印    逆                  矩阵格式
```

$$-2x+3x^2-x^3$$

$$-6x+11x^2-6x^3+x^4$$

$$-24x+50x^2-35x^3+10x^4-x^5$$

$$-120x+274x^2-225x^3+85x^4-15x^5+x^6$$

$$-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7$$

$$\begin{pmatrix} \frac{2-8x+6x^2-x^3}{-2x+3x^2-x^3} & \frac{-2+3x-x^2}{-2x+3x^2-x^3} & \frac{2x-x^2}{-2x+3x^2-x^3} & \frac{x-x^2}{-2x+3x^2-x^3} \\ \frac{-2+3x-x^2}{-2x+3x^2-x^3} & \frac{2-3x+x^2}{-2x+3x^2-x^3} & 0 & 0 \\ \frac{2x-x^2}{-2x+3x^2-x^3} & 0 & \frac{-2x+x^2}{-2x+3x^2-x^3} & 0 \\ \frac{x-x^2}{-2x+3x^2-x^3} & 0 & 0 & \frac{-x+x^2}{-2x+3x^2-x^3} \end{pmatrix}$$

$$\begin{pmatrix} \frac{6-28x+29x^2-10x^3+x^4}{-6x+11x^2-6x^3+x^4} & \frac{-6+11x-6x^2+x^3}{-6x+11x^2-6x^3+x^4} & \frac{6x-5x^2+x^3}{-6x+11x^2-6x^3+x^4} & \frac{3x-4x^2+x^3}{-6x+11x^2-6x^3+x^4} & \frac{2x-3x^2+x^3}{-6x+11x^2-6x^3+x^4} \\ \frac{-6+11x-6x^2+x^3}{-6x+11x^2-6x^3+x^4} & \frac{6-11x+6x^2-x^3}{-6x+11x^2-6x^3+x^4} & 0 & 0 & 0 \\ \frac{6x-5x^2+x^3}{-6x+11x^2-6x^3+x^4} & 0 & \frac{-6x+5x^2-x^3}{-6x+11x^2-6x^3+x^4} & 0 & 0 \\ \frac{3x-4x^2+x^3}{-6x+11x^2-6x^3+x^4} & 0 & 0 & \frac{-3x+4x^2-x^3}{-6x+11x^2-6x^3+x^4} & 0 \\ \frac{2x-3x^2+x^3}{-6x+11x^2-6x^3+x^4} & 0 & 0 & 0 & \frac{-2x+3x^2-x^3}{-6x+11x^2-6x^3+x^4} \end{pmatrix}$$

$$\begin{pmatrix} \frac{24-124x+155x^2-75x^3+15x^4-x^5}{-24x+50x^2-35x^3+10x^4-x^5} & \frac{-24+50x-35x^2+10x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & \frac{24x-26x^2+9x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & \frac{12x-19x^2+8x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & \frac{8x-14x^2+7}{-24x+50x^2-35} \\ \frac{-24+50x-35x^2+10x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & \frac{24-50x+35x^2-10x^3+x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & 0 & 0 \\ \frac{24x-26x^2+9x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & \frac{-24x+26x^2-9x^3+x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & 0 \\ \frac{12x-19x^2+8x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & 0 & \frac{-12x+19x^2-8x^3+x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 \\ \frac{8x-14x^2+7x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & 0 & 0 & \frac{-8x+14x^2-}{-24x+50x^2-35} \\ \frac{6x-11x^2+6x^3-x^4}{-24x+50x^2-35x^3+10x^4-x^5} & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{120-668x+949x^2-565x^3+160x^4-21x^5+x^6}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & \frac{-120+274x-225x^2+85x^3-15x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & \frac{120x-154x^2+71x^3-14x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & \frac{60x-107x^2}{-120x+274x^2-2} \\ \frac{-120+274x-225x^2+85x^3-15x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & \frac{120-274x+225x^2-85x^3+15x^4-x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & \\ \frac{120x-154x^2+71x^3-14x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & \frac{-120x+154x^2-71x^3+14x^4-x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & \\ \frac{60x-107x^2+59x^3-13x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & 0 & \frac{-60x+107x^2}{-120x+274x^2-2} \\ \frac{40x-78x^2+49x^3-12x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & 0 & \\ \frac{30x-61x^2+41x^3-11x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & 0 & \\ \frac{24x-50x^2+35x^3-10x^4+x^5}{-120x+274x^2-225x^3+85x^4-15x^5+x^6} & 0 & 0 & \end{pmatrix}$$

$$\begin{pmatrix} \frac{720-4248x+6636x^2-4564x^3+1610x^4-301x^5+28x^6-x^7}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & \frac{-720+1764x-1624x^2+735x^3-175x^4+21x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & \frac{720x-1044x^2+580x^3-155x^4}{-720x+1764x^2-1624x^3+735x^4-1} \\ \frac{-720+1764x-1624x^2+735x^3-175x^4+21x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & \frac{720-1764x+1624x^2-735x^3+175x^4-21x^5+x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 \\ \frac{720x-1044x^2+580x^3-155x^4+20x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & \frac{-720x+1044x^2-580x^3+155x}{-720x+1764x^2-1624x^3+735x^4-1} \\ \frac{360x-702x^2+461x^3-137x^4+19x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & 0 \\ \frac{240x-508x^2+372x^3-121x^4+18x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & 0 \\ \frac{180x-396x^2+307x^3-107x^4+17x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & 0 \\ \frac{144x-324x^2+260x^3-95x^4+16x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & 0 \\ \frac{120x-274x^2+225x^3-85x^4+15x^5-x^6}{-720x+1764x^2-1624x^3+735x^4-175x^5+21x^6-x^7} & 0 & 0 \end{pmatrix}$$

T_5

5. 定义函数，它对参数 n 生成矩阵

$$\begin{pmatrix} 0 & 1 & 2 & \cdots & n-1 \\ 1 & 0 & 1 & \cdots & n-2 \\ 1 & 2 & 0 & \cdots & n-3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & 0 \end{pmatrix}$$

In[∘]:=
```
Clear["Global`*"]
清除
f[n_] := Table[If[i ≤ j, j - i, j], {i, n}, {j, n}]
         表格   如果
f[10] // MatrixForm // Print
         矩阵格式       打印
```

$$\begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 & 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 & 6 & 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 0 & 1 & 2 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 0 \end{pmatrix}$$

T_6

6. 作一个函数，它对任何的一维数表求出其正序数与反序数(正序数，后一个元素比前一个元素大的数的个数，例如:1,3,6,9;反序数的定义与之相反)。

In[∘]:=
```
Clear["Global"]
清除
f[x_List] := Sum[Sum[Boole[x[[i]] > x[[j]]], {j, 1, i}], {i, 2, Length[x]}]
             求和 求和 布尔                                   长度
f[{1, 3, 6, 9}]
```

Out[∘]=
```
6
```

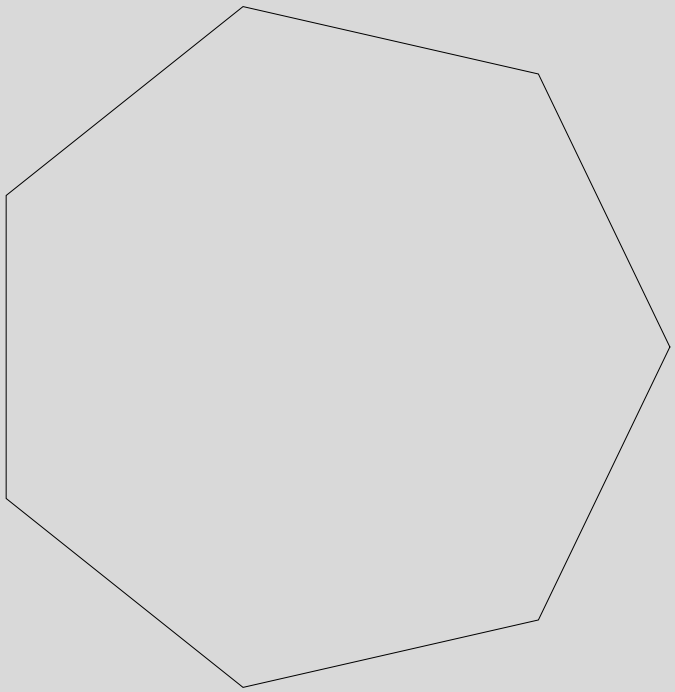
T_7

7. 定义绘制正 n 边形的作图函数。

In[ ]:=

```
Clear["Global"]
清除

f[n_] := Graphics[Line[Table[{Cos[2 k π / n], Sin[2 k π / n]}, {k, 0, n}]]]
         图形      线段  表格    余弦          正弦

f[7]
```

Out[ ]=



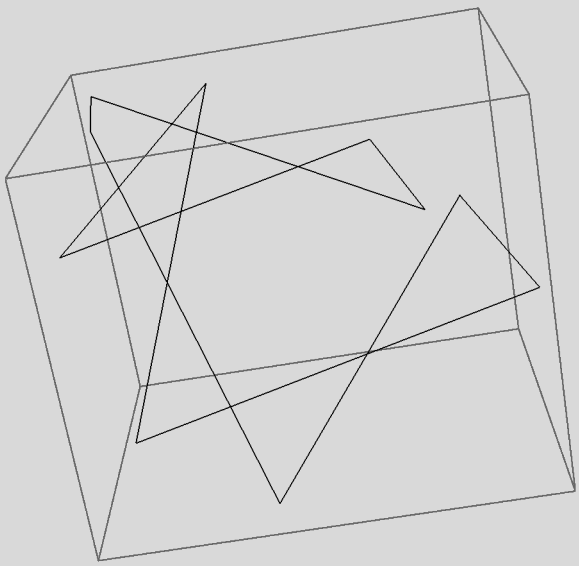
T_8

8. 定义在单位立方体中随机生成 n 边形的函数。

In[•]:=

```
Clear["Global`*"]
清除
f[n_] :=
 Graphics3D[{Line[list = Table[RandomReal[{-1/2, 1/2}, 3], {k, 1, n}]],
 三维图形        线段          表格    伪随机实数
   Line[{list[[1]], list[[Length[list]]]}]}]
   线段                        长度
f[
 10]
```

Out[•]=



T_9

9. 随机形成 n 个 100 以内的整数数表，定义函数计算数表的算术平均、几何平均和调和平均。其中：

$$\text{算术平均} = \frac{a_1 + a_2 + \cdots + a_n}{n}$$

$$\text{几何平均} = \sqrt[n]{a_1 \cdot a_2 \cdot \cdots \cdot a_n}$$

$$\text{调和平均} = \frac{n}{1/a_1 + 1/a_2 + \cdots + 1/a_n}$$

In[•]:=
```
Clear["Global`*"]
清除
n = 10;
a = RandomInteger[100, n]
    伪随机整数
arimean[s_List] := Total[s] / Length[s]
                   总计        长度
arimean[a]
geomean[s_List] := Product[s[[k]], {k, 1, Length[s]}] ^ (1 / Length[s])
                   乘积                 长度              长度
geomean[a]
harmean[s_List] := Length[s] / Total[1 / s]
                   长度         总计
harmean[a]
```

Out[•]= {62, 22, 26, 63, 33, 99, 88, 23, 20, 31}

Out[•]= $\frac{467}{10}$

Out[•]= $2^{4/5}\sqrt{3}\ 11^{2/5}\times 31^{1/5}\times 10\,465^{1/10}$

Out[•]= $\frac{77\,859\,600}{2\,284\,573}$

T_{10}

10. 定义对任意矩阵做 3 种初等行变换或初等列变换的函数。

In[•]:=

```
Clear["Global`*"]
清除
f1[a_, i_, j_] := Module[{b}, b = a; b[[{i, j}]] = a[[{j, i}]]; b]
                  模块
f2[a_, i_, x_] := Module[{b}, b = a; b[[i]] *= x; b]
                  模块
f3[a_, i_, j_, x_] := Module[{b}, b = a;
                      模块
  b[[i]] -= x * b[[j]];
  b](*第i行减去第j行的x倍*)
n = 5;
a = RandomInteger[10, {n, n}];
    伪随机整数
a // MatrixForm
     矩阵格式
f1[a, 1, 2] // MatrixForm
               矩阵格式
f2[a, 1, 4] // MatrixForm
               矩阵格式
f3[a, 1, 2, 5] // MatrixForm
                  矩阵格式
```

Out[•]//MatrixForm=

$$\begin{pmatrix} 5 & 7 & 8 & 6 & 0 \\ 10 & 2 & 9 & 8 & 6 \\ 6 & 7 & 2 & 5 & 10 \\ 10 & 5 & 2 & 2 & 9 \\ 4 & 4 & 1 & 0 & 7 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 10 & 2 & 9 & 8 & 6 \\ 5 & 7 & 8 & 6 & 0 \\ 6 & 7 & 2 & 5 & 10 \\ 10 & 5 & 2 & 2 & 9 \\ 4 & 4 & 1 & 0 & 7 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} 20 & 28 & 32 & 24 & 0 \\ 10 & 2 & 9 & 8 & 6 \\ 6 & 7 & 2 & 5 & 10 \\ 10 & 5 & 2 & 2 & 9 \\ 4 & 4 & 1 & 0 & 7 \end{pmatrix}$$

Out[•]//MatrixForm=

$$\begin{pmatrix} -45 & -3 & -37 & -34 & -30 \\ 10 & 2 & 9 & 8 & 6 \\ 6 & 7 & 2 & 5 & 10 \\ 10 & 5 & 2 & 2 & 9 \\ 4 & 4 & 1 & 0 & 7 \end{pmatrix}$$

T_{11}

11. 定义函数 $g(x)$，并计算 $g(15)$，$g(5.2)$，$g(15)$。

$$g(x)=\begin{cases}\lg x, & x>10\\ e^x+1, & -10\leqslant x\leqslant 10\\ |x|, & x<-10\end{cases}$$

修改成g'(15)

In[•]:=
```
Clear["Global`*"]
清除
g[x_] :=
 Piecewise[{{Log10[x], x > 10}, {E^x + 1, -10 ≤ x ≤ 10}, {Abs[x], x < -10}}]
 分段函数      常用对数              自然常数                  绝对值
{g[15], g[5.2], g'[15]}
```

Out[•]=
$$\left\{\frac{\text{Log}[15]}{\text{Log}[10]},\ 182.272,\ \frac{1}{15\,\text{Log}[10]}\right\}$$

T_{12}

12. 定义函数 $f(x,y)$，并计算 $f(0.1,0.1)$，$f(-0.1,0.1)$，$f(0.1,-0.1)$，$f(-0.1,-0.1)$。

$$f(x)=\begin{cases}\sin x+\cos y, & x\geqslant 0, y>0\\ x+y, & x\geqslant 0, y\leqslant 0\\ x^y, & x<0, y>0\\ x-y, & x<0, y\geqslant 0\end{cases}$$

修改成f(x, y)

In[•]:=
```
f[x_, y_] := Piecewise[{{Sin[x] + Cos[y], x ≥ 0 && y > 0},
             分段函数        正弦      余弦
   {x + y, x ≥ 0 && y ≤ 0}, {x^y, x < 0 && y > 0}, {x - y, x < 0 && y ≥ 0}}]
{f[0.1, 0.1], f[-0.1, 0.1], f[0.1, -0.1], f[-0.1, -0.1]}
```

Out[•]=
```
{1.09484, 0.755451 + 0.245461 i, 0., 0}
```

T_{13}

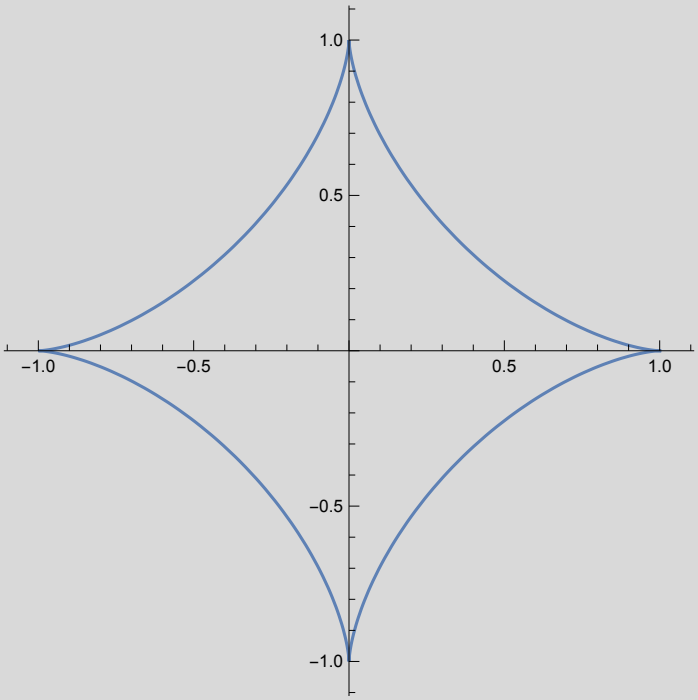
13. 定义函数 A，计算 $x(t)=a\cos^3 t$，$y(t)=a\sin^3 t$ 所围区域的面积。

$$A=\frac{1}{2}\int_L (x\,dy-y\,dx)$$

In[•]:=

```
Clear["Global`*"]
清除
x[t_] := a Cos[t]^3
          余弦
y[t_] := a Sin[t]^3
          正弦
ParametricPlot[{x[t], y[t]} /. a → 1, {t, 0, 2 π}]
绘制参数图
Integrate[x[t] × y'[t] - y[t] × x'[t], {t, 0, 2 π}] / 2
积分
```

Out[•]=



Out[•]=

$$\frac{3\,a^2\,\pi}{8}$$

第 8 章 程序设计

T_1

1. 按 5 个数一行的方式，输出 100 至 1000 之间的能被 3 或 11 整除的所有自然数。

In[ ]:=

```
Clear["Global`*"]
list = Table[If[Mod[x, 3] == 0 || Mod[x, 11] == 0, x, 0], {x, 100, 1000}];
pos = Position[list, 0];
list = Delete[list, pos];
n = Length[list];
A = ConstantArray[0, {Ceiling[n / 5], 5}];
For[i = 1, i < Ceiling[n / 5], i++, Do[A[[i, j]] = list[[5 * (i - 1) + j]], {j, 5}]]
Do[A[[i, j]] = list[[5 * (i - 1) + j]], {j, 5 - (5 * Ceiling[n / 5] - n)}]
A // TableForm
```

Out[ ]//TableForm=

102	105	108	110	111
114	117	120	121	123
126	129	132	135	138
141	143	144	147	150
153	154	156	159	162
165	168	171	174	176
177	180	183	186	187
189	192	195	198	201
204	207	209	210	213
216	219	220	222	225
228	231	234	237	240
242	243	246	249	252
253	255	258	261	264
267	270	273	275	276
279	282	285	286	288
291	294	297	300	303
306	308	309	312	315
318	319	321	324	327
330	333	336	339	341
342	345	348	351	352
354	357	360	363	366
369	372	374	375	378
381	384	385	387	390
393	396	399	402	405
407	408	411	414	417
418	420	423	426	429
432	435	438	440	441
444	447	450	451	453
456	459	462	465	468
471	473	474	477	480
483	484	486	489	492
495	498	501	504	506
507	510	513	516	517
519	522	525	528	531
534	537	539	540	543
546	549	550	552	555

```
558    561    564    567    570
572    573    576    579    582
583    585    588    591    594
597    600    603    605    606
609    612    615    616    618
621    624    627    630    633
636    638    639    642    645
648    649    651    654    657
660    663    666    669    671
672    675    678    681    682
684    687    690    693    696
699    702    704    705    708
711    714    715    717    720
723    726    729    732    735
737    738    741    744    747
748    750    753    756    759
762    765    768    770    771
774    777    780    781    783
786    789    792    795    798
801    803    804    807    810
813    814    816    819    822
825    828    831    834    836
837    840    843    846    847
849    852    855    858    861
864    867    869    870    873
876    879    880    882    885
888    891    894    897    900
902    903    906    909    912
913    915    918    921    924
927    930    933    935    936
939    942    945    946    948
951    954    957    960    963
966    968    969    972    975
978    979    981    984    987
990    993    996    999    0
```

T_2

2. 删除一个数列中的重复元素。

In[ ]:=
```
Clear["Global`*"]
a = {1, 2, 3, 2, 1, 5, 5, 3, 5, 6, 9, 0, 0, 6, 4, 3, 3, 3};
Union[a]
```

Out[ ]=
```
{0, 1, 2, 3, 4, 5, 6, 9}
```

T_3

3. 计算 $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$直到误差小于$10^{-16}$为止。

$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \frac{e^{\theta x}}{(n+1)!} x^{n+1}$ $(-\infty < x < \infty, 0 < \theta < 1)$

In[∘]:=
```
Clear["Global`*"]
x = 1;
FindRoot[Max[{Exp[x], 1}]/((n + 1)!) * x^(n + 1) == 10^-16, {n, 10}]
N[Sum[x^k/k!, {k, 0, Ceiling[n /. %]}], 25]
N[E^x, 25]
```

Out[∘]= {n → 17.4933}

Out[∘]= 2.718281828459045226708117

Out[∘]= 2.718281828459045235360287

T_4

4. 用弦截法求方程 $x^3-2x^2+7x+4=0$ 的根，要求误差小于10^{-16}。

$$x_0=-1,\quad x_1=1,\quad x_k=\frac{x_{k-2}f(x_{k-1})-x_{k-1}f(x_{k-2})}{f(x_{k-1})-f(x_{k-2})},\quad k\geqslant 2$$

In[∘]:=
```
Clear["Global`*"]
f[x_] := x^3 - 2 x^2 + 7 x + 4
x0 = -1; x1 = 1;
a = {{x0, f[x0]}, {x1, f[x1]}};
For[i = 1, i < 50, i++, len = Length[a];
 m = (a[[len, 2]] * a[[len - 1, 1]] - a[[len - 1, 2]] * a[[len, 1]]) /
   (a[[len, 2]] - a[[len - 1, 2]]);
 n = f[m];
 a = Join[a, {{m, n}}];
 If[Abs[n] < 10^-16, Break[]]]
N[a, 10] // MatrixForm
```

Out[∘]//MatrixForm=

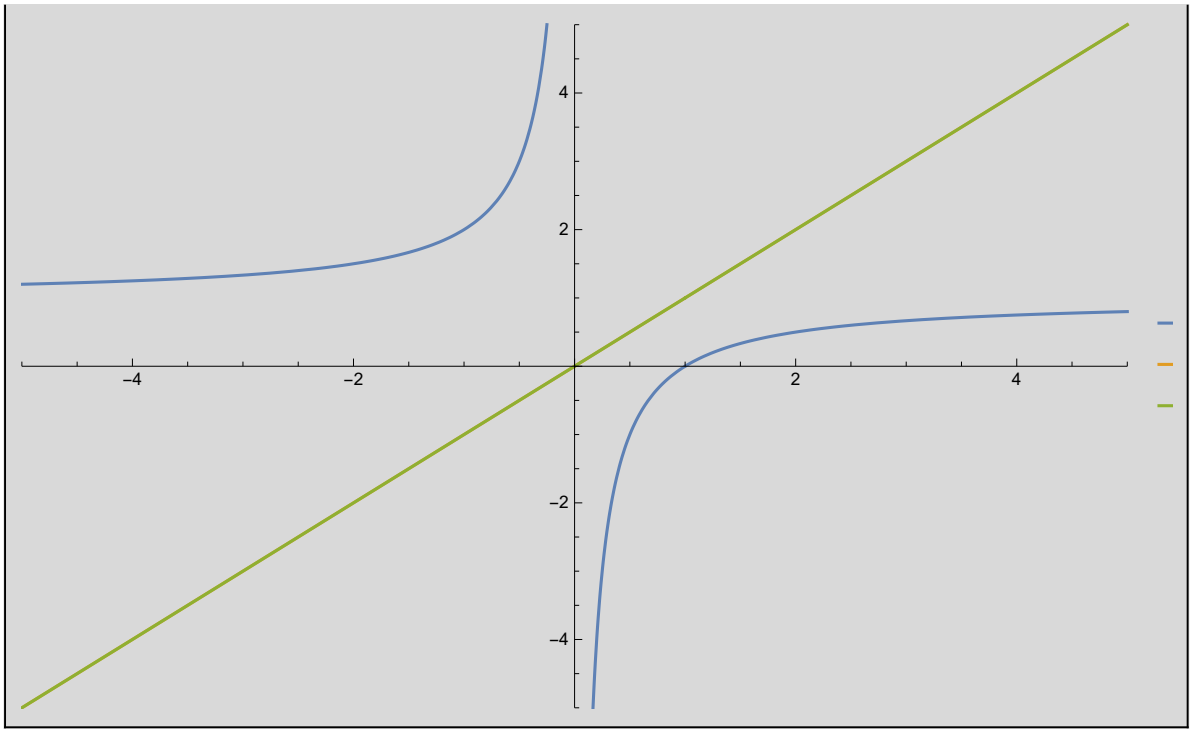
$$\begin{pmatrix} -1.000000000 & -6.000000000 \\ 1.000000000 & 10.00000000 \\ -0.2500000000 & 2.109375000 \\ -0.5841584158 & -0.9709298545 \\ -0.4788297561 & 0.07985073926 \\ -0.4868338736 & 0.002765300983 \\ -0.4871210068 & -8.220023512\times 10^{-6} \\ -0.4871201558 & 8.432000035\times 10^{-10} \\ -0.4871201559 & 2.570784177\times 10^{-16} \\ -0.4871201559 & -8.040042319\times 10^{-27} \end{pmatrix}$$

T_5

5. 定义函数 $f_1(x)=\dfrac{1}{1-x}$，$f_n(x)=f(f_{n-1}(x))$，$n\geqslant 2$，画出 $f_5(x)$，$f_{15}(x)$，$f_{30}(x)$的图像。观察并分析$\lim\limits_{n\to\infty} f_n(x)$的性质。

In[∘]:=
```
Clear["Global`*"]
s = 5;
f[n_, x_] := Nest[1 / (1 - #) &, x, n]
Simplify[Table[f[n, x], {n, 1, 10}]]
Plot[{f[5, x], f[15, x], f[30, x]}, {x, -s, s}, ImageSize → Large,
 PlotLegends → "Expressions", PlotRange → {{-s, s}, {-s, s}}] (*f15(x)和f30(x)重合*)
```

Out[∘]=
$$\left\{\frac{1}{1-x}, \frac{-1+x}{x}, x, \frac{1}{1-x}, \frac{-1+x}{x}, x, \frac{1}{1-x}, \frac{-1+x}{x}, x, \frac{1}{1-x}\right\}$$

Out[∘]=


T_6

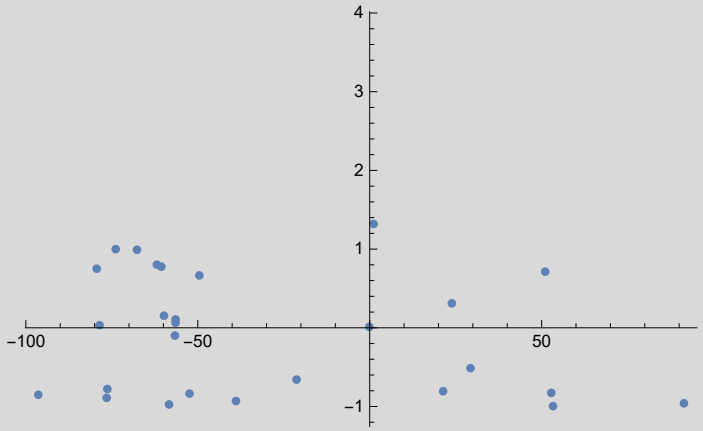
6. 随机生成在$[-100,100]$以内的 30 个实数 x_i，并绘出$(x_i, f(x_i))$的散点图。其中

$$f(x)=\begin{cases}\sin x, & -100\leqslant x\leqslant -20\\ x^2, & -20<x<20\\ \cos x, & 20\leqslant x\leqslant 100\end{cases}$$

In[ ]:=

```
Clear["Global`*"]
n = 30;
f[x_] :=
 Piecewise[{{Sin[x], -100 ≤ x ≤ -20}, {x^2, -20 < x < 20}, {Cos[x], 20 ≤ x ≤ 100}}]
a = RandomReal[{-100, 100}, n];
list = Table[{a[[k]], f[a[[k]]]}, {k, 1, n}];
ListPlot[list]
```

Out[ ]=

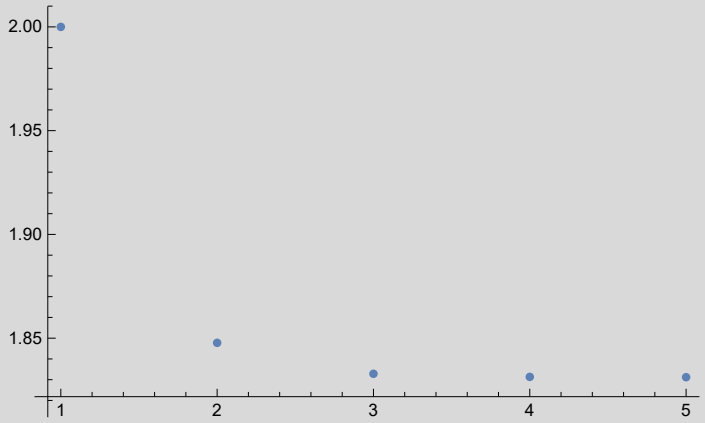


T_7

7. 求数列 $x_1 = 2, x_n = \sqrt{2 + \sqrt{x_{n-1}}}$ 的极限，并画出数列散点图。

In[∘]:=
```
Clear["Global`*"]
f[n_] := Nest[Sqrt[2 + Sqrt[#]] &, 2, n - 1]
ListPlot[Table[f[n], {n, 1, 5}], PlotRange → Full]
Table[N[f[n], 10], {n, 1, 10}]
Solve[x == Sqrt[2 + Sqrt[x]], x] (*计算精确收敛值*)
N[%, 10]
```

Out[∘]=



Out[∘]=
```
{2.000000000, 1.847759065, 1.832845606, 1.831345485, 1.831194184,
 1.831178920, 1.831177380, 1.831177225, 1.831177209, 1.831177207}
```

Out[∘]=

$$\left\{\left\{x \to \frac{1}{3}\left(-1+\left(\frac{79}{2}-\frac{3\sqrt{249}}{2}\right)^{1/3}+\left(\frac{1}{2}\left(79+3\sqrt{249}\right)\right)^{1/3}\right)\right\}\right\}$$

Out[∘]=
```
{{x → 1.831177207}}
```

T_8

8. 随机生成元素在[-10,10]以内的 3 阶可逆方阵,并计算它的逆矩阵。

In[∘]:=
```
Clear["Global`*"]
A = RandomReal[{-10, 10}, {3, 3}];
A // MatrixForm
Inverse[A] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -3.66692 & 8.55888 & 8.1472 \\ 9.40983 & -7.2578 & 5.76758 \\ 2.1883 & 1.8671 & -2.35068 \end{pmatrix}$$

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.0115071 & 0.0646135 & 0.198416 \\ 0.0635342 & -0.016841 & 0.178882 \\ 0.0611762 & 0.0467735 & -0.0986164 \end{pmatrix}$$

T_9

9. 随机生成元素在[－10,10]以内的 4 阶实方阵,并计算它的特征值和特征向量。

In[ ]:=
```
Clear["Global`*"]
A = RandomReal[{-10, 10}, {4, 4}];
A // MatrixForm
Eigensystem[A] // TableForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 9.99158 & 7.22705 & -2.83288 & 5.431 \\ -7.0714 & 7.90719 & 9.25287 & 5.59267 \\ -0.310422 & 2.44703 & -5.32284 & 5.51817 \\ -7.96412 & -5.84138 & -1.64714 & -9.04306 \end{pmatrix}$$

Out[ ]//TableForm=

7.90679 + 8.23119 i	7.90679 − 8.23119 i	−6.14036 + 2.97179 i	−6.1403
−0.047942 + 0.539005 i	−0.047942 − 0.539005 i	0.249596 − 0.00390649 i	0.24959
−0.753923 + 0. i	−0.753923 + 0. i	−0.30764 − 0.268053 i	−0.3076
−0.121918 − 0.0682518 i	−0.121918 + 0.0682518 i	0.717171 + 0. i	0.71717
0.141144 − 0.315169 i	0.141144 + 0.315169 i	0.0442147 + 0.504879 i	0.04421

T_{10}

10. 生成计算矩阵的 3 种初等变换的程序包。

```
<< "C:\\Users\\znz78\\MatrixTransposition.wl"
Clear[A];
A = RandomInteger[{-10, 10}, {4, 5}];
f1[A, 1, 2] // MatrixForm
f2[A, 1, 10] // MatrixForm
f3[A, 1, 2, 1] // MatrixForm

(*
MatrixTransposition.wl的内容

 BeginPackage["Global`"];
f1::usage="交换矩阵的i, j两行";
f2::usage="将矩阵第i行的全体元素乘以系数x";
f3::usage="将矩阵的第i行减去第j行的x倍";
f1[a_,i_,j_]:=Module[{b},b=a;b[[{i,j}]]=a[[{j,i}]];b];
f2[a_,i_,x_]:=Module[{b},b=a;b[[i]]*=x;b];
f3[a_,i_,j_,x_]:=Module[{b},b=a;b[[i]]-=x*b[[j]];b];
EndPackage[];*)
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 & 1 & 2 & 2 & 6 \\ 10 & -5 & -6 & -5 & 9 \\ -7 & 3 & -3 & -10 & -1 \\ 5 & 1 & 8 & 7 & -5 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 100 & -50 & -60 & -50 & 90 \\ 2 & 1 & 2 & 2 & 6 \\ -7 & 3 & -3 & -10 & -1 \\ 5 & 1 & 8 & 7 & -5 \end{pmatrix}$$

Out[ ]//MatrixForm=

$$\begin{pmatrix} 8 & -6 & -8 & -7 & 3 \\ 2 & 1 & 2 & 2 & 6 \\ -7 & 3 & -3 & -10 & -1 \\ 5 & 1 & 8 & 7 & -5 \end{pmatrix}$$

T_{11}

11. 计算任意向量 $\boldsymbol{x}=(x_1,\cdots,x_n)$ 的 3 种范数

$$\|\boldsymbol{x}\|_1=\sum_{k=1}^{n}|x_k|,\quad \|\boldsymbol{x}\|_2=\sqrt{\sum_{k=1}^{n}|x_k|^2},\quad \|\boldsymbol{x}\|_\infty=\max_{1\leqslant k\leqslant n}|x_k|$$

In[∘]:=
```
Clear["Global`*"]
x = RandomReal[{-10, 10}, 5]
norm1[x_] := Total[Abs[x]]
norm2[x_] := Sqrt[Total[Abs[x]^2]]
norminf[x_] := Max[Abs[x]]
{norm1[x], norm2[x], norminf[x]}
```

Out[∘]=
```
{-8.45035, -3.95438, -6.40883, -7.32394, 2.82057}
```

Out[∘]=
```
{28.9581, 13.7737, 8.45035}
```

T_{12}

12. 计算任意 $m\times n$ 实矩阵 A 的范数 $\|A\|_2=\sqrt{\rho(A^{\mathrm{T}}A)}$，其中 $\rho(A^{\mathrm{T}}A)$ 表示 A 的最大特征根。

In[∘]:=
```
Clear["Global`*"]
m = 4; n = 5;
A = RandomReal[{-10, 10}, {m, n}];
A // MatrixForm
Sqrt[Max[Abs[Eigenvalues[Transpose[A].A]]]]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -6.23096 & -2.30737 & -3.94974 & -1.60042 & 5.7568 \\ -9.05501 & 7.10564 & -2.73253 & -8.18923 & 8.95617 \\ 2.13861 & 1.50849 & -4.40247 & 6.12664 & 4.02126 \\ -4.48181 & 9.27409 & -8.46465 & 4.66172 & 7.25624 \end{pmatrix}$$

Out[∘]=
```
22.0067
```

T_{13}

13. 对数据 (x_i, y_i)，$i=1,2,\cdots,n$，定义线性拟合和二次拟合。

In[∘]:=

```
(*可以利用系统函数
 Fit[data,{1,x},x]线性拟合
 Fit[data,{1,x,x^2},x]二次拟合*)

data = {{-3, 4}, {-2, 2}, {-1, 3}, {0, 0}, {1, -1}, {2, -2}, {3, -5}};
Fit[data, {1, x}, x]
Fit[data, {1, x, x^2}, x]
fig1 = Plot[{%, %%}, {x, -3.1, 3.1}];
fig2 = ListPlot[data];
Show[fig1, fig2]
(*-----------------------------------------------------------------*)
(*下面从最小二乘法出发自定义拟合函数
 linear为线性拟合, parabola为二次拟合*)
linear[data_] := Module[{A, B, n, sol}, n = Length[data];
  A = (n                    Sum[data[[k, 1]], {k, 1, n}]
       Sum[data[[k, 1]], {k, 1, n}]   Sum[data[[k, 1]]^2, {k, 1, n}]);
  B = (Sum[data[[k, 2]], {k, 1, n}]
       Sum[data[[k, 1]] data[[k, 2]], {k, 1, n}]);
  sol = LinearSolve[A, B];
  sol[[1, 1]] + sol[[2, 1]] * x]
(*-----------------------------------------------------------------*)
parabola[data_] := Module[{A, B, n, sol}, n = Length[data];
  A = (n                    Sum[data[[k, 1]], {k, 1, n}]   Sum[data[[k, 1]]^2, {k, 1, n}]
       Sum[data[[k, 1]], {k, 1, n}]   Sum[data[[k, 1]]^2, {k, 1, n}]   Sum[data[[k, 1]]^3, {k, 1, n}]
       Sum[data[[k, 1]]^2, {k, 1, n}]   Sum[data[[k, 1]]^3, {k, 1, n}]   Sum[data[[k, 1]]^4, {k, 1, n}]);
  B = (Sum[data[[k, 2]], {k, 1, n}]
       Sum[data[[k, 1]] data[[k, 2]], {k, 1, n}]
       Sum[data[[k, 1]]^2 data[[k, 2]], {k, 1, n}]);
  sol = LinearSolve[A, B];
  sol[[1, 1]] + sol[[2, 1]] * x + sol[[3, 1]] * x^2]
(*-----------------------------------------------------------------*)
fit1 = N[linear[data]]
fit2 = N[parabola[data]]
Show[Plot[{fit1, fit2}, {x, -3.1, 3.1}], ListPlot[data]]
```

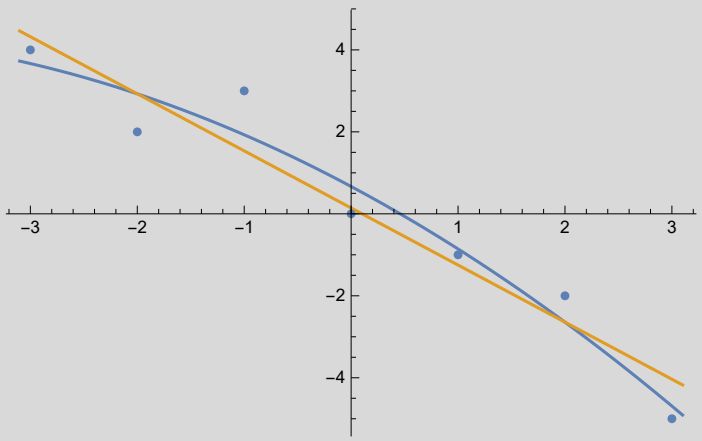
Out[∘]=

$0.142857 - 1.39286\,x$

Out[∘]=

$0.666667 - 1.39286\,x - 0.130952\,x^2$

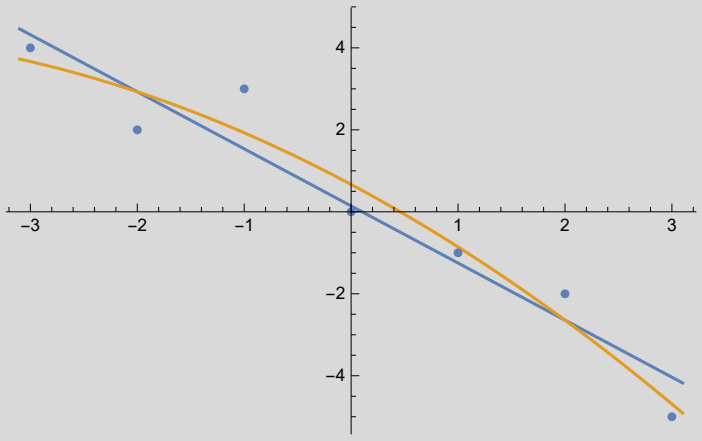
Out[•]=



Out[•]= 0.142857 - 1.39286 x

Out[•]= 0.666667 - 1.39286 x - 0.130952 x^2

Out[•]=



T_{14}

14. 对数据(x_i, y_i), $i=1,2,\cdots,n$,定义 Hermite 插值和三次样条插值函数。

Hermite插值: 给定n + 1个节点{x_i, i=0, 1, ..., n}及该点的函数值和一阶导数值, 构造(2n+1)次Hermite插值多项式使得该多项式在节点下的函数值与一阶导数值与给定值相同.

In[ ]:=

```
(*Hermite插值*)
Clear["Global`*"]
f[x_] := Cos[x] + Sin[5 x]
data = Table[{x, f[x], f'[x]}, {x, -1, 1, 0.1}]
hermite[data_] := Module[{n}, n = Length[data];
  Sum[data[[i, 3]] ((x - data[[i, 1]]) (Prod[(x - data[[j, 1]])/(data[[i, 1]] - data[[j, 1]])], {j, Drop[Table[k, {k, n}], {i}]})^2) +
      data[[i, 2]] ((Prod[(x - data[[j, 1]])/(data[[i, 1]] - data[[j, 1]])], {j, Drop[Table[k, {k, n}], {i}]})^2
        (1 - 2 (x - data[[i, 1]]) Sum[1/(data[[i, 1]] - data[[j, 1]])], {j, Drop[Table[k, {k, n}], {i}]}))], {i, 1, n}]]
fit = Expand[hermite[data]]
fig1 = Plot[{f[x], fit}, {x, -1.1, 1.1}, PlotLegends → "Expressions"];
fig2 = ListPlot[Table[{data[[k, 1]], data[[k, 2]]}, {k, 1, Length[data]}]];
Show[fig1, fig2]
Plot[f[x] - fit, {x, -1.1, 1.1}, PlotLabel → "Error"]
```

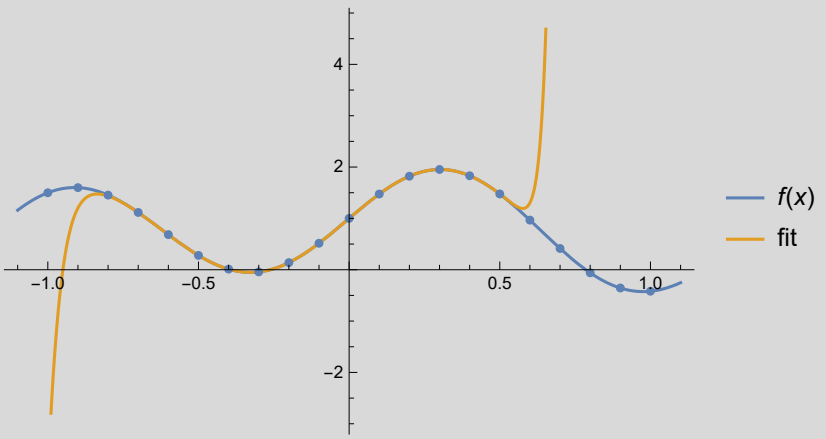
Out[ ]=

```
{{-1., 1.49923, 2.25978}, {-0.9, 1.59914, -0.270652},
 {-0.8, 1.45351, -2.55086}, {-0.7, 1.11563, -4.03807}, {-0.6, 0.684216, -4.38532},
 {-0.5, 0.27911, -3.52629}, {-0.4, 0.0117636, -1.69132},
 {-0.3, -0.0421585, 0.649206}, {-0.2, 0.138596, 2.90018},
 {-0.1, 0.515579, 4.48775}, {0., 1., 5.}, {0.1, 1.47443, 4.28808},
 {0.2, 1.82154, 2.50284}, {0.3, 1.95283, 0.0581658}, {0.4, 1.83036, -2.47015},
 {0.5, 1.47605, -4.48514}, {0.6, 0.966456, -5.5146}, {0.7, 0.414059, -5.3265},
 {0.8, -0.0600958, -3.98557}, {0.9, -0.35592, -1.83731}, {1., -0.418622, 0.57684}}
```

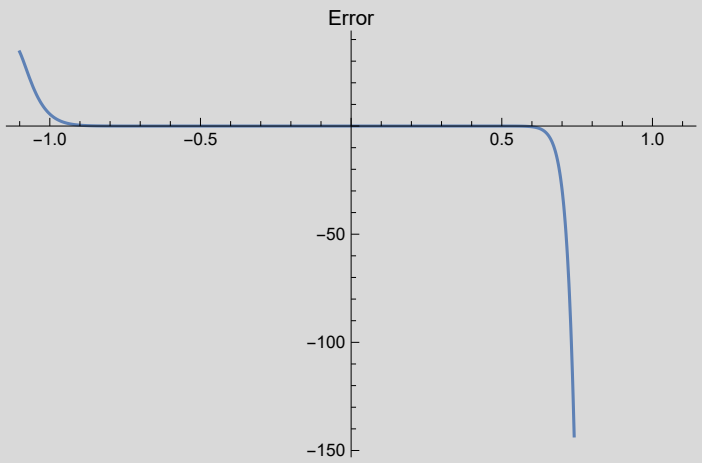
Out[ ]=

$$1. + 5.\,x - 0.5\,x^2 - 20.8333\,x^3 + 0.0416667\,x^4 + 26.0417\,x^5 - 0.0013889\,x^6 - 15.501\,x^7 + 0.0000166297\,x^8 + 5.38219\,x^9 - 0.000800848\,x^{10} - 1.22795\,x^{11} - 0.018074\,x^{12} + 0.164703\,x^{13} + 0.0980225\,x^{14} + 0.824219\,x^{15} + 2.00146\,x^{16} - 6.02832\,x^{17} - 74.1797\,x^{18} - 358.664\,x^{19} - 1142.75\,x^{20} - 2549.81\,x^{21} - 3635.19\,x^{22} - 1220.88\,x^{23} + 9804.5\,x^{24} + 33\,038.\,x^{25} + 65\,816.5\,x^{26} + 98\,660.\,x^{27} + 121\,010.\,x^{28} + 127\,526.\,x^{29} + 118\,418.\,x^{30} + 96\,620.\,x^{31} + 67\,376.\,x^{32} + 38\,564.\,x^{33} + 17\,302.\,x^{34} + 5744.\,x^{35} + 1285.\,x^{36} + 152.\,x^{37} + 3.125\,x^{38} + 1.875\,x^{39} + 0.59375\,x^{40} - 0.078125\,x^{41}$$

Out[ ]=



Out[•]=



In[•]:=

```
(*三次样条插值函数
 可使用系统函数Interpolation*)
Off[InterpolatingFunction::dmval]
Clear["Global`*"]
f[x_] := Sin[x] + Cos[5 x];
data = Table[{x, f[x]}, {x, -2, 2, 0.3}]
fit = Interpolation[data]
Show[Plot[{f[x], fit[x]}, {x, -2.1, 2.1}, PlotLegends → "Expressions"],
 ListPlot[data]]
```

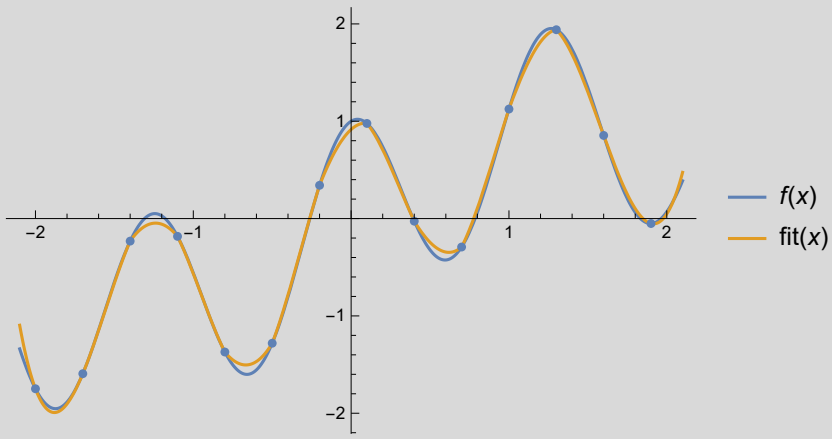
Out[•]=

```
{{-2., -1.74837}, {-1.7, -1.59368}, {-1.4, -0.231547},
 {-1.1, -0.182538}, {-0.8, -1.371}, {-0.5, -1.28057},
 {-0.2, 0.341633}, {0.1, 0.977416}, {0.4, -0.0267285}, {0.7, -0.292239},
 {1., 1.12513}, {1.3, 1.94015}, {1.6, 0.854074}, {1.9, -0.0508721}}
```

Out[•]=

```
InterpolatingFunction[ Domain: {{-2., 1.9}} Output: scalar ]
```

Out[•]=



15. 用复化梯形公式计算定积分

$$\int_a^b f(x)\mathrm{d}x \approx \frac{h}{2}\left(f(a)+2\sum_{k=1}^{n-1} f(a+kh)+f(b)\right),\quad h=\frac{b-a}{n}$$

In[∘]:=
```
Clear["Global`*"]
f[x_] := Sin[x] + Log[1/x] + E^(-x^2)
a = 1; b = 2; n = 10;
NIntegrate[f[x], {x, a, b}] (*精确值*)
h = (b - a)/n;
h * (f[a] + 2 * Sum[f[a + k * h], {k, 1, n - 1}] + f[b]) / 2 // N(*复化梯形近似值*)
```

Out[∘]=
```
0.705412
```

Out[∘]=
```
0.705584
```

T_{16}

16. 用 Newton 迭代公式 $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$，求方程 $f(x)=0$ 在 x_0 附近的根。

In[∘]:=
```
Clear["Global`*"]
f[x_] := x^3 - 77/10 * x^2 + 192/10 * x - 153/10
x0 = 1; error = 10^-6;
a = {{x0, f[x0]}};
For[i = 1, i < 50, i++, x = x0 - f[x0] / f'[x0];
 a = Join[a, {{x, f[x]}}];
 If[Abs[x - x0] < error, Break[]];
 x0 = x]
N[a, 10] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1.000000000 & -2.800000000 \\ 1.411764706 & -0.7270710360 \\ 1.623241688 & -0.1454925749 \\ 1.692299634 & -0.01316824361 \\ 1.699910369 & -0.0001514977835 \\ 1.699999988 & -2.088348212\times 10^{-8} \\ 1.700000000 & -3.970139248\times 10^{-16} \end{pmatrix}$$

T_{17}

17. 用 Gauss-Seidel 迭代求解 $\begin{cases} 10x_1-2x_2-x_3=0 \\ -2x_1+10x_2-x_3=-21 \\ -x_1-2x_2+5x_3=-20 \end{cases}$，自取初始值，当 $\|\boldsymbol{X}^{(k+1)}-\boldsymbol{X}^{(k)}\|_\infty<10^{-4}$ 时迭代停止。

In[∘]:=

```
Clear["Global`*"]
A = {{10, -2, -1}, {-2, 10, -1}, {-1, -2, 5}};
b = {{0}, {-21}, {-20}};
x = ( 0  0  0 );(*输入迭代初值*)
errormax = 10^-4;
T = Input["请输入迭代次数"];
n = Length[A];(*矩阵阶数n*)
U = ConstantArray[0, {n, n}];
For[i = 1, i ≤ n, i++,
 For[j = i + 1, j ≤ n, j++, U[[i, j]] = A[[i, j]]]]
S = -Inverse[A - U].U;(*构造迭代矩阵S*)
f = Inverse[A - U].b;(*构造迭代余项f*)
Print["矩阵A的上三角矩阵:", U // MatrixForm,
 "Gauss-Seidel迭代矩阵S:", S // MatrixForm, "\t迭代余项f:", f // MatrixForm]
For[i = 1, i ≤ T, i++, AppendTo[x, N[S.x[[i]] + (Transpose[f])[[1]], 7]];
 If[Max[Abs[x[[-1]] - x[[-2]]]] < errormax, step = i;
  Break[]]] (*迭代结果的矩阵*)
error = ConstantArray[0, {step, n}];
For[i = 1, i ≤ step, i++, error[[i]] = x[[i + 1]] - x[[i]]] (*迭代误差的矩阵*)
Print["迭代结果Xn+1=S.Xn+f:\n"]
Print@TableForm[Delete[x, 1], TableHeadings → Automatic] (*不输出迭代初值*)
Print["\n\n迭代误差Xi-Xi-1:\n"]
Print@TableForm[error, TableHeadings → Automatic]
N[LinearSolve[A, b], 8] (*精确值*)
```

矩阵A的上三角矩阵: $\begin{pmatrix} 0 & -2 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ Gauss-Seidel迭代矩阵S: $\begin{pmatrix} 0 & \frac{1}{5} & \frac{1}{10} \\ 0 & \frac{1}{25} & \frac{3}{25} \\ 0 & \frac{7}{125} & \frac{17}{250} \end{pmatrix}$ 迭代余项f: $\begin{pmatrix} 0 \\ -\frac{21}{10} \\ -\frac{121}{25} \end{pmatrix}$

迭代结果Xn+1=S.Xn+f:

	1	2	3
1	0	-2.100000	-4.840000
2	-0.9040000	-2.764800	-5.286720
3	-1.081632	-2.844998	-5.354326
4	-1.104432	-2.856319	-5.363414
5	-1.107605	-2.857862	-5.364666
6	-1.108039	-2.858074	-5.364838
7	-1.108099	-2.858103	-5.364861

迭代误差Xi-Xi-1:

	1	2	3
1	0	-2.100000	-4.840000
2	-0.9040000	-0.664800	-0.44672
3	-0.177632	-0.080198	-0.06761
4	-0.022800	-0.011321	-0.00909
5	-0.003173	-0.001543	-0.00125
6	-0.000434	-0.000212	-0.00017
7	-0.000060	-0.000029	-0.00002

Out[∘]=

```
{{-1.1081081}, {-2.8581081}, {-5.3648649}}
```

T_{18}

18. 定义函数 $f(x)$，输出矩阵 $f(5)$，形式如下所示，其中 x 为奇数。

$$\begin{matrix} * & * & * & * & * \\ * & 0 & 0 & 0 & * \\ * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * \\ * & * & * & * & * \end{matrix}$$

In[∘]:=
```
Clear["Global`*"]
f[n_] := Module[{A, mid = (n + 1) / 2}, A = ConstantArray[0, {n, n}];
  For[i = 0, i ≤ (n - 1) / 2, i++,
   If[OddQ[i],
    A[[mid + i, All]] = 0;
    A[[mid - i, All]] = 0;
    A[[All, mid + i]] = 0;
    A[[All, mid - i]] = 0,
    A[[mid + i, All]] = "*";
    A[[mid - i, All]] = "*";
    A[[All, mid + i]] = "*";
    A[[All, mid - i]] = "*"]]; A]
f[5] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} * & * & * & * & * \\ * & 0 & 0 & 0 & * \\ * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * \\ * & * & * & * & * \end{pmatrix}$$

T_{19}

19. 定义函数 $g(y)$，输出矩阵 $g(5)$ 的形式如下：

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 16 & 17 & 18 & 19 & 6 \\ 15 & 24 & 25 & 20 & 7 \\ 14 & 23 & 22 & 21 & 8 \\ 13 & 12 & 11 & 10 & 9 \end{matrix}$$

In[ ]:=
```
Clear["Global`*"]
g[y_] := Module[{A = ConstantArray[0, {y, y}], x = 1, i = 0, j = 0, time, k},
  For[k = y; time = 0, k > 0, k -= 2; time++,
   If[OddQ[y],
    If[time == Floor[y / 2], A[[++i, ++j]] = x; Break[]],
    If[time == Ceiling[y / 2], Break[]]];
   i++; j++;
   Do[A[[i, j]] = x++; j++, {k - 1}];
   Do[A[[i, j]] = x++; i++, {k - 1}];
   Do[A[[i, j]] = x++; j--, {k - 1}];
   Do[A[[i, j]] = x++; i--, {k - 1}];
  ];
  A]
g[5] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 16 & 17 & 18 & 19 & 6 \\ 15 & 24 & 25 & 20 & 7 \\ 14 & 23 & 22 & 21 & 8 \\ 13 & 12 & 11 & 10 & 9 \end{pmatrix}$$

T_{20}

20. 编写程序包计算矩阵的 $\| A \|_1$(1 范数)和 $\| A \|_\infty$(∞ 范数)：

$$\| A \|_1 = \max_k \sum_{i=1}^{n} | a_{ik} |$$

$$\| A \|_\infty = \max_i \sum_{k=1}^{n} | a_{ik} |$$

In[ ]:=

```
<< "C:\\Users\\znz78\\MatrixNorm.wl"
Clear[A];
A = {{1, -3, 4, 0, 9}, {-4, 8, 9, -4, 8}, {-5, -2, -9, 0, 5}, {-4, 5, 8, -9, 0}};
{Norm1[A], NormInf[A]}

(*
BeginPackage["Global`"];
Norm1::usage="计算矩阵的1范数";
NormInf::usage="计算矩阵的∞范数";
Norm1[x_]:=Module[{n},n=Dimensions[x][[2]];
  Max[Table[Total[Abs[x[[All,j]]]],{j,1,n}]]];
NormInf[x_]:=Module[{m},m=Dimensions[x][[1]];
  Max[Table[Total[Abs[x[[i,All]]]],{i,1,m}]]];
EndPackage[];
*)
```

Out[ ]=

```
{30, 33}
```